INTRODUCTION

The following tables of Legendre Polynomials, $P_n(x)$, have been prepared to meet the needs of workers in various branches of mathematics and physics.

Tables for $x=0(0{\cdot}01)1$. When the accompanying tables were designed (in 1932) the existing tables for this range and interval were:

(1) British Association: Report of the Committee on Mathematical Tables for 1879. This extends to $P_7(x)$, and all values are exact; no differences are given.

(2) Tallqvist, H. Tafeln der Kugelfunctionen $P_n(x)$ und ihrer abgeleiteten Functionen. *Acta Societatis Scientiarum Fennicae*, tome XXXII, no. 6 (Helsingfors, 1904). This includes all the values given in (1) and also $P_8(x)$. All the derivatives are tabulated.

(3) Hayashi, K. *Tafeln der Besselschen, Theta-, Kugel- und anderer Funktionen* (Berlin: Springer, 1930). These tables extend to $P_8(x)$, with no differences or derivatives.

These three tables were compared with each other. No errors were found in (1) and (2) but five occur in Hayashi. He remarks "Die Tafeln der Kugelfunktionen sind aufs neue von mir berechnet. Ein Vergleichen mit den vorhandenen Tafeln für diese Funktionen zeigte ziemlich viele Unstimmigkeiten".

In the present tables, which extend to $n=12$, $P_2(x)$ and $P_3(x)$ are exact to five and seven decimals respectively; for other values of n, $P_n(x)$ is rounded off to seven decimals. $P_4(x)$ is exact when x is a multiple of 0·02; in the remaining cases it can be made exact by adding 0·4375 units of the seventh decimal; thus

$$P_4(0{\cdot}01)=+0{\cdot}37462\ 50437\ 5 \quad \text{and} \quad P_4(0{\cdot}35)=-0{\cdot}01872\ 26562\ 5.$$

$P_2(x)$, $P_3(x)$ and $P_4(x)$ were built up from their constant differences on a Burroughs Class 11 machine. $P_5(x)$, $P_6(x)$ and $P_7(x)$ were taken from the 1879 Report of the British Association. $P_8(x)$ is based on Tallqvist and Hayashi, the differences being computed specially for this work. $P_9(x)$ was found to nine decimals by the recurrence formula, and checked by differences. As a further check, $P_8(x)$ and $P_9(x)$ were computed directly at interval 0·2 from the definitions. $P_{10}(x)$, $P_{11}(x)$ and $P_{12}(x)$ were computed by A. J. Thompson from the definitions, without reference to any previous work.

Tables for $x=1(0{\cdot}01)6$. These tables, which extend to $n=12$, are completely new. They give seven significant figures throughout, and usually eight when the first is 1 or 2. Exact values at interval 1.0 were first prepared from the definitions. Then exact values at interval 0·2 were computed as far as $n=9$, by summation on a Burroughs machine from the constant difference; these values were used later as independent checks. $P_2(x)$, which is exact to five decimals, was built up from its constant second difference of 30; $P_3(x)$ was built up from its second differences, utilising cycles in the end figures; $P_4(x)$, $P_5(x)$ and $P_6(x)$ were built up in the same way, and rounded off as required. The remaining values were computed by the recurrence formula, and checked by differencing on a National machine.*

The printer's copy was prepared on the Burroughs machine already mentioned, by building up from the second, fourth or sixth difference as required. By this means it is possible to ensure the complete accuracy and legibility of every figure.

Tables for $x = 6\ (0{\cdot}1)11$. These tables, on p. A 42, are an extension at a larger interval to $x = 11$ and $n = 6$. The values of $P_2(x)$ and $P_3(x)$ are exact.

