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Kazuaki Taira

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KAZUAKI TAIRA  
*Waseda University, Japan*



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To the memory of my mother  
Yasue Taira (1918–2014)

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## Preface to the Second Edition

The purpose of this monograph is to provide a careful and accessible exposition of a functional analytic approach to initial boundary value problems for semilinear parabolic differential equations of second order. Unlike many other books on analytic semigroups, it focuses on the relationship between the three interrelated subjects in analysis: elliptic boundary value problems and parabolic initial boundary value problems, with emphasis on the general study of analytic semigroups. This semigroup approach can be traced back to the pioneering work of Fujita–Kato [18] on the Navier–Stokes equation in fluid mechanics.

More precisely, we study a class of *degenerate* elliptic boundary value problems for second-order, uniformly elliptic differential operators in the framework of  $L^p$  Sobolev spaces which includes as particular cases the Dirichlet and Robin problems. We prove that this class of boundary value problems provides an interesting example of analytic semigroups in the  $L^p$  topology. We confined ourselves to the simple but important boundary condition. This makes it possible to develop our basic machinery with a minimum of bother and the principal ideas can be presented concretely and explicitly.

This monograph will have great appeal to both advanced students and researchers as an effective introduction to the three interrelated subjects in analysis: analytic semigroups, elliptic boundary value problems and initial boundary value problems for semilinear parabolic differential equations. Moreover, this book will provide a method for the study of elliptic boundary value problems, a powerful method clearly capable of further extensive development.

The first edition of this book (1995) was based on the lecture notes given at the University of Turin and the University of Bologna in Italy (1988) under the sponsorship of the Italian “Consiglio Nazionale delle

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Ricerche” and at the University of the Philippines (1989) in the course of the JSPS–DOST exchange program. The first edition was found useful by a number of people from different areas in analysis.

Our approach is distinguished by the extensive use of the techniques characteristic of recent developments in the theory of partial differential equations. The main technique used is the calculus of pseudo-differential operators which may be considered as a modern theory of classical potentials. The theory of pseudo-differential operators continues to be one of the most influential works in modern history of analysis. Several recent developments in the theory of pseudo-differential operators have made possible further progress in the study of elliptic boundary value problems and hence the study of semilinear parabolic initial boundary value problems. The presentation of these new results is the main purpose of this book.

This second edition is an expanded and revised version of a set of lecture notes for the graduate courses given by the author both at Hiroshima University (1995–1997) and at the University of Tsukuba (1998–2009) which were addressed to the advanced undergraduates and beginning-graduate students with interest in functional analysis and partial differential equations.

This second edition has been revised to streamline some of the analysis and to give better coverage of important examples and applications.

- (1) This edition includes one new chapter (Chapter 2), three appendices (Appendices A, B and C) and eight re-worked and expanded chapters (Chapters 1, 3–9). For example, in Section 6.6 we establish the *Lopatinski–Shapiro ellipticity condition* for boundary value problems in the framework of vector bundles over a compact smooth Riemannian manifold with boundary, and state the most fundamental theorem for elliptic boundary value problems.
- (2) In the last Chapter 9, we prove *global* existence and uniqueness theorems for a class of initial boundary value problems for semilinear parabolic differential equations if an *a priori* bound for nonlinear terms can be found.
- (3) In order to make the material in Chapters 6–9 accessible to a broad spectrum of readers in analysis, I have added *Introduction and Main Results* as Chapter 1. In this introductory chapter, I have attempted to describe how our problems can be solved, by using the mathematics I present in Chapters 2–5. The proofs of main theorems (Theorems 1.1–1.4) are flowcharted (Summary

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of the contents in Section 1.6). Moreover, this second edition is amply illustrated and all the figures are provided with appropriate captions.

- (4) To make the book more up-to-date, additional references have been included in References.

This second edition may be considered as a short introduction to the more advanced book “Semigroups, boundary value problems and Markov processes” second edition (2014), which was published in the Springer Monographs in Mathematics series. For graduate students working in functional analysis and partial differential equations, this book may serve as a concise introduction to these two interrelated fields of analysis. For graduate students about to major in the subject and mathematicians in the field looking for a coherent overview, it will provide a powerful method for the analysis of elliptic boundary value problems in the framework of  $L^p$  Sobolev spaces.

I would like to thank the referees who made many comments which were crucial in improving the exposition, both mathematically and stylistically. My special thanks go to the editorial staff of Cambridge University Press for their unfailing helpfulness and cooperation during the production of this second edition.

Last but not least, I owe a great debt of gratitude to my family who gave me moral support during the preparation of this book.

Tsuchiura, January 2016

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Kazuaki Taira

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## Preface to the First Edition

This monograph is devoted to the functional analytic approach to initial boundary value problems for semilinear parabolic differential equations. First, we study non-homogeneous boundary value problems for second-order elliptic differential operators, in the framework of Sobolev spaces of  $L^p$  type, which include as particular cases the Dirichlet and Neumann problems. We prove that these boundary value problems provide an example of analytic semigroups in the  $L^p$  topology. The essential point in the proof is to define a function space which is a tool well suited to investigating our boundary conditions. By virtue of the theory of analytic semigroups, we can apply this result to the study of the initial boundary value problems for semilinear parabolic differential equations in the framework of  $L^p$  spaces.

This monograph grew out of a set of lecture notes “On initial boundary value problems for semilinear parabolic differential equations” for graduate courses given at the University of Tsukuba in winter 1994/95. In order to make this monograph accessible to a broad readership, I have tried to start from scratch. In the preparatory chapters, we even prove fundamental results like a generation theorem for analytic semigroups in functional analysis and Sobolev imbeddings theorems in partial differential equations. Furthermore, we summarize the basic definitions and results about the  $L^p$  theory of pseudo-differential operators which is considered as a modern theory of potentials. The  $L^p$  theory of pseudo-differential operators forms a most convenient tool in the study of elliptic boundary value problems in the framework of Sobolev spaces of  $L^p$  type. The material in these preparatory chapters is given for completeness, to minimize the necessity of consulting too many outside references. This makes the monograph fairly self-contained.

This work was begun at the University of Turin and the University

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of Bologna in May 1988 under the sponsorship of the Italian “Consiglio Nazionale delle Ricerche” and a major part of the work was done at the University of the Philippines in the course of the JSPS–DOST exchange program from January 1989 to March 1989 while I was on leave from the University of Tsukuba. I take this opportunity to express my gratitude to all these institutions for their hospitality and support.

Thanks are also due to the editorial staff of Cambridge University Press for their unfailing helpfulness and cooperation during the production of the book.

I hope that this monograph will lead to a better insight into the study of initial boundary value problems for semilinear parabolic differential equations. For probabilistic information on the topics discussed here, I would like to call attention to my previous book *Boundary Value Problems and Markov Processes*, Lecture Notes in Mathematics, No. 1499, Springer-Verlag, 1991.

Higashi-Hiroshima, June 1995

Kazuaki Taira