

Index

- a priori* bound, 8, 12, 21, 59, 101, 304, 312, 313
a priori estimate, 14, 18, 45, 177, 211, 283
 abstract Cauchy problem, 7, 11, 303
 abstract linear Cauchy problem, 11, 58, 84
 abstract semilinear Cauchy problem, 11, 59, 91
 adjoint of a closed operator, 55
 adjoint of a pseudo-differential operator, 14, 202
 adjoint of an operator, 54
 adjoint operator, 54, 55
 Agmon's method, 17, 18, 258, 282
 algebra, 202
 algebraic complement, 40
 amplitude, 189, 192, 195, 199, 205
 analytic semigroup, 6, 58, 64, 296
 annihilator, 35
 antilinear, 52
 Ascoli–Arzelà theorem, 152
 asymptotic expansion, 185, 199, 202, 203, 205, 212, 213
 atlas, 251
 Baire category theorem, 39
 Banach space, 30
 Banach's closed graph theorem, 39, 42, 257
 Banach's closed range theorem, 39
 Banach's open mapping theorem, 38
 Banach–Steinhaus theorem, 28
 base space, 321
 Besov space, 4, 177, 182
 Besov space boundedness theorem, 14, 176, 206
 Bessel function, 180
 Bessel inequality, 53
 Bessel potential, 178, 180
 bidual space, 36
 bijective, 257, 294
 boundary differential operator, 250
 boundary point lemma, 266, 319
 boundary value problem, 250, 252
 bounded convergence topology, 28
 bounded linear functional, 28, 35
 bounded linear operator, 28, 32
 bounded set, 27
 bounded variation, 316
 bundle atlas, 251, 321
 bundle map, 251
 bundle trivialization, 251
 canonical projection, 321
 Cauchy (convergence) condition, 26
 Cauchy sequence, 26, 30
 Cauchy's theorem, 61, 66, 68, 69, 71, 73, 75
 characteristic set, 212, 278
 characterization of ellipticity, 219
 classical pseudo-differential operator, 201
 classical pseudo-differential operators on a manifold, 208
 classical symbol, 185
 closable, 38
 closable operator, 38
 closed extension, 38
 closed graph theorem, 39, 42, 80
 closed linear operator, 60, 81
 closed operator, 38, 218, 235, 268, 272, 276, 284, 292, 294, 304
 closed range, 39, 43, 45, 218, 236, 268, 272, 292, 293, 294
 closed range theorem, 39, 273, 277, 300, 301
 closed set, 24
 closed subspace, 26

- compact convergence topology, 28
- compact injection, 45
- compact operator, 40, 183, 272, 275, 294
- complemented subspace, 40
- complete metric space, 94
- complete orthonormal system, 53
- complete symbol, 199, 203
- completely continuous, 41
- completely continuous operator, 183
- completeness of a normed space, 30
- completeness of a quasinormed space, 26
- complex parameter, 18, 282
- composition of pseudo-differential operators, 14, 202
- conjugate linear, 52
- conormal derivative, 2, 226, 319
- continuity of a linear operator, 27
- continuous operator, 27
- contraction mapping, 94
- contraction mapping principle, 12, 59, 94, 99
- converge strongly, 26, 30
- converge weakly, 52
- cotangent bundle, 203, 321
- criterion for hypoellipticity, 14, 177, 209

- definition of Besov spaces, 181
- definition of Fourier integral operators, 191
- definition of pseudo-differential operators, 195
- definition of pseudo-differential operators on a manifold, 206
- definition of Sobolev spaces, 181
- degenerate boundary condition, 5
- degenerate boundary value problem, 1
- degenerate elliptic boundary value problem, 227
- degenerate oblique derivative problem, 214
- delta function, 171, 189
- densely defined, 218, 235, 268, 272, 276, 292, 294
- densely defined operator, 60, 81
- diagonal, 196, 198, 206
- differential operator, 250
- Dirac measure, 12, 109, 171, 189
- Dirichlet condition, 4
- Dirichlet problem, 14, 217
- Dirichlet–Neumann operator, 15, 216, 234, 239, 260, 268, 271, 287, 292, 293
- distribution having sectional traces, 172, 174
- distribution kernel, 192, 196, 198, 206
- divergence, 235
- divergence theorem, 240