Interpolation. Interpolation may be performed by the methods described in the Association's *Mathematical Tables*, vol. I,† p. vi, or in *Interpolation and Allied Tables* (London: H.M. Stationery Office, 1936). The method described on p. xi of the first-mentioned volume or on pp. 928–9 of the latter volume for eliminating higher-order differences that do not exceed a certain limiting value (1000 for δ^4 and 10,000 for δ^6) has been adopted here; modified differences are denoted by $*\delta^2$ and $*\delta^4$. All the differences necessary for interpolation to the full accuracy of the table are given. Modified differences are to be used in exactly the same way as unmodified differences. Where the modification begins or ends in the middle of a page all the differences provided are to be used; thus an interpolation of $P_7(x)$ for $x = 0{\cdot}795$ would be performed with only one difference (here $+738$) from the column δ^4.

L. J. COMRIE

* Comrie, L. J., "Scientific Applications of the National Accounting Machine." Supplement to the *Journal of the Royal Statistical Society*, **3**, 87 (1936).

† First edition, 1931. Detailed treatment of interpolation is unfortunately omitted from the second edition, 1946.

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LEGENDRE POLYNOMIALS

Legendre's differential equation:

$$(1-x^2)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+n(n+1)\,y=0.$$

Standardised polynomial solution for positive integral values of n:

$$y=P_n(x)=\frac{(2n)!}{2^n(n!)^2}\left[x^n-\frac{n(n-1)}{2(2n-1)}x^{n-2}+\frac{n(n-1)(n-2)(n-3)}{2.4(2n-1)(2n-3)}x^{n-4}-\dots\right].$$

Explicit expression of polynomials for $n=0$ to 12:

$P_0(x)=1,$

$P_1(x)=x,$

$P_2(x)=\frac{1}{2}(3x^2-1),$

$P_3(x)=\frac{1}{2}(5x^3-3x),$

$P_4(x)=\frac{1}{8}(35x^4-30x^2+3),$

$P_5(x)=\frac{1}{8}(63x^5-70x^3+15x),$

$P_6(x)=\frac{1}{16}(231x^6-315x^4+105x^2-5),$

$P_7(x)=\frac{1}{16}(429x^7-693x^5+315x^3-35x),$

$P_8(x)=\frac{1}{128}(6435x^8-12012x^6+6930x^4-1260x^2+35),$

$P_9(x)=\frac{1}{128}(12155x^9-25740x^7+18018x^5-4620x^3+315x),$

$P_{10}(x)=\frac{1}{256}(46189x^{10}-109395x^8+90090x^6-30030x^4+3465x^2-63),$

$P_{11}(x)=\frac{1}{256}(88179x^{11}-230945x^9+218790x^7-90090x^5+15015x^3-693x),$

$P_{12}(x)=\frac{1}{1024}(676039x^{12}-1939938x^{10}+2078505x^8-1021020x^6+225225x^4-18018x^2+231).$

Other definitions:

$$(1-2xz+z^2)^{-\frac{1}{2}}=P_0(x)+P_1(x)\,z+P_2(x)\,z^2+P_3(x)\,z^3+\dots\quad(z<1),$$

$$P_n(x)=\frac{1}{2^n n!}\left(\frac{d}{dx}\right)^n(x^2-1)^n.$$

Recurrence formula, used (inter alia) to obtain $P_n(x)$ for $n>12$:

$$(n+1)\,P_{n+1}(x)-(2n+1)\,xP_n(x)+nP_{n-1}(x)=0.$$

Derivative formulae:

$$(x^2-1)\,P'_n(x)=n\{xP_n(x)-P_{n-1}(x)\}=(n+1)\{P_{n+1}(x)-xP_n(x)\},$$

$$P'_n(x)=(2n-1)\,P_{n-1}(x)+(2n-5)\,P_{n-3}(x)+(2n-9)\,P_{n-5}(x)+\dots,$$

$$xP'_n(x)-P'_{n-1}(x)=nP_n(x),\qquad P'_{n+1}(x)-xP'_n(x)=(n+1)\,P_n(x),$$

$$P'_{n+1}(x)-P'_{n-1}(x)=(2n+1)\,P_n(x),\qquad P'_{n+1}(x)-2xP'_n(x)+P'_{n-1}(x)=P_n(x).$$

Special values:

$$P_{2n}(0)=(-1)^n\frac{(2n)!}{2^{2n}(n!)^2},\qquad P_{2n+1}(0)=0,\qquad P_n(1)=1,\qquad P_n(-x)=(-1)^n\,P_n(x).$$

