Chapter 1
Draw a diagram

You might have heard the saying 'a picture is worth a thousand words'. This means that a picture can show a lot of information without using language. Different types of graphs, Venn diagrams, maps, scale drawings, tree diagrams and two way tables are all mathematical 'pictures' that can show mathematical information in a clear way.

Drawing a picture is a very useful problem-solving strategy. You can use pictures to help you 'see' a problem and to work out what you need to do to solve it.

When you are given a word problem, you can 'translate' it into a more visual form (a diagram, graph, rough sketch or table) to help you see the problem more clearly. You can also use your picture to organise the information you are given and work out what you need to do to solve the problem.

For some problems a diagram will be provided. If so, remember that you can write on the diagram and add information to it to help you solve the problem.

When no diagram is provided you can draw your own.

Decide what type of picture will be most useful. Then draw a clear diagram that is large enough to work on. You can use different colours and highlighters to make it easier to see what you are doing. Rough sketches are acceptable but your sketch should look like the thing it describes. If there is a triangle in the problem, then your shape should be a triangle. If there is supposed to be a straight line, then your line should be straight. The actual sizes of sides and angles are not important in a rough sketch.

Label your diagram. If there is information provided in the question (such as the lengths of sides, or the sizes of angles) then write these on your diagram. This will often help when you are solving a problem.

Add new information that you work out. When you work out something new, add this to the diagram too.

So, in summary:

- draw a clear diagram
- label it
- add new information that you work out.

Here are three examples where drawing diagrams could help you:

Problem 1.1: A canteen offers a 'meal deal' that allows customers to choose a main course of fish, chicken or vegetables and a side order of either rice, fries, noodles or salad.

How many different meal combinations can you choose?

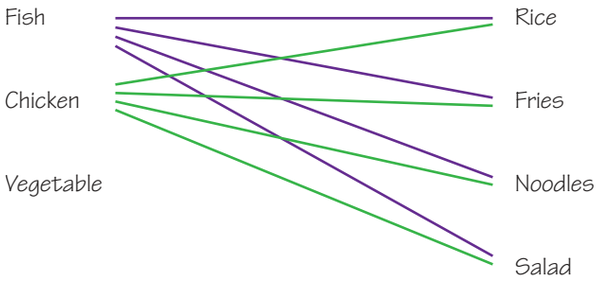
Tip

When we use the word 'diagram' here, we mean any visual representation of a problem. This can include rough sketches, graphs, number lines, tree diagrams, possibility diagrams, two way tables and Venn diagrams.

Tip

Drawing is a useful strategy to consider for problems involving combined probabilities, sets, loci, area and perimeter, ratio and proportion, fractional sharing, growth and decay, vectors, transformations, angles, distances and statistics.

You could work systematically and create a list, but a diagram would also help.



The diagram above shows all the options and the lines show some of the possible combinations.

There are four purple lines from fish to the four side options and four green lines from chicken to the four side options. This shows you that there are four choices for each main. You don't need to draw in the other lines to work out that there are 12 possible combinations.

You could use a possibility diagram like the one below to solve this problem.

Each tick, or each cell on the grid, represents one possibility. There are 12 ticks, so there are 12 possible meal combinations.

	Rice	Fries	Noodles	Salad
Fish	✓	✓	✓	✓
Chicken	✓	✓	✓	✓
Vegetable	✓	✓	✓	✓

Problem 1.2: A rectangle has sides of 10 cm and 8 cm to the nearest centimetre.

a What are the limits of accuracy for the area of this rectangle?

b What is the difference between the minimum and maximum values for:

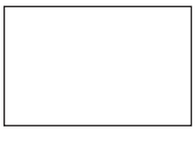
i the lengths of the sides

ii the area?

Draw a rough sketch of a rectangle. Label the sides and find the upper and lower bound of each measurement. This is the error interval.

$9.5 \leqslant L < 10.5$

10 cm

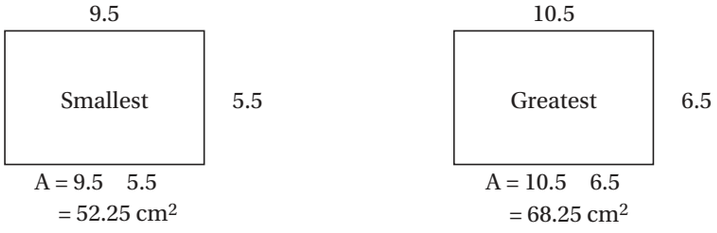


6 cm $\quad 5.5 \leqslant W < 6.5$

Tip

It can be useful to use a ruler to draw straight lines, even in sketches.

Sketch the smallest and greatest rectangles and find the area of each.



