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Functional Nanowires and Nanotubes

Editors Anna Fontcuberta i Morral, Kornelius Nielsch, Heiner Linke, Hyunjung Shin and Li Shi

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## Optical Properties

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### Composition and Size Effects on the Optical Properties of Isolated Silicon-Germanium Nanowires

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#### ABSTRACT

Silicon and Germanium nanowires (NWs) have shown a strong ability to enhance both the absorption and scattering of light. Tailoring the optical properties of Si or Ge NWs can be obtained by adjusting the nanowire diameter. Another parameter that can be used is the chemical composition of silicon-germanium ( $\text{Si}_{1-x}\text{Ge}_x$ -NWs) alloys. In this work, we perform a numerical study on the optical properties of single  $\text{Si}_{1-x}\text{Ge}_x$ -NWs based on the Lorenz-Mie theory. The effects of Ge composition, light polarization and angle of incidence on the nanowire optical properties are investigated.

#### INTRODUCTION

Semiconductor nanowires are promising novel materials for the next-generation of nano-electronics CMOS transistors, as well as for photovoltaic devices such as photodetectors and solar cells. Among all semiconducting nanostructures, silicon nanowires are nowadays the most studied because of the supremacy of silicon in CMOS technology or low cost solar cells [1-3]. Compared to their bulk counterparts, they show an elastic light scattering enhancement, Raman efficiency enhancement, or increased photoconductivity adjustable by tuning of nanowire diameter. These phenomena are due to morphology-dependent resonances so-called Mie resonances [4-10].

Some theoretical and experimental studies of the optical properties have been performed on pure silicon [4-7] or germanium nanowires [8-10]. The results showed a branched enhancement of absorption and/or scattering resulting from multiple wave interferences inside nanowires of diameters above a given wavelength threshold, and an apparently conventional behavior dominated by direct electronic transitions below this threshold.

For pure Si-NWs or Ge-NWs, the resonance phenomena, leading to electric-field enhancement, occur in a limited spectral region. We propose in this paper, a theoretical study on the optical properties of individual  $\text{Si}_{1-x}\text{Ge}_x$  NWs, which offer the chemical composition  $x$  as supplementary tunable parameter. Our calculations show that this parameter leads to high efficiencies over the whole wavelength range of sunlight from near ultraviolet to near infrared.

THEORY

We used the Lorenz-Mie theory [11] applied to the geometry of a nanowire which is described as an infinitely long cylinder of radius  $a$ , immersed in vacuum, as illustrated schematically in Fig. 1. The plane of incidence is defined by the incident wave vector and the nanowire axis, and  $\xi$  is the incidence angle.

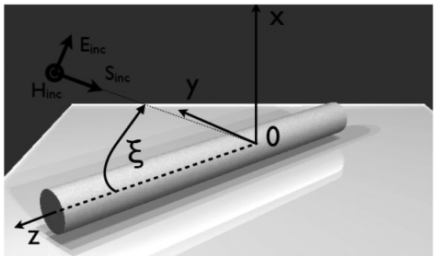


Figure 1- Schematic geometry of light scattering by an infinite cylinder

The calculation is carried out within the linear, homogenous, isotropic scattering medium assumption. In the framework of this theory, the electromagnetic field ( $\mathbf{E}$ ,  $\mathbf{H}$ ) are solutions of the vector wave equation:

$$\Delta \mathbf{E} + k^2 \mathbf{E} = \mathbf{0} \text{ and } \Delta \mathbf{H} + k^2 \mathbf{H} = \mathbf{0} \quad (1)$$

