CAMBRIDGE

Cambridge University Press
978-1-316-64422-5 — A Level Mathematics for AQA Student Book 1 (AS/Year 1)
Stephen Ward , Paul Fannon
Excerpt
More Information

1 Proof and mathematical communication

In this chapter you will learn how to:

- use appropriate terms to describe mathematical objects, such as identity and equation
- use a counter example to disprove a mathematical idea
- apply some techniques for proving a mathematical idea – deduction and exhaustion.

Before you start…

GCSE	You should know the definition of the square root function.	1 Write down the value of $\sqrt{9}$.
GCSE	You should be able to manipulate algebraic expressions.	2 Factorise $4x^2-1$.
GCSE	You should know basic angle facts.	3 a What is the sum of the angles in a triangle? b What is the sum of the exterior angles of any polygon?
GCSE	You should know the definition of rational and irrational numbers.	4 Which of these numbers are irrational? $\pi, 0.\dot{3}, 0.5, \sqrt{2}$
GCSE	You should be able to work with function notation.	5 If $f(x)=2x^2-3$ find $f(3)$.

Beyond any doubt

Mathematicians are particularly valued for their ability to communicate their ideas precisely and to support their assertions with formal arguments, called **proofs**. In this chapter you will look at the language used by mathematicians and some ways in which they prove their ideas.

Did you know?

The first known use of the equals (=) sign occurs in Robert Recorde's 1557 book, *The Whetstone of Witte*. He explains that he used two parallel lines, 'because no two things can be more equal'.

Section 1: Mathematical structures and arguments

Mathematical ideas may be represented in many different ways, such as tables, diagrams, graphs or words. One of the most fundamental representations is an **equation**. For example:

$$x^2-1=8$$

An equation is only true for some values of x, in this case, $x=\pm3$.

© in this web service Cambridge University Press www.cambridge.org

Cambridge University Press
978-1-316-64422-5 — A Level Mathematics for AQA Student Book 1 (AS/Year 1)
Stephen Ward , Paul Fannon
Excerpt
More Information

Another similar mathematical structure is called an **identity**.

There are some rules that only apply to identities. For example, if two polynomials are identically equal the coefficients of corresponding variables must be the same.

WORKED EXAMPLE 1.1

$2x^2+12x-3 \equiv a(x+p)^2+q$

Find the values of a, p and q.

$$\begin{aligned}2x^2+12x-3 &\equiv a(x+p)^2+q\\ &= a(x^2+2xp+p^2)+q\\ &= ax^2+2apx+ap^2+q\end{aligned}$$

Multiply out the brackets to allow coefficients to be compared.

Coefficient of x^2: $2=a$

Coefficient of x: $12=2ap$

Constant term: $-3=ap^2+q$

Compare coefficients.

$$\begin{aligned}4p&=12\\ p&=3\end{aligned}$$

Substitute $a=2$ into the second equation.

$$\begin{aligned}2\times3^2+q&=-3\\ q&=-21\end{aligned}$$

Substitute $a=2$, $p=3$ into the third equation.

You can manipulate both equations and inequalities by doing the same thing to both sides. Generally, you will do this by writing lines of working one under another. In more formal work, you can emphasise the logic of the argument by using special symbols.

Key point 1.2

- The symbol $\Rightarrow$ means that a subsequent statement follows from the previous statement.
  $P\Rightarrow Q$ means 'P implies Q' or 'if P is true then Q is true' or 'P is sufficient for Q'.
- The symbol $\Leftarrow$ means that the previous statement follows from the subsequent statement.
  $P\Leftarrow Q$ means 'P is implied by Q' or 'if Q is true then P is true' or 'P is necessary for Q'.
- The symbol $\Leftrightarrow$ means that a subsequent statement is equivalent to the previous one.
  $P\Leftrightarrow Q$ means 'P is equivalent to Q' or 'Q is true if and only if P is true'.
  This can also be written as 'Q iff P'.