- domain, 23
- domain of class C^2 , 318
- domain of definition, 23
- double of a manifold, 170, 178, 217
- dual space, 28, 35
- dual spaces of a normed factor space, 36

- eigenspace, 42, 56
- eigenvalue, 42, 56
- eigenvalue space, 42
- eigenvalue vector, 42
- eigenvector, 42
- elementary symmetric polynomial, 166
- elliptic boundary condition, 254
- elliptic boundary value problem, 16, 250, 254, 255
- elliptic differential operator, 254, 267, 270, 285, 318
- elliptic pseudo-differential operator, 202
- elliptic pseudo-differential operators on a manifold, 208
- elliptic symbol, 186
- ellipticity of a symbol, 186
- equicontinuous, 29
- equivalent norms, 30
- existence and uniqueness theorem, 4, 5, 84, 87, 226
- existence and uniqueness theorem for the Dirichlet problem, 226
- existence and uniqueness theorem for the Neumann problem, 226
- existence and uniqueness theorem of the Cauchy problem, 87
- existence of a volume potential, 219
- existence theorem, 266
- exponential matrix, 316
- exponential order at infinity, 315
- extension of an operator, 23
- extension operator, 161

- Fefferman–Phong inequality, 14, 177, 211
- fibre, 250, 255, 321
- finite codimension, 218, 236, 268, 273, 277, 293
- finite codimensional range, 43
- finite dimension, 218, 236, 268, 275, 293
- finite dimensional null space, 43, 45
- finite dimensional space, 33
- finite index, 18, 273, 293, 294
- fixed point, 94, 99
- formulation of the boundary value problem, 232
- formulation of the degenerate elliptic boundary value problem, 2
- Fourier coefficient, 53
- Fourier expansion, 54

- Fourier integral distribution, 189
Fourier integral operator, 184, 192
Fourier inversion formula, 200
Fourier transform, 178, 179
Fréchet space, 26, 179, 184, 185
fractional power, 11, 58, 71, 75, 77, 78, 304
fractional powers and imbedding theorems, 304
Fredholm alternative, 42
Fredholm boundary operator, 15, 235, 260, 268, 271, 278, 287, 288, 293
Fredholm integral equation, 15, 216, 233
Fredholm operator, 43, 218, 236, 256, 269, 272, 293
Friedrichs' mollifier, 149
Fubini's theorem, 77, 199
function rapidly decreasing at infinity, 179
fundamental neighborhood system, 24
fundamental solution, 285
fundamental theorem for elliptic boundary value problems, 16, 255
- Gårding's inequality, 14, 177, 211
Gagliardo–Nirenberg inequality, 306
generalized Hölder's inequality, 246
generalized Sobolev space, 177
generation theorem for analytic semigroups, 5, 6, 58, 60, 63, 282, 296
global *a priori* estimate, 237, 256, 282, 284
global existence and uniqueness theorem, 8, 9, 12, 21, 59, 102, 304, 312, 313
global existence and uniqueness theorem for the Cauchy problem, 101
globally hypoelliptic, 14, 177, 208, 214
Gram–Schmidt orthogonalization, 53
graph norm, 92, 305
Gronwall's inequality, 12, 59, 103
growth condition, 8, 9, 313
- Hölder space, 110
Hahn–Banach extension theorem, 34
Hamilton map, 212, 279
Hessian, 212, 279
Hilbert space, 48, 49
Hilbert–Schmidt expansion formula, 57
Hilbert–Schmidt theorem, 56
Hilbert–Schmidt theory, 55
Hille–Yosida theory of analytic semigroups, 11, 58
homogeneous case, 84
homogeneous Cauchy problem, 84
homogeneous principal symbol, 202
homogeneous principal symbol of pseudo-differential operators on a manifold, 208
homotopy of symbols, 286
Hopf's boundary point lemma, 17, 266, 319
hypoelliptic, 208, 213, 275, 276
hypoelliptic pseudo-differential operator, 208
- imbedding properties of X_α , 305
imbedding property of fractional powers, 20
index, 218, 236, 269
index index, 293
index of an operator, 43
index zero, 18, 273, 292
initial value problem, 317
injective, 18, 265
inner product, 48
inner product space, 48
interpolation inequality, 113, 244
interpolation inequality for traces, 16, 243, 244
interpolation theorem, 112
invariance of pseudo-differential operators under change of coordinates, 203
inverse, 23
inverse Fourier transform, 178, 179
inverse operator, 23
isometry, 31
isomorphism, 5, 31
- Jacobian matrix, 204
jump formula, 12, 109, 171, 172, 175
- L^p approach to elliptic boundary value problems, 215
 L^p boundedness of pseudo-differential operators, 206
 L^p theory of pseudo-differential operators, 176
Laplace inversion formula, 316
Laplace transform, 11, 58, 59, 315
Laplace–Beltrami operator, 278
Laplacian, 171, 219, 220, 222, 225, 234, 260, 271, 286, 288
Lebesgue's dominated convergence theorem, 69, 77, 105
left elementary transformation, 254
Lie group of linear automorphisms, 321
linear, 23
linear Cauchy problem, 11, 84
linear functional, 23
linear operator, 23, 27
local chart, 251

