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G. Lewingdon Parsons
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**ELEMENTARY DIFFERENTIAL
AND INTEGRAL
CALCULUS**

by

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PART I
ELEMENTARY
DIFFERENTIAL CALCULUS

PREFACE

THE present volume is intended for the general purpose of preparing candidates for the Higher Certificate Examinations. Like its companion volume, it owes its origin to lecture notes given in preparation for this examination, and can thus be said to have stood, in part at any rate, the test of experience. The author feels that in Differential Calculus (just as in Integral Calculus) there is too wide a gap between the easy introductions to the Calculus, which do not go far enough, and the advanced text-books, in which the main principles are in danger of being lost in a mass of subsidiary detail. To the mathematical specialist these details are important and often essential, but the author is strongly of opinion that even the would-be mathematical specialist will acquire mathematical precision much more easily if he has previously worked through a book such as this, by way of general introduction to the subject. To those, on the other hand, who wish to regard the Calculus as a working tool and not as a subject for exhaustive enquiry, the finer points are unnecessary, and it may be hoped that they, too, will find most of their needs supplied in the present volume.

A few applications to Algebra, Mechanics, etc., are given; these simple methods are given in so few elementary text-books that it seemed worth while to include them. The order of the different parts of the subject is a matter of taste, and some of the examples may well be omitted by all but the most intelligent pupils. A short historical

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sketch has been added. Much of the history of the invention of the Calculus is obscure, and much more is controversial, but the author feels strongly that some attempt ought to be made to teach it.

The Oxford and Cambridge Joint Board have again kindly granted permission to use questions set in their Higher Certificate Examinations, and such examples are denoted in the text by an asterisk.

In conclusion the author's sincere thanks are due to the publishers, whose accurate work and skilful cooperation have greatly facilitated the labour of preparing the book for the press.

G. L. P.

August, 1927.

A number of corrections have been made in the second edition. An appendix on Newton's Approximation and a set of Miscellaneous Examples have been added.

G. L. P.

November, 1933

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