Key point 1.1

An identity is a relation that is true *for all values* of the unknown. It is indicated by the $\equiv$ symbol. For example:

$$x^2-1\equiv(x-1)(x+1)$$

Two statements connected by an identity symbol are **congruent** expressions.

Tip

Whenever you are simplifying an expression, technically you should use an identity symbol. However, it is common in mathematics to use an equals sign instead.

Tip

A **coefficient** of a variable is the constant in front of that variable. For example, in the quadratic $2x^2-3x$, 2 is the coefficient of x^2 and -3 is the coefficient of x.

Gateway to A Level

See Gateway to A Level Section A for a reminder of expanding brackets.

Fast forward

In Chaper 3 you will see that you often have to write quadratics in the form $a(x+p)^2+q$. The method of comparing coefficients provides an alternative to the procedure suggested in Chapter 3. You can choose to use whichever one you prefer.

© in this web service Cambridge University Press www.cambridge.org

Cambridge University Press
978-1-316-64422-5 — A Level Mathematics for AQA Student Book 1 (AS/Year 1)
Stephen Ward , Paul Fannon
Excerpt
More Information

WORKED EXAMPLE 1.2

Insert a $\Rightarrow$, $\Leftarrow$ or a $\Leftrightarrow$ symbol in each position marked by $*$.

a $2x+1=9$
$* \, 2x=8$
$* \, x=4$

b $x=4$
$* \, x^2=16$

a $2x+1=9$ — These statements are equivalent: the logic flows both ways.
$\Leftrightarrow 2x=8$
$\Leftrightarrow x=4$ — Again $2x=8$ and $x=4$ are equivalent.

b $x=4$ — $x=4$ implies that $x^2=16$ but the reverse is not true as $x^2=16$ implies $x=\pm4$ (not just $x=4$).
$\Rightarrow x^2=16$

When you are solving equations, each line of your working needs to be equivalent to the last if the 'solutions' you get at the end are to be valid.

WORKED EXAMPLE 1.3

A student is attempting to solve the equation $\sqrt{x+6}=x$.

a Find the error with this working.

$$\sqrt{x+6}=x$$
$$x+6=x^2$$
$$x^2-x-6=0$$
$$(x-3)(x+2)=0$$
$$x=3 \text{ or } x=-2$$

b Solve the equation correctly.

a $\sqrt{x+6}=x$ — Look at each line in turn to see whether a $\Leftrightarrow$ symbol is valid.

$\Rightarrow x+6=x^2$ — The first line implies the second, but the second does not imply the first, so they are not equivalent.
They are not equivalent since:
$x+6=x^2$
$\Rightarrow \pm\sqrt{x+6}=x$

This leads to one incorrect solution, coming from $-\sqrt{x+6}=x$ — All subsequent lines are equivalent, so one of the solutions is correct.

b Therefore the negative solution is false. — Since the LHS of the original equation is positive, the RHS must be positive too.
The correct solution is $x=3$.

Gateway to A Level

See Gateway to A Level Sections B and C for a reminder of solving quadratic equations by factorising.

Tip

Squaring an equation is a common way of introducing incorrect solutions, since it prevents lines of working being equivalent.

Tip

LHS and RHS are standard abbreviations for 'left-hand side' and 'right-hand side'.

© in this web service Cambridge University Press www.cambridge.org

CAMBRIDGE

Cambridge University Press
978-1-316-64422-5 — A Level Mathematics for AQA Student Book 1 (AS/Year 1)
Stephen Ward , Paul Fannon
Excerpt
More Information

In practice it is often easier not to worry about whether every line is equivalent, but to be aware that any 'solutions' need to be checked by substituting them back into the original equation. Any that are not correct can then just be deleted.

Dividing by zero can remove solutions, in the same way that squaring can introduce them.

Fast forward

The problem of false solutions will also arise when you solve equations involving logarithms, in Chapter 7.

WORKED EXAMPLE 1.4

Insert an appropriate $\Rightarrow$, $\Leftarrow$ or a $\Leftrightarrow$ symbol in the position marked by $*$. Hence explain why the solution is incomplete.