Orthogonal relation:

$$\int_{-1}^{1}P_m(x)\,P_n(x)\,dx=0,\ \text{when}\ m\neq n;\qquad\int_{-1}^{1}\{P_n(x)\}^2\,dx=\frac{2}{2n+1}.$$

Expression of x^{n-1} in terms of Legendre polynomials:

$$x^{n-1}=\frac{2^{n-1}\{(n-1)!\}^2}{(2n-1)!}\left[(2n-1)\,P_{n-1}(x)+(2n-5)\frac{(2n-1)}{2}P_{n-3}(x)\right.$$
$$\left.+(2n-9)\frac{(2n-1)(2n-3)}{2.4}P_{n-5}(x)+(2n-13)\frac{(2n-1)(2n-3)(2n-5)}{2.4.6}P_{n-7}(x)+\dots\right].$$

Illustrations of accuracy in use of recurrence formula:

(i) $11P_{11}(\cdot73)=21(\cdot73)\,P_{10}(\cdot73)-10P_9(\cdot73)=21(\cdot73)(\cdot20299\ 76)-10(\cdot31199\ 70)=-\cdot00801\ 68$,
hence $P_{11}(\cdot73)=-\cdot00072\ 88$ ($-\cdot00072$ 8**7** in the tables);

(ii) $12P_{12}(\cdot73)=23(\cdot73)\,P_{11}(\cdot73)-11P_{10}(\cdot73)=23(\cdot73)(-\cdot00072\ 87)-11(\cdot20299\ 76)=-2\cdot24520\ 85$,
hence $P_{12}(\cdot73)=-\cdot18710\ 07$ ($-\cdot18710$ 0**8** in the tables).

Everett's interpolation formula ($\phi=1-\theta$):

$$u_\theta=\phi u_0+\frac{\phi(\phi^2-1)}{3!}\delta^2u_0+\frac{\phi(\phi^2-1)(\phi^2-4)}{5!}\delta^4u_0+\dots+\theta u_1+\frac{\theta(\theta^2-1)}{3!}\delta^2u_1+\frac{\theta(\theta^2-1)(\theta^2-4)}{5!}\delta^4u_1+\dots.$$