A = 9.5 5.5
= 52.25 cm^2

A = 10.5 6.5
= 68.25 cm^2

Write the values as an error interval for the area, A, of the rectangle using the correct notation.

a The limits of accuracy for the area are $52.25 \text{ cm}^2 \leqslant A < 68.25 \text{ cm}^2$

Your sketches show that the difference between the minimum and maximum values of the length and width is 1 cm.

You can subtract to find the difference between the minimum and maximum area.

b **i** 1 cm **ii** 16 cm^2

Here is an example where a table with highlighting is useful:

Problem 1.3: Amman says, "If I write out numbers in rows of six, all of the prime numbers will either be in the column that has 1 at the top, or in the column that has 5 at the top".

Can you tell if he is right?

You need to have some numbers to look at here so a diagram will be important.

Highlight a few prime numbers.

The table shows that the numbers in each column increase by six as you move down.

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36

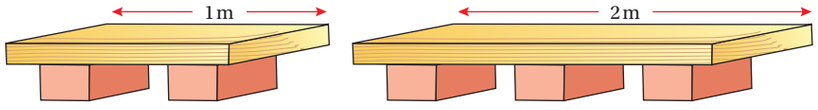
3
9
15
21
27
33

You know that 2 is the only even prime number. So you can eliminate all numbers in the 2nd, 4th and 6th columns, except for the 2 in the first row. (Adding six to an even number will always give an even answer.)

Adding six is the same as adding two threes, so all the numbers in the column with 3 at the top must multiples of 3 and therefore not prime, except for the 3 in the first row.

This means that besides the first row, any prime numbers must be in the first or fifth columns so Amman is right.

In the local cement factory, the cement bags are placed on pallets made of planks of wood and bricks.



The number of bricks needed to make the pallet is calculated as 'one more than the length of the plank in metres'.

a What length of pallet uses five bricks?

b If the pallet is 7 m long, how many bricks are used in it?

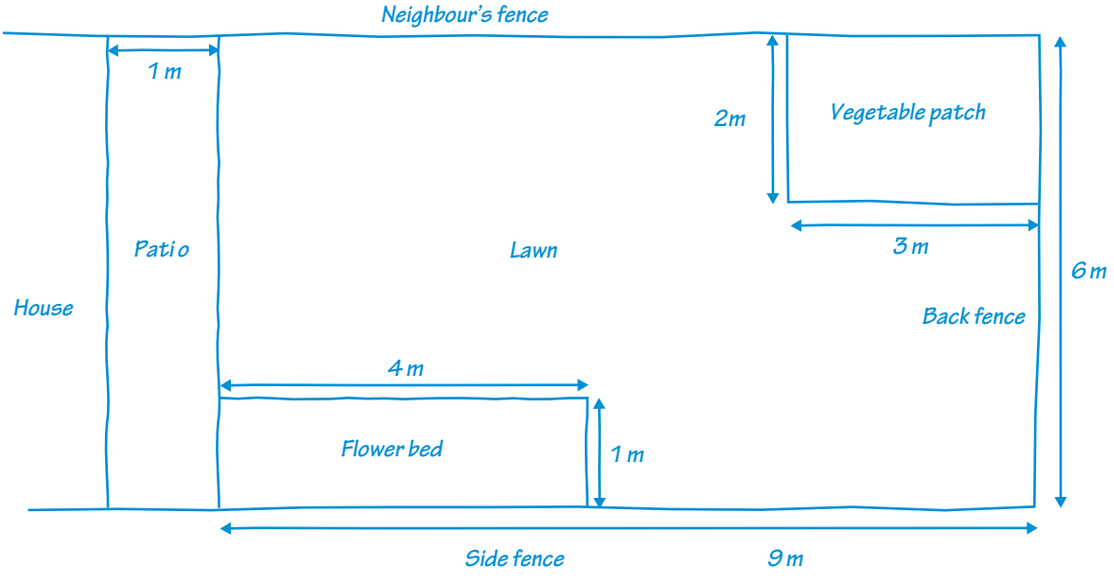
The factory needs pallets with a total length of 15 m for the next batch of cement. It has planks of wood that are 4 m long and 3 m long.

c What combinations of planks can they have?

d How many bricks would be needed for each combination?

Sanjita wants to plant a cherry tree in her garden. She needs to make sure there is a circular area of lawn with diameter 3m around the base of the tree, so that all of the fruit will fall onto the lawn area.

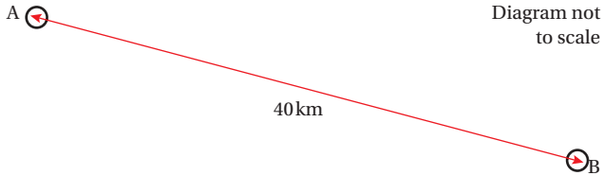
Here is a sketch, not drawn to scale, of Sanjita's garden.



Where could the tree be planted to meet her conditions?