$x^2 = 6x$

$* \, x = 6$ Divide by x.

The missing symbol is a $\Leftarrow$ — If $x = 6$ then $x^2 = 6x$, but the reverse is not always true.

So $x = 6$ is only one solution – there is also the possibility that $x = 0$.

EXERCISE 1A

1. Where appropriate insert a $\Rightarrow$, $\Leftarrow$ or a $\Leftrightarrow$ symbol in each space.

   a **i** Shape P is a rectangle __ shape P is a square.
   ii Shape Q is a quadrilateral __ shape Q is a rhombus.

   b **i** n is even __ n is a whole number.
   ii n is a prime number __ n is a whole number.

   c **i** A triangle has two equal sides __ a triangle has two equal angles.
   ii Two circles have the same area __ two circles have the same radius.

   d **i** $x^2 - 2x - 3 = 0$ __ $x = 3$ **ii** $x^2 - 2x + 1 = 0$ __ $x = 1$

   e **i** Sam can do 10 press-ups __ Sam can do 100 press-ups.
   ii Niamh is over 21 __ Niamh is over 18.

   f **i** Neither A nor B is true __ A is false and B is false.
   ii A and B are not both true __ A and B are both not true.

   g **i** Chris is a boy __ Chris is a footballer.
   ii Shape X is a right-angled triangle __ shape X is an isosceles triangle.

Did you know?

It is assumed here that A is either true or false. The study of this type of logic is called Boolean Algebra.

2. $x^3 - 4x^2 - 3x + 18 = (x + a)(x - b)^2$ for all x.

   Find the values of a and b.

3. $x^4 + 8x^3 + 2x + 16 = (x^3 + a)(x + b)$ for all x.

   Find the values of a and b.

4. What is the flaw in this working?

   Question: For $x = 4$, find the value of $2x + 2$.

   Working: $2 \times 4 = 8 + 2 = 10$

© in this web service Cambridge University Press www.cambridge.org

CAMBRIDGE

Cambridge University Press
978-1-316-64422-5 — A Level Mathematics for AQA Student Book 1 (AS/Year 1)
Stephen Ward , Paul Fannon
Excerpt
More Information

5 Where is the flaw in this argument?

Suppose $1 = 3$

$-1 = 1$ Subtract 2 from both sides.

$1 = 1$ Square both sides.

Therefore the first line is true.

6 **a** Consider the equation:

$\sqrt{x^2+9} = 3x-7$

Add appropriate symbols ($\Leftrightarrow, \Leftarrow$ or $\Rightarrow$) in each position marked by $*$ in the solution.

$* \; x^2 + 9 = 9x^2 - 42x + 49$ Square both sides.

$* \; 0 = 8x^2 - 42x + 40$ Subtract $x^2 + 9$ from both sides.

$* \; 0 = 4x^2 - 21x + 20$ Divide both sides by 2.

$* \; 0 = (4x-5)(x-4)$ Factorise.

$* \; x = \frac{5}{4}$ or $x = 4$

b Hence explain the flaw in the solution given.

7 **a** Insert the appropriate symbol ($\Rightarrow$, $\Leftarrow$ or $\Leftrightarrow$) in each position marked by $*$.

$\frac{1}{x^2} = \frac{2}{x}$

$* \; x = 2x^2$

$* \; 0 = 2x^2 - x$

$* \; 0 = x(2x-1)$

$* \; x = 0$ or $x = \frac{1}{2}$

b Hence explain the error in the working.

8 **a** Insert the appropriate symbol ($\Rightarrow$, $\Leftarrow$ or $\Leftrightarrow$) in each position marked by $*$.

$x^2 + 3x = 4x + 12$

$* \; x(x+3) = 4(x+3)$

$* \; x = 4$

b Hence explain the error in the working.

9 Where is the flaw in the following argument?
Suppose two numbers a and b are equal.

$a = b$

$a^2 = ab$ Multiply by a.

$a^2 - b^2 = ab - b^2$ Subtract b^2.

$(a-b)(a+b) = b(a-b)$ Factorise.