x	$P_2(x)$†	$P_3(x)$	δ^2	$P_4(x)$	δ^2	$P_5(x)$	$^*\delta^2$	$P_6(x)$	$^*\delta^2$
0·00	−0·50000	−0·00000 00	0	+0·37500 00	−75 00	+0·00000 00	0	−0·31250 00	+131 31
·01	·49985	·01499 75	+ 1 50	·37462 50	74 93	·01874 13	− 5 27	·31184 39	131 05
·02	·49940	·02998 00	3 00	·37350 07	74 79	·03743 00	10 48	·30987 81	130 34
·03	·49865	·04493 25	4 50	·37162 85	74 51	·05601 39	15 70	·30660 97	129 19
·04	·49760	·05984 00	6 00	·36901 12	74 16	·07444 08	20 91	·30205 03	127 51
0·05	−0·49625	−0·07468 75	+ 7 50	+0·36565 23	−73 67	+0·09265 87	− 26 06	−0·29621 66	+125 43
·06	·49460	·08946 00	9 00	·36155 67	73 11	·11061 61	31 16	·28912 95	122 86
·07	·49265	·10414 25	10 50	·35673 00	72 41	·12826 20	36 23	·28081 47	119 81
·08	·49040	·11872 00	12 00	·35117 92	71 64	·14554 58	41 19	·27130 26	116 35
·09	·48785	·13317 75	13 50	·34491 20	70 73	·16241 78	46 12	·26062 78	112 45
0·10	−0·48500	−0·14750 00	+15 00	+0·33793 75	−69 75	+0·17882 88	− 50 94	−0·24882 93	+108 10
·11	·48185	·16167 25	16 50	·33026 55	68 63	·19473 06	55 65	·23595 06	103 33
·12	·47840	·17568 00	18 00	·32190 72	67 44	·21007 60	60 31	·22203 93	98 18
·13	·47465	·18950 75	19 50	·31287 45	66 11	·22481 86	64 79	·20714 70	92 60
·14	·47060	·20314 00	21 00	·30318 07	64 71	·23891 35	69 18	·19132 94	86 63
0·15	−0·46625	−0·21656 25	+22 50	+0·29283 98	−63 17	+0·25231 68	− 73 46	−0·17464 61	+ 80 33
·16	·46160	·22976 00	24 00	·28186 72	61 56	·26498 58	77 57	·15716 02	73 64
·17	·45665	·24271 75	25 50	·27027 90	59 81	·27687 94	81 53	·13893 85	66 63
·18	·45140	·25542 00	27 00	·25809 27	57 99	·28795 80	85 32	·12005 11	59 27
·19	·44585	·26785 25	28 50	·24532 65	56 03	·29818 37	88 98	·10057 15	51 65
0·20	−0·44000	−0·28000 00	+30 00	+0·23200 00	−54 00	+0·30752 00	− 92 41	−0·08057 60	+ 43 71
·21	·43385	·29184 75	31 50	·21813 35	51 83	·31593 25	95 69	·06014 39	35 50
·22	·42740	·30338 00	33 00	·20374 87	49 59	·32338 85	98 75	·03935 72	27 07
·23	·42065	·31458 25	34 50	·18886 80	47 21	·32985 74	101 61	− ·01830 02	18 42
·24	·41360	·32544 00	36 00	·17351 52	44 76	·33531 06	104 25	+ ·00294 06	9 56
0·25	−0·40625	−0·33593 75	+37 50	+0·15771 48	−42 17	+0·33972 17	−106 66	+0·02427 67	+ 53
·26	·39860	·34606 00	39 00	·14149 27	39 51	·34306 66	108 85	·04561 78	− 8 65
·27	·39065	·35579 25	40 50	·12487 55	36 71	·34532 35	110 76	·06687 22	17 96
·28	·38240	·36512 00	42 00	·10789 12	33 84	·34647 32	112 46	·08794 69	27 34
·29	·37385	·37402 75	43 50	·09056 85	30 83	·34649 88	113 87	·10874 81	36 81
0·30	−0·36500	−0·38250 00	+45 00	+0·07293 75	−27 75	+0·34538 62	−114 99	+0·12918 12	− 46 29
·31	·35585	·39052 25	46 50	·05502 90	24 53	·34312 42	115 87	·14915 14	55 80
·32	·34640	·39808 00	48 00	·03687 52	21 24	·33970 41	116 41	·16856 37	65 25
·33	·33665	·40515 75	49 50	+ ·01850 90	17 81	·33512 04	116 69	·18732 36	74 67
·34	·32660	·41174 00	51 00	− ·00003 53	14 31	·32937 04	116 63	·20533 70	84 00
0·35	−0·31625	−0·41781 25	+52 50	−0·01872 27	−10 67	+0·32245 47	−116 25	+0·22251 07	− 93 18
·36	·30560	·42336 00	54 00	·03751 68	6 96	·31437 71	115 55	·23875 29	102 19
·37	·29465	·42836 75	55 50	·05638 05	− 3 11	·30514 46	114 50	·25397 35	111 04
·38	·28340	·43282 00	57 00	·07527 53	+ 81	·29476 77	113 12	·26808 42	119 60
·39	·27185	·43670 25	58 50	·09416 20	4 87	·28326 03	111 36	·28099 94	127 92
0·40	−0·26000	−0·44000 00	+60 00	−0·11300 00	+ 9 00	+0·27064 00	−109 24	+0·29263 60	−135 90
·41	·24785	·44269 75	61 50	·13174 80	13 27	·25692 80	106 74	·30291 42	143 55
·42	·23540	·44478 00	63 00	·15036 33	17 61	·24214 93	103 84	·31175 77	150 74
·43	·22265	·44623 25	64 50	·16880 25	22 09	·22633 29	100 58	·31909 45	157 57
·44	·20960	·44704 00	66 00	·18702 08	26 64	·20951 15	96 88	·32485 66	163 83
0·45	−0·19625	−0·44718 75	+67 50	−0·20497 27	+31 33	+0·19172 21	− 92 77	+0·32898 13	−169 61
·46	·18260	·44666 00	69 00	·22261 13	36 09	·17300 58	88 24	·33141 10	174 78
·47	·16865	·44544 25	70 50	·23988 90	40 99	·15340 79	83 27	·33209 40	179 36
·48	·15440	·44352 00	72 00	·25675 68	45 96	·13297 81	77 86	·33098 47	183 21
·49	·13985	·44087 75	73 50	·27316 50	51 07	·11177 05	71 99	·32804 46	186 37
0·50	−0·12500	−0·43750 00	+75 00	−0·28906 25	+56 25	+0·08984 38	− 65 70	+0·32324 22	−188 77