- local existence and uniqueness theorem, 7, 8, 12, 59, 94
- local existence and uniqueness theorem for the Cauchy problem, 93, 94
- localized Sobolev space, 184
- Lopatinski–Shapiro ellipticity condition, 4, 16, 250, 251, 254, 255
- mapping property, 293
- maximum principle, 258, 265, 318
- mean curvature, 234
- Melin’s inequality, 14, 177, 213, 277
- metrizable, 25
- minimal closed extension, 38, 55
- mollifier, 149
- moment inequality, 79, 80
- multiple layer, 175
- multiplicity, 42
- neighborhood, 24
- Neumann problem, 233
- Neumann series, 33, 42
- Newtonian potential, 219
- nilpotent, 279
- non-characteristic, 174
- non-degenerate, 4
- non-homogeneous case, 85
- non-homogeneous Cauchy problem, 100
- non-trivial continuous linear functional, 34
- norm, 29
- normal coordinate, 170, 177, 217
- normed factor space, 30
- normed linear space, 29, 30
- null space, 23, 39, 218, 226, 236, 268, 275, 293
- open mapping theorem, 38
- open set, 24
- operator norm, 32
- ordinary differential equation, 253
- orthogonal, 49
- orthogonal projection, 51
- orthogonality, 49
- orthonormal set, 53
- oscillatory integral, 13, 176, 188, 189, 199
- outward normal, 3, 240
- parallelogram law, 49
- parametrix, 44, 203, 209, 257, 263, 264
- Parseval identity, 54
- partial Fourier transform, 199, 253
- partition of unity, 163
- Peetre’s criterion, 10, 45, 272
- Peetre’s lemma, 294
- phase function, 186, 189, 192, 195
- plane-wave expansion, 189
- point spectrum, 42
- Poisson operator, 15, 220, 225, 226, 267, 270, 287
- positive density, 170, 178, 217
- pre-Hilbert space, 48
- Preliminaries from functional analysis, 22
- principal symbol, 203, 253, 254
- principle of uniform boundedness, 33
- product space of normed linear spaces, 30
- properly supported, 197
- pseudo-differential operator, 195
- pseudo-differential operators on a manifold, 206
- pseudo-local property, 196
- quasinorm, 26
- quasinormed linear space, 24
- quasinormed space, 26
- range, 23
- real parameter, 17
- reduction to the boundary, 232, 293
- reflexivity, 52
- regular distribution with respect to one variable, 13, 109, 172, 173
- regularity property, 236, 256, 258
- regularity theorem, 258
- regularization, 149
- regularizer, 196
- Rellich–Kondrachov theorem, 183, 272, 274, 294
- residue theorem, 62, 68, 74
- resolvent, 6, 42, 75, 78, 304
- resolvent set, 6, 42, 304
- resonance theorem, 33
- restriction of an operator, 23
- Riemannian integrable, 315
- Riemannian metric, 234
- Riemannian vector bundle, 250, 255
- Riesz’s representation theorem, 51
- Riesz–Schauder theorem, 42
- Riesz–Schauder theory, 40
- Robin condition, 4
- scalar field, 23
- Schwartz space, 179, 189
- Schwarz inequality, 48, 281
- second dual space, 36
- second fundamental form, 235
- second-order elliptic differential operator, 17, 18, 19, 269, 282, 318
- second-order parabolic differential equation, 7

