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978-1-316-62086-1 - Analytic Semigroups and Semilinear Initial Boundary Value Problems:
Second Edition

Kazuaki Taira

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