$a + b = b$ Cancel $(a-b)$.

$2b = b$ Use the fact that $a = b$.

$2 = 1$ Divide by b.

10 Do you agree with this statement?

Either A or B is true $\Leftrightarrow$ A and B are not both true.

© in this web service Cambridge University Press www.cambridge.org

Cambridge University Press
978-1-316-64422-5 — A Level Mathematics for AQA Student Book 1 (AS/Year 1)
Stephen Ward , Paul Fannon
Excerpt
More Information

Section 2: Inequality notation

From previous studies, you know that solving a linear inequality is just like solving an equation, as long as you don't multiply or divide by a negative number. The answer is written as an inequality.

For example:

$$2x - 5 \geqslant 9$$
$$\Leftrightarrow 2x \geqslant 14$$
$$\Leftrightarrow x \geqslant 7$$

This solution can be written in **set notation** as: $\{x : x \geqslant 7\}$.

This is read as 'x such that x is bigger than or equal to 7'.

It can also be written in **interval notation** as: $[7, \infty)$.

This means that the solution lies in the interval from 7 (included) to infinity (not included).

Key point 1.3

- $x \in (a, b)$ means $a < x < b$
- $x \in [a, b]$ means $a \leqslant x \leqslant b$
- $x \in [a, b)$ means $a \leqslant x < b$
- $x \in (a, b]$ means $a < x \leqslant b$

Tip

The $\in$ symbol in set notation means 'is in the set...' or 'belongs to the set...'.

Two different intervals can be combined, using set theory notation.

Key point 1.4

- $A \cup B$ is the **union** of A and B. It means the solution is in A or B or both.
- $A \cap B$ is the **intersection** of A and B. It means the solution is in both A and B.

WORKED EXAMPLE 1.5

Write each inequality in set notation.

a $1 \leqslant x < 7$
b $x > 1$ or $x < -2$

a $\{x : x \geqslant 1\} \cap \{x : x < 7\}$ — x is **both** greater than or equal to 1, **and** less than 7 so it is in the intersection.

b $\{x : x > 1\} \cup \{x : x < -2\}$ — x is greater than 1 **or** less than -2 so it is in the union.

If there is no solution to the inequality then x is in the empty set, written as $x \in \varnothing$.

© in this web service Cambridge University Press www.cambridge.org

CAMBRIDGE
Cambridge University Press
978-1-316-64422-5 — A Level Mathematics for AQA Student Book 1 (AS/Year 1)
Stephen Ward , Paul Fannon
Excerpt
More Information

EXERCISE 1B

1. Write each inequality in set notation and interval notation.

   a i $x > 7$ ii $x < 6$ b i $x \leqslant 10$ ii $x \geqslant 5$

   c i $0 < x \leqslant 1$ ii $5 < x < 7$ d i $x > 5$ or $x \leqslant 0$ ii $x \geqslant 10$ or $x < 2$

2. Write each interval as an inequality in x.

   a i $[1, 4)$ ii $(2, 8]$

   b i $[1, 3]$ ii $(2, 4)$

   c i $(-\infty, 5)$ ii $[12, \infty)$

   d i $\{x : 0 < x < 10\} \cap \{x : x \geqslant 8\}$ ii $\{x : 1 < x < 4\} \cap \{x : x \geqslant 3\}$

Section 3: Disproof by counter example

It is not usually possible to prove that something is always true by looking at examples. However, it is possible to use examples to prove that something is not always true. This is called a **counter example**.

WORKED EXAMPLE 1.6

Disprove by counter example that $(x+1)^2 \equiv x^2 + 1$ for all x.

Take $x = 2$.

When searching for a counter example, try different types of number. Here, any non-zero number will work.

LHS: $(2+1)^2 = 9$

RHS: $2^2 + 1 = 5$

So $x = 2$ is a counter example.