3

The diagram represents towns A and B in a mountainous region.



The mountain rescue helicopters from both towns will always be sent to rescue any casualty within a radius of 25 km of town A or town B. The fire and rescue team from town B will travel to any accident scene closer to town B than town A.

Shade the region that the helicopters and town B's fire and rescue team will both cover.

4

A rectangle has length $(2x + 3)$ and width $(x - 1)$.

a Write an expression for the perimeter of the rectangle.

b Write an expression for the area of the rectangle.

The area of the rectangle is 250 cm^2.

c How long is the longest side?

d What is the perimeter of the rectangle?

Tip

$253 = 23 \times 11$

5

The probability that Hamza catches the 6.30 am train to the city is 0.7.

If he misses the train he will be late for work.

The probability the train will be late is 0.15.

If the train is late he will be late for work.

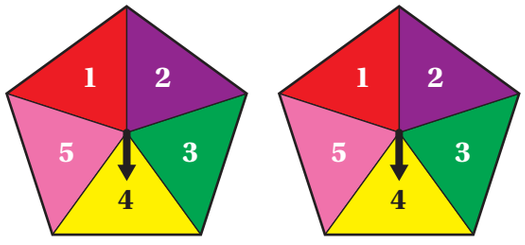
What is the probability Hamza will be on time for work on a particular day?

Tip

What type of diagram might help here?

6 ★☆☆

Two five-sided spinners are numbered 1 to 5. When the arrows are spun, your total score is calculated by adding the two numbers that the spinners land on.



a Draw a suitable diagram to show all possible outcomes when spinning these spinners.

b What is the highest score you could get?

c What is the probability of getting a total score of 8?

7 ★☆☆

The vertices of a quadrilateral are A, B, C and D.

A has coordinates (2, 1).

$\overrightarrow{AB} = \begin{pmatrix}2\\3\end{pmatrix}$, $\overrightarrow{BC} = \begin{pmatrix}4\\0\end{pmatrix}$, $\overrightarrow{AD} = \begin{pmatrix}4\\0\end{pmatrix}$

a Write a column vector for $\overrightarrow{CD}$.

b Compare $\overrightarrow{CD}$ with $\overrightarrow{AB}$. What do you notice? Can you explain?

c What type of quadrilateral is ABCD?

Tip

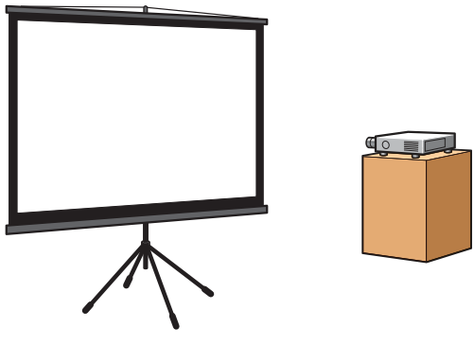
Use squared paper to draw your diagram.

8 ★☆☆

A projector is placed 1 m from a screen. When the projector is turned on, the image produced is only 20 cm high.

How far back should the projector be moved to produce an image that exactly fills the screen, which is approximately 1.5 m in height?

(Assume that no other adjustments are made to the projector.)



9

A factory manager planned to install a new hot drinks machine for the factory workers. He thought tea would be the most popular hot drink.

The workers did a survey to check what the preferred hot drink was among them. Each person could choose one drink from hot chocolate, tea or coffee.

Eight women wanted hot chocolate.

A total of 16 workers wanted tea, of which seven were men.

10 men and 12 women chose coffee.

There were 25 men in total.

Was tea the most popular hot drink?

Tip

What type of diagram might be helpful?

10

A ship sails 17.6 km on a bearing of 270° and then 15.4 km due south.

What is the shortest distance back to where it started?

11

Maria needs to make a long-distance journey. She is looking for the cheapest car hire.

Whacky Wheels has a standard charge of $35, then 15¢ for every kilometre driven.

Wheelies Rentals has a charge of 23¢ per kilometre travelled, but no standard charge.

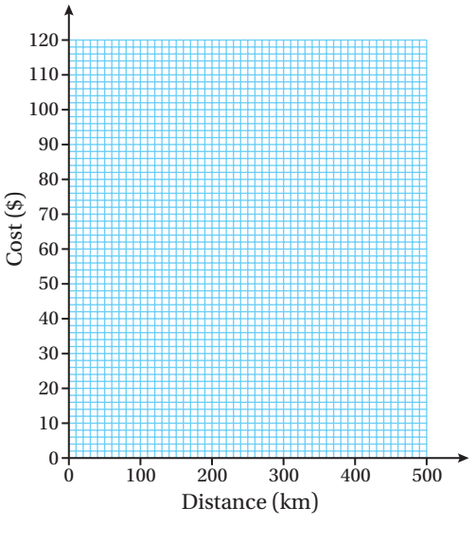
a Complete the charges graph for both car hire companies.

b Maria thinks the return journey will be 300 km. Which company would be cheaper to use?

c Maria made a mistake in her route plan and the return journey was 500 km. How much money would Maria have saved by using the other hire company?