* These differences have been modified; see page A 4.

† For $P_2(x)$, δ^2 is +30 throughout.

x	$P_2(x)$†	$P_3(x)$	δ^2	$P_4(x)$	δ^2	$P_5(x)$	*δ^2	$P_6(x)$	*δ^2
0·50	−0·12500	−0·43750 00	+ 75 00	−0·28906 25	+ 56 25	+0·08984 38	− 65 70	+0·32324 22	− 188 77
·51	·10985	·43337 25	76 50	·30439 75	61 57	·06726 11	58 85	·31655 37	190 30
·52	·09440	·42848 00	78 00	·31911 68	66 96	·04409 07	51 61	·30796 38	190 98
·53	·07865	·42280 75	79 50	·33316 65	72 49	+ ·02040 52	43 81	·29746 58	190 72
·54	·06260	·41634 00	81 00	·34649 13	78 09	− ·00371 75	35 56	·28506 24	189 47
0·55	−0·04625	−0·40906 25	+ 82 50	−0·35903 52	+ 83 83	−0·02819 48	− 26 75	+0·27076 62	− 187 18
·56	·02960	·40096 00	84 00	·37074 08	89 64	·05293 87	17 47	·25460 02	183 78
·57	− ·01265	·39201 75	85 50	·38155 00	95 59	·07785 63	− 7 60	·23659 85	179 25
·58	+ ·00460	·38222 00	87 00	·39140 33	101 61	·10284 90	+ 2 72	·21680 66	173 47
·59	·02215	·37155 25	88 50	·40024 05	107 77	·12781 34	13 69	·19528 23	166 45
0·60	+0·04000	−0·36000 00	+ 90 00	−0·40800 00	+114 00	−0·15264 00	+ 25 13	+0·17209 60	− 158 06
·61	·05815	·34754 75	91 50	·41461 95	120 37	·17721 42	37 20	·14733 17	148 27
·62	·07660	·33418 00	93 00	·42003 53	126 81	·20141 54	49 80	·12108 73	137 07
·63	·09535	·31988 25	94 50	·42418 30	133 39	·22511 75	63 03	·09347 51	124 28
·64	·11440	·30464 00	96 00	·42699 68	140 04	·24818 83	76 80	·06462 30	109 92
0·65	+0·13375	−0·28843 75	+ 97 50	−0·42841 02	+146 83	−0·27048 99	+ 91 23	+0·03467 47	− 93 93
·66	·15340	·27126 00	99 00	·42835 53	153 69	·29187 81	106 25	+ ·00379 03	76 20
·67	·17335	·25309 25	100 50	·42676 35	160 69	·31220 27	121 88	− ·02785 28	56 66
·68	·19360	·23392 00	102 00	·42356 48	167 76	·33130 73	138 16	·06005 91	35 29
·69	·21415	·21372 75	103 50	·41868 85	174 97	·34902 91	155 09	·09261 47	− 11 97
0·70	+0·23500	−0·19250 00	+105 00	−0·41206 25	+182 25	−0·36519 88	+ 172 67	−0·12528 63	+ 13 35
·71	·25615	·17022 25	106 50	·40361 40	189 67	·37964 06	190 87	·15782 06	40 75
·72	·27760	·14688 00	108 00	·39326 88	197 16	·39217 24	209 81	·18994 35	70 26
·73	·29935	·12245 75	109 50	·38095 20	204 79	·40260 49	229 38	·22135 96	102 07
·74	·32140	·09694 00	111 00	·36658 73	212 49	·41074 23	249 66	·25175 09	136 07
