ELECTRICAL AND OPTICAL WAVE-MOTION

THE MATHEMATICAL ANALYSIS OF ELECTRICAL AND OPTICAL WAVE-MOTION

ON THE BASIS OF MAXWELL'S EQUATIONS

BY

H. BATEMAN, M.A., Ph.D.

Late Fellow of Trinity College, Cambridge;
Johnston Research Scholar, Johns Hopkins University, Baltimore

Cambridge:
at the University Press
1915

CAMBRIDGE
UNIVERSITY PRESS

University Printing House, Cambridge CB2 8BS, United Kingdom

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning and research at the highest international levels of excellence.

www.cambridge.org
Information on this title: www.cambridge.org/9781316626122

First published 1915
First paperback edition 2016

A catalogue record for this publication is available from the British Library

ISBN 978-1-316-62612-2 Paperback

PREFACE

THIS book is intended as an introduction to some recent developments of Maxwell's electromagnetic theory which are directly connected with the solution of the partial differential equation of wave-motion. The higher developments of the theory which are based on the dynamical equations of motion are not considered at all. Even with this limitation the subject is a vast one, and to bring the work of perusing the literature within my power I have omitted an account of the modern theory of relativity which has been expounded very clearly in several recent publications.

For a thorough understanding of the present subject a very extensive knowledge of mathematics is necessary, but there are parts of the subject in which a reader with only a limited mathematical equipment may soon feel at home and perhaps do useful original work. With the idea of enabling such a reader to obtain a quick grasp of the nature of the subject and the results obtained, I have thought it advisable to state without proof a number of relations of which adequate demonstrations can only be obtained by means of complicated and difficult analysis. I have also endeavoured to keep the analysis as elementary as possible, but in some places where the work is perfectly straightforward a few details are omitted.

The book is far from being a complete treatise on the subject, for I have not given any existence theorems to show that the solutions of certain problems exist and are unique, and no attempt has been made to enter into the details of numerical computations. There are many parts of the subject indeed to which a pure mathematician might make useful additions; in particular, I might direct attention to p. 21, line 2, and p. 101, where there are one or two matters which require further discussion.

Chapter VIII and paragraph 5 contain some of my own contributions to the subject. At present there seem to be several different directions in which future developments may be made, and so it seems unwise to give a hasty judgment concerning the physical significance of the results. Ideas which naturally present themselves are that the aether can be regarded as built up from singular curves of the type considered in § 43, and that §§ 41 and 44 may throw some light on the question of the difference between positive and negative elementary electric charges. I hope to discuss an hypothesis relating to the first idea in a future note, but am unable to give any support at present to the second idea.

I gratefully acknowledge my indebtedness to Sir Joseph Larmor who read the manuscript before it was revised and made some helpful suggestions, to Prof. Ames who read the greater portion of the manuscript, to Prof. Morley and Mr Hassé who helped me with their advice and vigilance in reading the proof-sheets, and to the officers and staff of the University Press for their careful work and constant consideration shown in matters connected with the printing. For the correctness of the new formulae and examples I alone am responsible; if any errors are discovered I shall be grateful if my readers will inform me.

HARRY BATEMAN.

October, 1914.

ADDITIONS AND CORRECTIONS

p. 28. Formula (30) is due to Lamé. Cf. A. E. H. Love, *The Mathematical Theory of Elasticity*, 2nd edition, p. 55.

p. 101. An asymptotic expression for $T_m^n(s)$ when n is a large positive integer can be derived from a formula given by L. Fejér in 1909. This formula is accessible in a paper by O. Perron, *Arkiv der Mat. u. Phys.* (1914).

p. 118. The factor c in front of the double integrals should be omitted.

p. 120. Delete the minus sign in the second of equations (277).

p. 127. Line 8. This statement is incorrect, the equations are poristic, the special case is the only one which can occur.

p. 132. Line 20. On account of the porism just mentioned, the hope may be abandoned.

p. 150. Ex. 13. For equations (10) of § 5 read equations (2) of § 2.

p. 154. Ex. 24. The equation should read

$$\frac{\partial}{\partial a}\left[\frac{\sigma}{t-\tau}\cos(a-\epsilon)\frac{\partial V}{\partial a}+\sigma\sin(a-\epsilon)\frac{\partial V}{\partial t}\right]+\frac{\partial}{\partial\theta}\left[(t-\tau)\frac{\partial V}{\partial a}\right]$$
$$+\frac{\partial}{\partial t}\left[\sigma\sin(a-\epsilon)\frac{\partial V}{\partial a}+(t-\tau)\frac{\partial V}{\partial\theta}-\sigma(t-\tau)\cos(a-\epsilon)\frac{\partial V}{\partial t}\right]=0.$$

CONTENTS

CHAP.		PAGE
I.	FUNDAMENTAL IDEAS	1
II.	GENERAL SURVEY OF THE DIFFERENT METHODS OF SOLVING THE WAVE-EQUATION	25
III.	POLAR COORDINATES	35
IV.	CYLINDRICAL COORDINATES	69
V.	THE PROBLEM OF DIFFRACTION	82
VI.	TRANSFORMATIONS OF COORDINATES APPROPRIATE FOR THE TREATMENT OF PROBLEMS CONNECTED WITH A SURFACE OF REVOLUTION	95
VII.	HOMOGENEOUS SOLUTIONS OF THE WAVE-EQUATION . . .	110
VIII.	ELECTROMAGNETIC FIELDS WITH MOVING SINGULARITIES .	115
IX.	MISCELLANEOUS THEORIES	141
	LIST OF AUTHORS QUOTED	155
	INDEX	158