- second-order uniformly elliptic differential operator, 2
- section of a vector bundle, 250, 321
- section of Besov class, 255
- section of Sobolev class, 255
- sectional trace, 172, 175
- Seeley extension operator, 300
- Seeley extension theorem, 12, 108, 161
- self-adjoint operator, 55
- semigroup property, 61
- semilinear Cauchy problem, 11, 91, 93
- semilinear Cauchy problem for problem (*), 308
- semilinear initial boundary value problem, 7, 303
- seminorm, 25, 110, 111, 179, 184, 185, 284
- sesquilinearity, 48
- sharp Gårding inequality, 14, 177, 210, 211
- simple convergence topology, 28
- singular support, 189
- Sobolev imbedding theorem, 108, 148, 153, 159, 168, 305
- Sobolev space, 4, 110, 181
- solution of the Cauchy problem, 84, 93
- space X_α of fractional powers, 304
- space of continuous functions, 8, 94
- space of continuously differentiable functions, 8, 94
- space of fractional powers, 92
- spectrum, 42
- strong convergence, 26
- strong dual space, 35, 37
- strong limit, 33
- strong maximum principle, 17, 219, 222, 223, 266, 318
- strong topology, 32, 35
- strongly elliptic, 15, 239
- strongly uniform elliptic differential operator, 19, 282, 285
- subelliptic, 209
- subelliptic pseudo-differential operator, 209
- subprincipal symbol, 213, 279, 280
- summary of the contents, 10
- surface potential, 219, 220
- surjective, 18, 266
- symbol, 184
- symbol class, 184
- tangent bundle, 321
- tempered distribution, 179
- topological complement, 40
- topological space, 23
- topology, 24
- topology of linear operators, 27
- total space, 321
- trace map, 170
- trace theorem, 12, 108, 168, 170, 227, 231
- transition map, 203
- transpose of a compact operator, 41
- transpose of a derivative, 203
- transpose of a differential operator, 187, 191
- transpose of a Fredholm operator, 45
- transpose of a matrix, 204
- transpose of a pseudo-differential operator, 14, 202, 272, 276, 295
- transpose of an operator, 37
- transpose operator, 37, 194, 273, 276, 299
- transpose pseudo-differential operator, 272, 276, 295
- trivial line bundle, 321
- trivialization, 251
- typical fibre, 250, 255, 321
- uniform topology of operators, 32
- uniformly elliptic differential operator, 2
- uniformly Hölder continuous, 110
- uniqueness theorem, 224, 265
- uniqueness theorem for the Dirichlet problem, 224
- Vandermonde determinant, 165
- variation of constants formula, 11, 58, 85, 86, 87
- vector bundle, 16, 250, 320
- volume potential, 219, 300
- weak* dual space, 35
- weak* topology, 35
- zero extension, 171, 173, 175
- zero index, 18, 273, 292, 293