0·75	+0·34375	−0·07031 25	+112 50	−0·35009 77	+220 33	−0·41638 18	+ 270 61	−0·28077 70	+ 172 54
·76	·36640	·04256 00	114 00	·33140 48	228 24	·41931 38	292 34	·30807 32	211 41
·77	·38935	− ·01366 75	115 50	·31042 95	236 29	·41932 12	314 70	·33325 06	252 83
·78	·41260	+ ·01638 00	117 00	·28709 13	244 41	·41618 02	337 86	·35589 49	296 84
·79	·43615	·04759 75	118 50	·26130 90	252 67	·40965 93	361 70	·37556 58	343 56
0·80	+0·46000	+0·08000 00	+120 00	−0·23300 00	+261 00	−0·39952 00	+ 386 34	−0·39179 60	+ 393 05
·81	·48415	·11360 25	121 50	·20208 10	269 47	·38551 60	411 69	·40409 05	445 38
·82	·50860	·14842 00	123 00	·16846 73	278 01	·36739 37	437 83	·41192 58	500 64
·83	·53335	·18446 75	124 50	·13207 35	286 69	·34489 17	464 72	·41474 91	558 96
·84	·55840	·22176 00	126 00	·09281 28	295 44	·31774 10	492 45	·41197 71	620 35
0·85	+0·58375	+0·26031 25	+127 50	−0·05059 77	+304 33	−0·28566 44	+ 520 91	−0·40299 57	+ 684 96
·86	·60940	·30014 00	129 00	− ·00533 93	313 29	·24837 72	550 20	·38715 87	752 83
·87	·63535	·34125 75	130 50	+ ·04305 20	322 39	·20558 65	580 31	·36378 72	824 10
·88	·66160	·38368 00	132 00	·09466 72	331 56	·15699 12	611 25	·33216 84	898 79
·89	·68815	·42742 25	133 50	·14959 80	340 87	·10228 19	642 98	·29155 51	977 09
0·90	+0·71500	+0·47250 00	+135 00	+0·20793 75	+350 25	−0·04114 12	+ 675 58	−0·24116 43	+1058 97
·91	·74215	·51892 75	136 50	·26977 95	359 77	+ ·02675 69	709 04	·18017 69	1144 64
·92	·76960	·56672 00	138 00	·33521 92	369 36	·10174 70	743 35	·10773 61	1234 12
·93	·79735	·61589 25	139 50	·40435 25	379 09	·18417 22	778 52	− ·02294 69	1327 50
·94	·82540	·66646 00	141 00	·47727 67	388 89	·27438 42	814 57	+ ·07512 48	1424 95
0·95	+0·85375	+0·71843 75	+142 50	+0·55408 98	+398 83	+0·37274 36	+ 851 54	+0·18745 36	+1526 51
·96	·88240	·77184 00	144 00	·63489 12	408 84	·47962 00	889 36	·31505 52	1632 27
·97	·91135	·82668 25	145 50	·71978 10	418 99	·59539 17	928 12	·45898 74	1742 34
·98	·94060	·88298 00	147 00	·80886 07	429 21	·72044 63	967 78	·62035 12	1856 88
0·99	0·97015	0·94074 75	148 50	0·90223 25	439 57	0·85518 04	1008 38	0·80029 20	1975 86
1·00	+1·00000	+1·00000 00	+150 00	+1·00000 00	+450 00	+1·00000 00	+1049 91	+1·00000 00	+2099 54

* These differences have been modified; see page A 4.

† For $P_2(x)$, δ^2 is +30 throughout.