The electromagnetic fields are expanded on the basis of the vector cylindrical harmonics  $\mathbf{M}_n$  and  $\mathbf{N}_n$  defined by:

$$\mathbf{M}_n = \nabla \times (\hat{u}_z \psi_n) \text{ and } \mathbf{N}_n = \frac{\nabla \times \mathbf{M}_n}{k} \quad (2)$$

where  $\psi_n$ , the solutions of the scalar wave equation, are given by the relation:

$$\psi_n(r, \theta, z) = R_n(r) e^{in\theta} e^{-ik \cos \xi z} \quad (3)$$

in which  $R_n$  are solutions of the radial Bessel equation. For incident and internal fields  $R_n(r) = J_n(kr)$ , the Bessel functions of the first kind, while for scattered fields  $R_n(r) = H_n^{(1)}(kr)$ , the Hankel function of the first kind. From the continuity of the tangential field components at the nanowire-vacuum interface,  $r = a$ , the expansion coefficients of the scattered field are derived. For the two independent states of polarization of the incident light, transverse magnetic (TM) and transverse electric (TE), these coefficients are given by:

$$\begin{aligned} a_n &= \frac{C_n V_n - B_n D_n}{W_n V_n + i D_n^2} & \text{and } b_n &= \frac{W_n B_n + i C_n D_n}{W_n V_n + i D_n^2} & \text{For TM mode} \\ a_n &= -\frac{A_n V_n - i C_n D_n}{W_n V_n + i D_n^2} & \text{and } b_n &= -i \frac{C_n W_n + A_n D_n}{W_n V_n + i D_n^2} & \text{For TE mode} \end{aligned}$$

with

$$\begin{aligned} A_n &= i\Omega \sin \xi [\Omega \sin \xi J_n'(\eta) J_n(\Omega \sin \xi) - \eta J_n(\eta) J_n'(\Omega \sin \xi)], \\ B_n &= \Omega \sin \xi [m^2 \Omega \sin \xi J_n(\eta) J_n(\Omega \sin \xi) - \eta J_n(\eta) J_n(\Omega \sin \xi)], \\ C_n &= n \cos \xi \eta J_n(\eta) J_n(\Omega \sin \xi) \left( \frac{\Omega^2 \sin^2 \xi}{\eta^2} - 1 \right), \\ D_n &= n \cos \xi \eta J_n(\eta) H_n^{(1)}(\Omega \sin \xi) \left( \frac{\Omega^2 \sin^2 \xi}{\eta^2} - 1 \right), \\ V_n &= \Omega \sin \xi [m^2 \Omega \sin \xi J_n'(\eta) H_n^{(1)}(\Omega \sin \xi) - \eta J_n(\eta) H_n^{(1)'}(\Omega \sin \xi)], \\ W_n &= i\Omega \sin \xi [\eta J_n(\eta) H_n^{(1)'}(\Omega \sin \xi) - \Omega \sin \xi J_n'(\eta) H_n^{(1)}(\Omega \sin \xi)], \end{aligned}$$

where  $\Omega = \frac{2\pi a}{\lambda}$  is the size parameter,  $\eta = \Omega \sqrt{m^2 - \cos^2 \xi}$  and  $m$  is the NW complex refractive index. The ability of nanowires to absorb or scatter incident light can be characterized by the absorption and scattering efficiencies,  $Q_{\text{sca}}$  and  $Q_{\text{abs}}$ , expressed as follows:

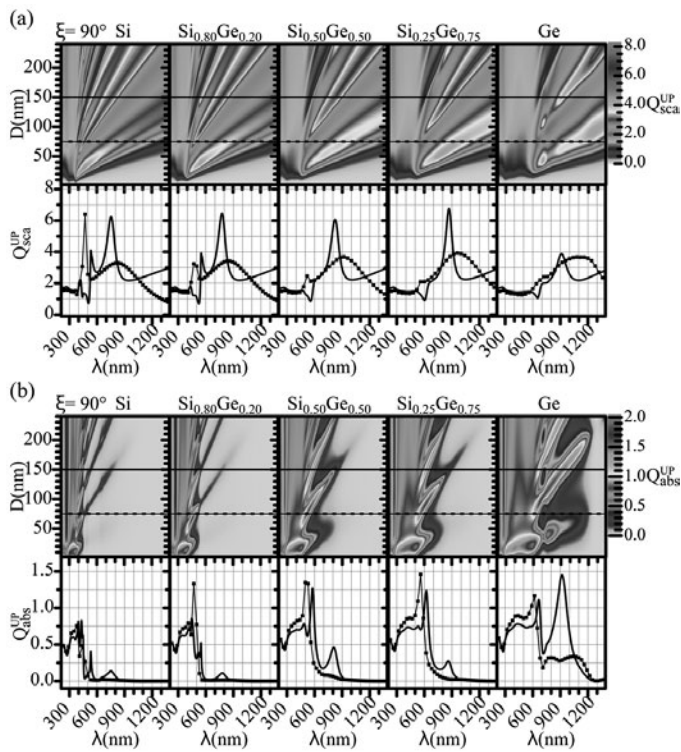
$$\begin{aligned} Q_{\text{sca}}^{\text{TM}} &= \frac{2}{\Omega} \left[ |b_0|^2 + 2 \sum_{n=1}^{\infty} |b_n|^2 + |a_n|^2 \right] \\ Q_{\text{sca}}^{\text{TE}} &= \frac{2}{\Omega} \left[ |a_0|^2 + 2 \sum_{n=1}^{\infty} |a_n|^2 + |b_n|^2 \right] \\ Q_{\text{abs}}^{\text{TM}} &= \frac{2}{\Omega} \operatorname{Re} \left[ b_0 + 2 \sum_{n=1}^{\infty} b_n \right] - Q_{\text{sca}}^{\text{TM}} \\ Q_{\text{abs}}^{\text{TE}} &= \frac{2}{\Omega} \operatorname{Re} \left[ a_0 + 2 \sum_{n=1}^{\infty} a_n \right] - Q_{\text{sca}}^{\text{TE}} \end{aligned}$$

The unpolarized (UP) light configuration corresponds to the average of TE and TM modes, which gives for the efficiencies:  $Q_{\text{sca,abs}}^{\text{UP}} = \frac{1}{2} (Q_{\text{sca,abs}}^{\text{TM}} + Q_{\text{sca,abs}}^{\text{TE}})$ . In the following calculations, the  $\text{Si}_{1-x}\text{Ge}_x$  NW complex refractive index was assumed to be that of its relaxed bulk counterpart [12,13] irrespective of the NW diameter value.

## RESULTS

The computed efficiencies,  $Q_{\text{abs}}$  and  $Q_{\text{sca}}$ , in normal incidence ( $\xi=90^\circ$ ) for unpolarized incident light are reported in Fig. 2 for pure silicon, pure germanium and three  $\text{Si}_{1-x}\text{Ge}_x$  alloy NWs with diameters ranging from 2 to 250 nm. The results obtained for Si and Ge NWs are in good agreement with those published by Brönstrup *et al.* and Cao *et al.* [5-10].

Branched resonances, appearing in the map, correspond to highly confined modes, also identified as leaky mode resonances in optical fibers and other dielectric resonators [8]. The branched resonance distribution shifts towards the red part of the spectrum with increasing alloy composition  $x$ . Indeed, this corresponds to the redshift of the maximum of the refractive index (real part of  $m$ ). The high refractive index value allows the trapping of light in  $\text{Si}_{1-x}\text{Ge}_x$  nanowires, since the wavelength of light in vacuum, ranging from 300 to 1200 nm, diminishes