EXERCISE 1C

1. Disprove the statement $\sqrt{x^2 + 9} \equiv x + 3$.
2. Use a counter example to prove that $\sin 2x \not\equiv 2 \sin x$.
3. Use a counter example to prove that $\sqrt{x^2}$ is not always x.
4. Prove that the product of two prime numbers is not always odd.
5. Prove that the number of factors of a number is not always even.
6. Prove that the sum of two irrational numbers is not always irrational.
7. Use a counter example to disprove this statement.

   $x < 3 \Rightarrow x^2 < 9$
8. Prove that $n^2 + n + 41$ does not take prime values for all positive integers.
9. Are any two straight lines that never meet necessarily parallel?

Gateway to A Level

See Gateway to A Level Section D for a reminder of rational and irrational numbers.

© in this web service Cambridge University Press www.cambridge.org

Cambridge University Press
978-1-316-64422-5 — A Level Mathematics for AQA Student Book 1 (AS/Year 1)
Stephen Ward , Paul Fannon
Excerpt
More Information

Section 4: Proof by deduction

To prove that a result is true, you need to start with what is given in the question and form a series of logical steps to reach the required conclusion.

Algebra is a useful tool which allows you to express ideas in general terms. You will often need to use algebraic expressions for even and odd numbers.

Key point 1.5

It is common to express:

- an even number as $2n$, for any integer n
- an odd number as $2n+1$ (or $2n-1$), for any integer n.

Tip

The word **integer** just means 'whole number'.

Tip

Remember – one example (or even several examples) does not make a proof.

Elevate

See Support Sheet 1 for a further example on algebraic proof and more practice questions.

WORKED EXAMPLE 1.7

Prove that the product of an even and an odd number is always even.

Let the even number be $2n$, for some integer n. — Define a general even number...

Let the odd number be $2m+1$, for some integer m. — ...and a general odd number.

Do not use $2n+1$ as this would be the next integer up from $2n$, which is too specific.

$$2n(2m+1) = 2(2mn+n)$$
$$= 2k \quad \text{for some integer } k$$

Aim to write the product in the form $2k$ to show that it is even.

$\therefore$ the product of an even and an odd number is even. — Make a conclusion.

WORKED EXAMPLE 1.8

Prove that the difference between the squares of consecutive odd numbers is always a multiple of eight.

Let the smaller odd number be $2n-1$.

Let the larger odd number be $2n+1$.

Define two consecutive odd numbers. This time you **do** want n in both.

$$(2n+1)^2 - (2n-1)^2 = (4n^2+4n+1) - (4n^2-4n+1)$$
$$= 4n+4n$$
$$= 8n$$

Square each, and subtract the smaller from the larger.

$\therefore$ the difference between the squares of consecutive odd numbers is always a multiple of 8. — Make a conclusion.

© in this web service Cambridge University Press

CAMBRIDGE

Cambridge University Press
978-1-316-64422-5 — A Level Mathematics for AQA Student Book 1 (AS/Year 1)
Stephen Ward , Paul Fannon
Excerpt
More Information

EXERCISE 1D

1. Prove that if n is odd then n^2 is also odd.

2. Prove that the sum of an even number and an odd number is always odd.

3. Prove that the sum of any three consecutive integers is always a multiple of 3.

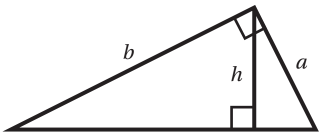
4. Prove that:

   a the sum of two consecutive multiples of 5 is always odd

   b the product of two consecutive multiples of 5 is always even.

5. Prove that the height, h, in the diagram is given by:

$$h = \frac{ab}{\sqrt{a^2 + b^2}}$$



6. Prove that the sum of the interior angles of a hexagon is 720°.

7. Prove that if a number leaves a remainder of 2 when it is divided by 3 then its square leaves a remainder of 1 when divided by 3.

Gateway to A Level

See Gateway to A Level Section E for a reminder of angle facts.

8. **a** Expand $(x+2)^2$.

   b Prove that $y = x^2 + 4x + 10 \Rightarrow y > 0$.