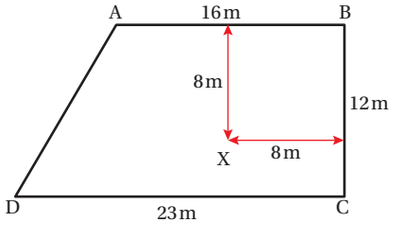
Tip

In this question you can use the axes that are given to help you draw the diagram.



12

ABCD is a field surrounded by fences AB, BC, CD and DA.



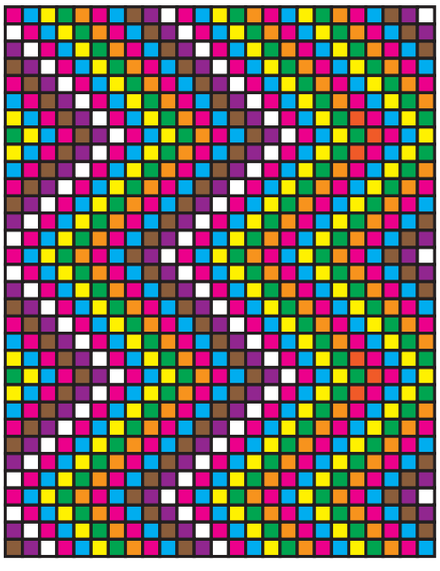
A goat is tied to the spike X on a rope measuring 3 m.

A bull is tied by an 8 m rope to the top of post A.

Find a route from corner D to corner B that would avoid both the bull and the goat.

13

This patchwork quilt is made from scraps of fabric.



Each patch is $(2x - 3)$ cm long and $(x + 3)$ cm wide. The area of the completed quilt is 2.8 m^2.

a There are 25 patches in each row. Write a possible expression for the width of the quilt.

b There are 32 patches in each column. Write a possible expression for the length of the quilt.

Tip

The diagram in the question is very detailed. Could a simpler diagram help?

c Write an expression for the area of the quilt, in the form $m(ax^2 + bx + c)$ where m is a constant.

d What are the dimensions of each patch? Give your answers in centimetres.

Tip

$2.8\,m^2 = 28\,000\,cm^2$. It might be easier to work in centimetres.

14 ★★☆

A square-based food container has a capacity of $1440\,cm^3$.

The base of the container has length x cm.

a Write an equation for the height of the container in terms of x.

The inside of the container (base and four sides) is to be lined with grease-proof paper with no overlaps.

b What is the formula, in terms of x, for the area of grease-proof paper needed?

c If the height of the container is 10 cm, what is the area of the base?

d What is the area of grease-proof paper needed?

15 ★★☆

A rectangular swimming pool is surrounded by a path made of mosaic tiles. The width of the path is x. It cost \$3196.80 to have the path tiled, at a rate of \$30 per square metre.

The pool itself measures 35 m by 30 m.

a Write an expression for the area of the tiled path in terms of x.

b Find the width of the path to the nearest centimetre.

16 ★★☆

Serrianne has taken up golf and goes to practise at the golf range twice a week. She uses one bucket of balls each time. In every bucket of 25 balls there are always 3 yellow balls; the rest are white.

Serrianne hits one ball (chosen at random) at a time.

a What is the probability that the first 3 balls she uses will all be yellow?

b What is the probability that the first 3 balls she uses will all be white?

c Calculate the probability that the first 3 balls Serrianne uses are a mixture of two yellows and one white.

17

To make the journey to work Abu must drive through two sets of traffic lights.

The probability of the first set being green is 0.7. If the first set is green, the probability of the second set also being green is 0.8. But if the first set is not green, the probability of the second set being green is 0.4.

a What is the probability that Abu does not have to stop on his journey to work tomorrow?

b What is the probability that Abu only has to stop once on his journey to work tomorrow?

18

Kalima and Jiao are very competitive and often have badminton and squash matches. The probability of Kalima winning at badminton is 0.85 and the probability of Kalima winning at squash is 0.35.

a What is the probability that the next time they play both matches, Kalima wins both?

b What is the probability that Kalima loses at badminton but wins at squash?

c What is the probability that both girls win one match each?

19

On a commercial flight to Tanzania the passengers were questioned about their malaria precautions. Only 70% of the passengers had obtained and started a course of anti-malaria tablets. The chances of getting malaria are $\frac{1}{200}$ if you take the tablets but $\frac{1}{50}$ if you are not taking the tablets. What is the probability that one passenger selected randomly will contract malaria?

Tip

What type of diagram would be helpful?