LEGENDRE POLYNOMIALS

x	$P_2(x)$†	$P_3(x)$	δ^2	$P_4(x)$	δ^2	$P_5(x)$	$*\delta^2$	$P_6(x)$	$*\delta^2$
1·00	1·00000	1·00000 00	150 00	1·00000 00	450 00	1·00000 00	1049 91	1·00000 00	2099 54
·01	1·03015	1·06075 25	151 50	1·10226 75	460 57	1·15532 04	1092 37	1·22071 20	2227 89
·02	1·06060	1·12302 00	153 00	1·20914 07	471 21	1·32156 63	1135 82	1·46371 18	2361 09
·03	1·09135	1·18681 75	154 50	1·32072 60	481 99	1·49917 21	1180 19	1·73033 16	2499 22
·04	1·12240	1·25216 00	156 00	1·43713 12	492 84	1·68858 16	1225 58	2·02195 29	2642 42
1·05	1·15375	1·31906 25	157 50	1·55846 48	503 83	1·89024 86	1271 89	2·34000 78	2790 70
·06	1·18540	1·38754 00	159 00	1·68483 67	514 89	2·10463 64	1319 26	2·68597 95	2944 33
·07	1·21735	1·45760 75	160 50	1·81635 75	526 09	2·33221 86	1367 59	3·06140 43	3103 23
·08	1·24960	1·52928 00	162 00	1·95313 92	537 36	2·57347 86	1416 94	3·46787 16	3267 71
·09	1·28215	1·60257 25	163 50	2·09529 45	548 77	2·82890 99	1467 31	3·90702 62	3437 68
1·10	1·31500	1·67750 00	165 00	2·24293 75	560 25	3·09901 62	1518 73	4·38056 8	3613 4
·11	1·34815	1·75407 75	166 50	2·39618 30	571 87	3·38431 17	1571 17	4·89025 5	3795 1
·12	1·38160	1·83232 00	168 00	2·55514 72	583 56	3·68532 08	1624 64	5·43790 3	3982 2
·13	1·41535	1·91224 25	169 50	2·71994 70	595 39	4·00257 83	1679 20	6·02538 5	4176 1
·14	1·44940	1·99386 00	171 00	2·89070 07	607 29	4·33662 98	1734 83	6·65463 9	4375 6
1·15	1·48375	2·07718 75	172 50	3·06752 73	619 33	4·68803 16	1791 52	7·32766 1	4581 5
·16	1·51840	2·16224 00	174 00	3·25054 72	631 44	5·05735 06	1849 28	8·04651 0	4793 9
·17	1·55335	2·24903 25	175 50	3·43988 15	643 69	5·44516 45	1908 19	8·81331 0	5012 9
·18	1·58860	2·33758 00	177 00	3·63565 27	656 01	5·85206 23	1968 16	9·63025 1	5238 3
·19	1·62415	2·42789 75	178 50	3·83798 40	668 47	6·27864 38	2029 26	10·49958 8	5470 6
1·20	1·66000	2·52000 00	180 00	4·04700 00	681 00	6·72552 0	2091 5	11·42364 4	5709 8
·21	1·69615	2·61390 25	181 50	4·26282 60	693 67	7·19331 3	2154 9	12·40481 1	5956 1
·22	1·73260	2·70962 00	183 00	4·48558 87	706 41	7·68265 7	2219 3	13·44555 2	6209 2
·23	1·76935	2·80716 75	184 50	4·71541 55	719 29	8·19419 6	2285 0	14·54839 9	6470 0
·24	1·80640	2·90656 00	186 00	4·95243 52	732 24	8·72858 7	2351 9	15·71595 9	6737 9
1·25	1·84375	3·00781 25	187 50	5·19677 73	745 33	9·28649 9	2419 8	16·95091 2	7013 5
·26	1·88140	3·11094 00	189 00	5·44857 27	758 49	9·86861 1	2489 0	18·25601 4	7296 4
·27	1·91935	3·21595 75	190 50	5·70795 30	771 79	10·47561 5	2559 3	19·63409 5	7587 6
