

A COURSE OF ANALYSIS

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EXTRACT FROM THE PREFACE TO THE FIRST EDITION

The main purpose of the book is to give a logical connected account of the subject, by starting with the definition of “Number” and proceeding in what appears to me to be a natural sequence of steps.

Since modern Analysis requires great precision of statement, and demands from the student a very clear understanding of its fundamental principles, I have aimed at presenting the subject in such a way as to make every important concept clearly understood. The examples at the end of each chapter have been chosen mainly to illustrate the fundamental concepts, and most of them have been taken from a collection which I have made of questions suitable for examination and exercise work for my students.

It is extremely difficult to acknowledge indebtedness to all the different sources in a work of this kind, and I am fully aware that I have benefited largely from most of the existing text-books and standard works on the subject, as well as from the lectures of Prof. J. E. Littlewood, F.R.S., and Mr S. Pollard of Trinity College, Cambridge. Since it is so often difficult to discover the rightful originator of particular theorems or modes of demonstration, no systematic attempt has been made to cite authorities; but where I have definitely borrowed from any recent work which appears to possess originality, acknowledgement has been made either in footnotes or in the text itself.

Prof. W. E. H. Berwick very kindly read through part of the first draft of the manuscript, and made some helpful suggestions for which I am grateful. My sincere thanks are due to Prof. G. N. Watson, F.R.S., for valuable criticisms and suggestions which have

helped very greatly to improve the form and presentation of the book; and to my colleague Mr W. M. Shepherd for his kindness in drawing all the diagrams and for his help with the proof-reading. I desire also to express my gratitude to the officials of the University Press both for their unfailing courtesy and for the excellence of their work.

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PREFACE TO THE EIGHTH IMPRESSION OF THE SECOND EDITION

The second edition of this book, based on a course of lectures on Analysis first prepared for the Honours students in the University College of North Wales, is largely a reproduction of the first edition. My experience of using the book with my own students has convinced me that it is already difficult enough for those who are using it as their first introduction to rigorous Analysis and so I have made no substantial changes in the subject-matter.

I have taken advantage of the opportunity presented by this reprint to correct one or two errors which had been overlooked previously.

I have included a set of Miscellaneous Examples at the end of the book. I wish to thank my colleague, Mr S. Moses, for his kindness in collecting these for me; and for providing Answers to the Examples.

E. G. P.

August 1962

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