9. Prove that the exterior angle in a triangle is the sum of the two opposite angles.

10. Prove that $n^2 + 3n + 2$ is never prime if n is a positive integer.

11. **a** Let n be a four-digit whole number with digits '$abcd$'. Explain why $n = 1000a + 100b + 10c + d$.

    b Prove that n is divisible by 9 if and only if $a + b + c + d$ is a multiple of 9.

    c Prove that n is divisible by 11 if and only if $a - b + c - d$ is divisible by 11.

12. By considering $\left(\sqrt{2}^{\sqrt{2}}\right)^{\sqrt{2}}$ prove that an irrational number raised to an irrational power can be rational.

Section 5: Proof by exhaustion

You should be aware that simply considering some examples does not constitute a mathematical proof. However, in some situations it is possible to check **all** possibilities, which can lead to a valid proof. This is called **proof by exhaustion**.

© in this web service Cambridge University Press www.cambridge.org

CAMBRIDGE

Cambridge University Press
978-1-316-64422-5 — A Level Mathematics for AQA Student Book 1 (AS/Year 1)
Stephen Ward , Paul Fannon
Excerpt
More Information

Did you know?

In traditional mathematics proof by exhaustion was not very common as there are usually too many possibilities to check. The use of computers has made it a much more viable method, but some mathematicians question whether just checking large numbers of possibilities by computer is a valid method of proof. One famous result that has been proved in this way is the **four colour theorem**.

WORKED EXAMPLE 1.9

Prove that 13 is a prime number.

13 is not divisible by 2, 3, 5, 7 or 11.

Since you can write any number as a product of its prime factors, you only need to check whether it has prime factors.

Therefore it must be a prime number.

Tip

You can be more efficient by only checking primes up to the square root of 13, since any factor above this would have to be paired with a factor that is less than the square root.

WORKED EXAMPLE 1.10

A whole number is squared and divided by 3. Prove that the remainder can only be 0 or 1.

You cannot check all whole numbers, but you can split them into three groups when considering division by 3: those that give no remainder, those that give a remainder 1 and those that give a remainder 2. You can then check what squaring does to numbers from each group.

Let n be a whole number.

Now use algebra to write each type of number.

Then n must be:

- a multiple of 3 ($n = 3k$), or
- one more than a multiple of 3 ($n = 3k + 1$), or
- two more than a multiple of 3 ($n = 3k + 2$).

If $n = 3k$ then:

Now check what happens when you square each type of number.

$$\begin{aligned} n^2 &= (3k)^2 \\ &= 9k^2 \\ &= 3(3k^2) \\ &= 3m \end{aligned}$$

which is a multiple of 3.

Continues on next page

© in this web service Cambridge University Press www.cambridge.org

CAMBRIDGE

Cambridge University Press
978-1-316-64422-5 — A Level Mathematics for AQA Student Book 1 (AS/Year 1)
Stephen Ward , Paul Fannon
Excerpt
More Information

If $n = 3k+1$ then:

$$
\begin{aligned}
n^2 &= (3k+1)^2 \\
&= 9k^2+6k+1 \\
&= 3(3k^2+2k)+1 \\
&= 3m+1
\end{aligned}
$$

which is one more than a multiple of 3.

If $n = 3k+2$ then:

$$
\begin{aligned}
n^2 &= (3k+2)^2 \\
&= 9k^2+12k+4 \\
&= 3(3k^2+4k+1)+1 \\
&= 3m+1
\end{aligned}
$$

which is one more than a multiple of 3.

So either there is no remainder or the remainder is 1.

Having checked each possible whole number, you can write the conclusion.

EXERCISE 1E

1. Prove that 11 is a prime number.
2. Prove that 83 is a prime number.
3. Prove that all regular polygons with fewer than seven sides have angles with a whole number of degrees.
4. Prove that no square number less than 100 ends in a 7.
5. Let $f(x)$ be the function that gives the number of factors of x. For example $f(10) = 4$ because it has factors 1, 2, 5 and 10. Prove that for any single-digit positive number $f(n) \leq n$.

Gateway to A Level

See Gateway to A Level Section F for a reminder of function notation.