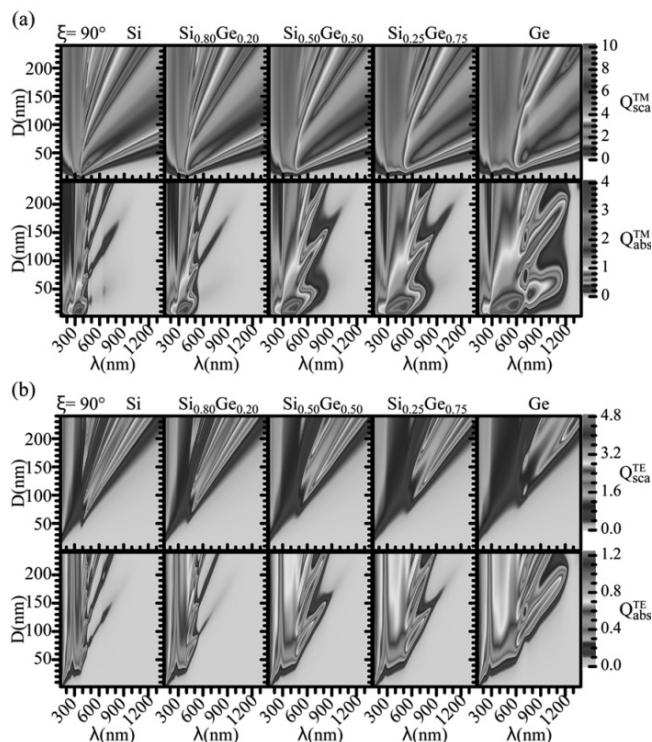


**Figure 2-** Unpolarized scattering (a) and absorption efficiencies (b) in Si<sub>1-x</sub>Ge<sub>x</sub> alloy nanowires as a function of the wavelength and the nanowire diameter: (top) Two-dimensional map and (bottom) efficiencies spectra for nanowire diameters 150 nm (solid line) and 75 nm (thin solid line with filled squares).

inside the NW and becomes of the same order of magnitude as the NW diameter. A wavelength threshold in the scattering efficiency, below which the branches disappear, can be noticed. For the absorption efficiency, a NW diameter cutoff is imposed in addition, below which  $Q_{abs}$  can reach high values. These two characteristics can be correlated with the penetration depth of the light in the nanowires as function of the wavelength. As expected, they show a wavelength redshift with increasing Ge composition in Si<sub>1-x</sub>Ge<sub>x</sub> alloy nanowires. Thinner NWs with diameters equivalent to the light penetration depth exhibit higher  $Q_{abs}$  due to the possible cumulated effects of both direct electronic transitions and optical resonances. The maximum of  $Q_{abs}$  is highly localized in Si, corresponding to the  $E'_0$  and  $E_1$  transitions located at 364 and 295 nm, respectively. The  $Q_{abs}$  maximum region is spread over a broader range of wavelengths with

increasing concentration of Ge, according to the degeneracy lifting of  $E'_0$ ,  $E_1$ ,  $E_1 + \Delta_1$ ,  $E_0 + \Delta_0$  [14].

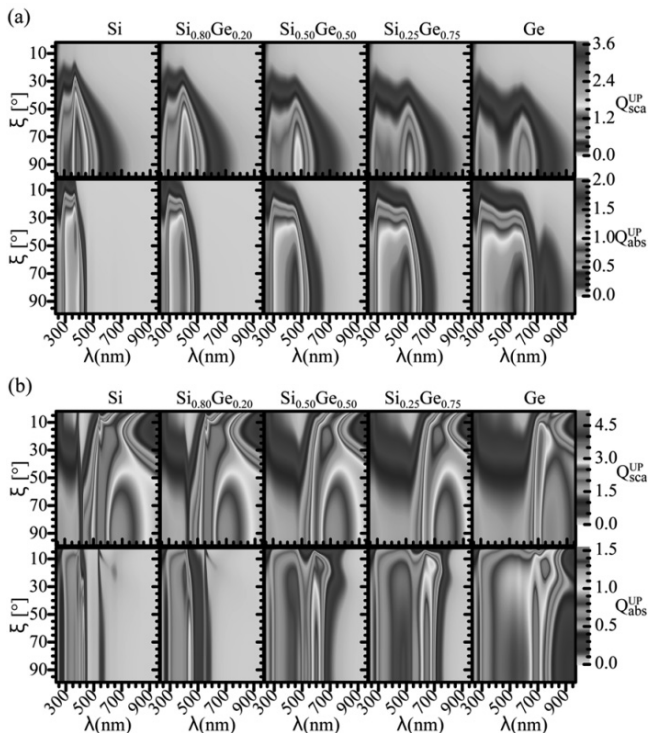
Thus, it is obvious that the specific optical properties of  $Si_{1-x}Ge_x$  NWs are of great interest for the optimization of both light scattering and absorption in solar cells or photodetectors. They are also very important for enhanced Raman scattering from and by nanowires. The benefit of sharp Mie resonances in Raman scattering was illustrated recently by Poborchii et al. [15], who investigated Si NWs of diameters less than 20 nm excited by a resonant laser line at 363.8 nm. Cao *et al.* showed similar Raman efficiency enhancement using non-resonant visible excitations, but for thicker nanowires [5]. Therefore, using  $Si_{1-x}Ge_x$  alloys should allow light absorption and scattering engineering, reaching high  $Q_{abs}$  and  $Q_{sca}$  values over a wide wavelength range but within a limited nanowire diameter range.



