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# ELEMENTARY INTEGRAL CALCULUS

*By*

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## PREFACE

**T**HE object of this little work is to provide an introduction to the subject of Integral Calculus for mathematical and scientific students. The plan of the book is largely based on the Syllabus for the Higher Certificate Examination and it owes its origin to a set of lectures given by the author in preparation for this examination. The author ventures to hope that, while reducing the ground to be covered, he has not left out anything which is fundamentally necessary to the opening stages of the subject. A previous knowledge of the differentiation of powers, products, sines, cosines, etc. is however assumed.

No attempt has been made to treat the subject from an absolutely mathematical standpoint. The author fails to see the value of discussing the validity of a process until the nature of the process and its usual applications are thoroughly grasped. In fact, the work is not by any means intended as the final reading for a prospective mathematical scholar.

It is the author's opinion that, in many books, sufficient stress is not laid on what he has ventured to call the Fundamental Theorem. Historically the discovery of this theorem by Newton and Leibnitz was the turning point of the whole theory. He has therefore reversed the conventional order of treatment of the subject, and introduced the notion of summation only after the student has learnt how to integrate. It is hoped that this will lead to a deeper understanding of the Fundamental Theorem, and hence to a more skilful use of it.

In view of the recent regulations permitting the Preliminary Examination in Natural Science at Oxford to be taken from school, a chapter on Differential Equations has been added, making the book a complete preparation for the Integral required. The addition of questions on rotational dynamics to the Mechanics Syllabus of the Higher Certificate Examination seems to indicate a move in this direction, whilst from the point of view of general education the popularity of wireless renders some mathematical knowledge on the subjects of “damping” and “resonance” highly desirable.

A short historical survey has been added, for the author believes that this is a subject which might, with advantage, be taught to a greater extent than is done to-day.

His acknowledgements are due to Mr T. W. Chaundy, M.A., Tutor of Christ Church, Oxford, who has, in the press of much more important work, been good enough to read the MS. and has made many valuable suggestions. The extent to which the author is indebted to many standard works will be apparent to any mathematician who reads this volume. He desires to mention in particular the works of Williamson, Gibson and Edwards.

Questions from the Higher Certificate Examination are included by kind permission of the Oxford and Cambridge Schools Examination Board, and are marked in the text with an asterisk.

In conclusion the author wishes to record his indebtedness to the readers of the Cambridge University Press, to whose careful and excellent work the elimination of many errors is due.

G. L. P.

*November, 1925.*

## CONTENTS

	PAGES
HISTORICAL SKETCH . . . . .	1—6
CHAPTER I	
DEFINITIONS AND STANDARD FORMS	
Definitions. §§ 1—4 . . . . .	7—9
Table of Forms. § 5 . . . . .	10—11
Examples I A . . . . .	11—12
Forms $\frac{\phi'(x)}{\phi(x)}$ , $[\phi(x)]^n \cdot \phi'(x)$ . § 6 . . . . .	12—13
Examples I B . . . . .	14
CHAPTER II	
INTEGRATION BY SUBSTITUTION, OR CHANGING THE SUBJECT OF INTEGRATION	
Definite Integrals. § 7 . . . . .	15—16
Integration by substitution. §§ 8—10 . . . . .	16—21
Examples II . . . . .	21—22
CHAPTER III	
INTEGRATION BY PARTS	
Rule for Integration by Parts. §§ 11—13 . . . . .	23—26
Important artifice. § 14 . . . . .	26—27
Forms $e^{ax} \frac{\cos bx}{\sin bx}$ . § 15 . . . . .	27—28
Examples III . . . . .	28
CHAPTER IV	
RATIONAL ALGEBRAIC FRACTIONS	
Reduction of Fractions. § 16 . . . . .	29—30
The form $\int \frac{ax+b}{Ax^2+Bx+C} dx$ . § 17 . . . . .	30—32
Worked Examples. § 18 . . . . .	32—34
Examples IV . . . . .	34

CHAPTER V

TRIGONOMETRICAL INTEGRALS AND  
 FORMULÆ OF REDUCTION

	PAGES
Odd power of sine or cosine. § 19 . . . . .	35—36
$\int \frac{dx}{a + b \sin x}$ , etc. § 20 . . . . .	36—37
Worked Examples. § 21 . . . . .	37—38
Examples VA . . . . .	38—39
Integrals of even order. § 22 . . . . .	39—40
Reduction formulæ for $\int \sin^n x dx$ and $\int \cos^n x dx$ . §§ 23—27 . . . . .	41—47
Formulæ for $\int \sin^p x \cos^q x dx$ . §§ 28—30. . . . .	47—50
Wallis' Formula for $\pi$ . § 31 . . . . .	50—51
Examples VB . . . . .	51—52

CHAPTER VI

THE FUNDAMENTAL "INVERSION THEOREM." APPLICATION  
 OF CALCULUS TO FINDING OF AREAS

The Fundamental Theorem. §§ 32—35 . . . . .	53—58
Areas of Curves. §§ 36—37 . . . . .	58—60
Worked Examples. § 38 . . . . .	60—64
Simpson's Rule for Area. § 39 . . . . .	64—65
Examples VI . . . . .	65—66

CHAPTER VII

FURTHER APPLICATIONS TO GEOMETRY: SECTORIAL AREAS,  
 VOLUMES OF REVOLUTION, FINDING THE LENGTH  
 OF A CURVE

Areas in Polar Coordinates. §§ 40—41 . . . . .	67—68
Volumes of Revolution, with worked examples. §§ 42—43 . . . . .	69—71
Examples VIIA . . . . .	72—73
Finding the Length of an Arc, with worked examples. §§ 44—46 . . . . .	73—76
Examples VII B . . . . .	76—77

CONTENTS

CHAPTER VIII

APPLICATION TO PROBLEM OF FINDING  
 CENTRES OF GRAVITY

	PAGES
Centre of Gravity. Principle. § 47 . . . . .	78
c.g. of areas and volumes of revolution, with worked examples. §§ 48—50 . . . . .	78—85
Pappus' Theorems. § 51 . . . . .	85—86
Volumes in Polar Coordinates. § 52 . . . . .	86—87
Examples VIII . . . . .	88—89

CHAPTER IX

FURTHER APPLICATIONS TO MECHANICS. RIGID DYNAMICS

Moments of Inertia. §§ 53—55 . . . . .	90—93
Analogies between Linear and Rotational Dynamics. § 56	93—94
Worked Example. § 57 . . . . .	94—98
Examples IX A . . . . .	98—99
Work done by a varying force. §§ 58—59 . . . . .	99—101
Centre of Pressure. § 60 . . . . .	101—103
Examples IX B . . . . .	103—104

CHAPTER X

DIFFERENTIAL EQUATIONS. A FEW TYPES

General Principles. §§ 61—62 . . . . .	105—107
Linear Equations with Constant Coefficients. §§ 63—67	107—112
Simple Harmonic Motion and Oscillations (including Damped Oscillation). §§ 68—70 . . . . .	112—116
Equations whose right-hand side is not zero. § 71 . . . . .	116
The Equation of Resonance. § 72 . . . . .	117—118
Examples X . . . . .	118—119
ANSWERS TO EXAMPLES . . . . .	120—127