6. Prove that n^2+2 is not divisible by 4 for the integers 1 to 5.
7. Prove that n^2+n is always even if $n \in \mathbb{Z}$.
8. Prove that when the square of a whole number is divided by 5, the remainder is always 0, 1 or 4.
9. Prove that $2x^3+3x^2+x$ is always divisible by 6 if x is an integer.

© in this web service Cambridge University Press www.cambridge.org

CAMBRIDGE

Cambridge University Press
978-1-316-64422-5 — A Level Mathematics for AQA Student Book 1 (AS/Year 1)
Stephen Ward , Paul Fannon
Excerpt
More Information

Checklist of learning and understanding

- Mathematical ideas may be expressed through descriptions such as diagrams, equations and identities.
- A mathematical argument can be described by a series of equations or identities put together in a logical order.
  These can be connected using implication symbols: $\Rightarrow$, $\Leftarrow$ or $\Leftrightarrow$.
  - The symbol $\Rightarrow$ means that a subsequent statement follows from the previous one.
  - The symbol $\Leftarrow$ means that the previous statement follows from the subsequent one.
  - The symbol $\Leftrightarrow$ means that a subsequent statement is equivalent to the previous one.
- An identity is a relation that is true **for all values** of the unknown. It is represented by the $\equiv$ symbol.
- Inequalities may be represented by set notation or interval notation.
- One counter example is sufficient to prove that a statement is not always true.
- An algebraic proof is often required to show that a statement is always true.
- Proof by exhaustion involves checking all possibilities; this can only be done if there is a small number of options, or the options can be split up into different cases.

© in this web service Cambridge University Press www.cambridge.org

Cambridge University Press
978-1-316-64422-5 — A Level Mathematics for AQA Student Book 1 (AS/Year 1)
Stephen Ward , Paul Fannon
Excerpt
More Information

Mixed practice 1

1. Which of these options provides a counter example to the statement: 'n^2+5 is even for every prime number n'?

   A $n=5$ **B** $n=4$ **C** $n=3$ **D** $n=2$

2. Prove that the product of any two odd numbers is always odd.

3. Prove that if n is even then n^2 is divisible by 4.

4. Prove that if $\frac{a}{b} \equiv \frac{c}{d}$ it does not follow that $a \equiv c$ and $b \equiv d$.

5. Prove this statement or disprove it with a counter example.

   'The sum of two numbers is always greater than their difference.'

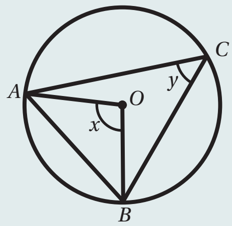
6. Prove that the product of two rational numbers is always rational.

7. Prove that the sum of the interior angles in an n-sided shape is $(180n - 360)°$.

8. Given that $x^3 + y^3 \equiv (x+y)(ax^2 + bxy + cy^2)$ find the values of a, b and c.

9. Prove this statement: n is odd $\Rightarrow n^2 + 4n + 3$ is a multiple of 4.

10. Prove that the angle, x, from a chord to the centre of a circle is twice the angle, y, to a point on the circumference in the major sector.



11. Prove that all cube numbers are either multiples of 9 or within one of a multiple of 9.

12. Prove each statement, or disprove it with a counter example.

    a ab is an integer $\Leftrightarrow a$ is an integer and b is an integer.

    b a is irrational and b is irrational $\Leftrightarrow ab$ is irrational.

13. Prove that the product of any three consecutive positive integers is a multiple of 6.

14. Prove that the difference between the squares of any two odd numbers is a multiple of 8.

15. **a** Prove that $n^2 - 79n + 1601$ is not always prime when n is a positive whole number.

    b Prove that $n^2 - 1$ is never prime when n is a whole number greater than 2.

16. $x = a^2 - b^2$ where a and b are both whole numbers. Prove that x is either odd or a multiple of 4.

Elevate

See Extension Sheet 1 for questions on another principle used in proof.

© in this web service Cambridge University Press www.cambridge.org