**Figure 3-** Absorption and scattering efficiencies as function of NW diameter and incident wavelength for different compositions  $x$  of Ge under normal incidence illumination and for polarized incident light: (a) TM mode and (b) TE mode.

The effect of light polarization on the scattering and absorption efficiencies of  $Si_{1-x}Ge_x$  NWs is illustrated in Fig. 3 for normal incidence illumination. As can be noted from this figure,

the maxima of both  $Q_{\text{abs}}$  and  $Q_{\text{sca}}$  under TM polarization are noticeably greater than those achieved under TE polarization [Fig. 2(a) vs Fig. 2(b)]. This can be attributed to the higher number of dipoles excited when the electric field is aligned with the NW axis (TM mode). In agreement with Fig. 2(a) where  $Q_{\text{abs}}$  reaches 400%, the enhanced Raman efficiency was clearly evidenced in the TM configuration for Si NWs of diameters less than 20 nm [15].



**Figure 4-** Unpolarized absorption and scattering efficiencies for two nanowire diameters and different alloy compositions  $x$  as function of the incidence angle  $\xi$  and the incident wavelength: (a)  $D_1=20$  nm (c)  $D_2= 100$  nm.

In Fig. 4, we present  $Q_{\text{abs}}$  and  $Q_{\text{sca}}$  as function of the angle of incidence  $\xi$  for different concentrations of Ge and for two diameters ( $D_1= 20$  nm and  $D_2= 100$  nm). For the smallest diameter  $D_1$ , there is sharp transition for angles smaller than  $40^\circ$  in the plot of  $Q_{\text{sca}}$  or  $30^\circ$  in the plot of  $Q_{\text{abs}}$ . These efficiencies do not change a lot while decreasing incident angles from normal incidence down to the above mentioned limits, and are constant and close to zero beyond these limits. The behavior is different in the case of the larger diameter  $D_2$ , where the scattering efficiency can be still high at low incidence angle, but for slightly different wavelengths. For



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instance in the case of  $D_2$ , the  $Q_{\text{sca}}$  maximum at small incidence angles (about 450-500% at  $20^\circ$ ) is red shifted by about 50 nm compared to the same  $Q_{\text{sca}}$  maximum at larger incidence angles (about 450-500% from  $35^\circ$  to  $90^\circ$ ), over the whole range of  $\xi$ . Thus, the scattering spectrum (or the color) of a NW excited by a wide range of wavevectors including low incidence angles, should not be different from the color of the same nanowire under normal incidence if its diameter is close to  $D_1 = 20$  nm. However, the observed color of larger nanowires would be different depending on the incidence angle distribution.

## CONCLUSION

The scattering and absorption efficiencies of single  $\text{Si}_{1-x}\text{Ge}_x$  nanowires have been calculated and their dependence on the NW diameter, alloy composition, light polarization and angle of incidence has been investigated using the Lorentz-Mie theory. We showed that scattering and absorption of light can be tuned via the alloy composition in addition to the nanowire diameter. High values of either  $Q_{\text{sca}}$  or  $Q_{\text{abs}}$  can be obtained over an extended wavelength range from near-UV to near-IR covering most of the sunlight spectrum. Highest efficiencies can be achieved with transverse magnetic polarization under normal incidence illumination.

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