·28	1·95760	3·32288 00	192 00	5·97505 12	785 16	11·10821 4	2630 9	21·08806 6	7886 3
·29	1·99615	3·43172 25	193 50	6·25000 10	798 67	11·76712 4	2703 8	22·62091 5	8193 2
1·30	2·03500	3·54250 00	195 00	6·53293 8	812 2	12·45307 4	2777 5	24·23571 1	8508 3
·31	2·07415	3·65522 75	196 50	6·82399 7	825 9	13·16680 2	2852 9	25·93560 5	8831 6
·32	2·11360	3·76992 00	198 00	7·12331 5	839 9	13·90906 1	2929 4	27·72383 1	9163 6
·33	2·15335	3·88659 25	199 50	7·43103 2	853 6	14·68061 6	3006 9	29·60370 8	9503 8
·34	2·19340	4·00526 00	201 00	7·74728 5	867 7	15·48224 3	3086 0	31·57863 9	9853 0
1·35	2·23375	4·12593 75	202 50	8·07221 5	881 8	16·31473 2	3166 3	33·65211 6	10211 1
·36	2·27440	4·24864 00	204 00	8·40596 3	896 1	17·17888 6	3247 6	35·82772 0	10577 7
·37	2·31535	4·37338 25	205 50	8·74867 2	910 4	18·07551 9	3330 6	38·10911 9	10954 3
·38	2·35660	4·50018 00	207 00	9·10048 5	924 8	19·00546 0	3414 5	40·50007 7	11339 3
·39	2·39815	4·62904 75	208 50	9·46154 6	939 3	19·96954 9	3500 0	43·00444 6	11734 4
1·40	2·44000	4·76000 00	210 00	9·83200 0	954 1	20·96864 0	3586 6	45·62617 6	12138 7
·41	2·48215	4·89305 25	211 50	10·21199 5	968 7	22·00360 0	3674 8	48·36931 1	12552 8
·42	2·52460	5·02822 00	213 00	10·60167 7	983 6	23·07531 0	3763 9	51·23799 2	12976 9
·43	2·56735	5·16551 75	214 50	11·00119 5	998 6	24·18466 2	3854 8	54·23646 0	13410 7
·44	2·61040	5·30496 00	216 00	11·41069 9	1013 7	25·33256 4	3946 9	57·36905 4	13854 9
1·45	2·65375	5·44656 25	217 50	11·83034 0	1028 8	26·51993 7	4040 1	60·64021 6	14309 4
·46	2·69740	5·59034 00	219 00	12·26026 9	1044 1	27·74771 4	4135 1	64·05449 1	14774 2
·47	2·74135	5·73630 75	220 50	12·70063 9	1059 4	29·01684 4	4231 0	67·61652 8	15249 9
·48	2·78560	5·88448 00	222 00	13·15160 3	1075 1	30·32828 7	4328 8	71·33108 3	15735 8
·49	2·83015	6·03487 25	223 50	13·61331 8	1090 5	31·68302 0	4427 5	75·20301 7	16233 5
1·50	2·87500	6·18750 00	225 00	14·08593 8	1106 2	33·08203 1	4528 1	79·23730 5	16741 2

* These differences have been modified; see page A 4.

† For $P_2(x)$, δ^2 is $+30$ throughout.

LEGENDRE POLYNOMIALS

x	$P_2(x)$†	$P_3(x)$	δ^2	$P_4(x)$	δ^2	$P_5(x)$	δ^2	$P_6(x)$	*δ^2
1·50	2·87500	6·18750 0	225 0	14·08593 8	1106 2	33·08203	4529	79·23730	16744
·51	2·92015	6·34237 8	226 4	14·56962 0	1122 1	34·52633	4629	83·43903	17259
·52	2·96560	6·49952 0	228 0	15·06452 3	1138 0	36·01692	4734	87·81338	17792
·53	3·01135	6·65894 2	229 6	15·57080 6	1154 0	37·55485	4837	92·36567	18335
·54	3·05740	6·82066 0	231 0	16·08862 9	1170 0	39·14115	4945	97·10133	18886
1·55	3·10375	6·98468 8	232 4	16·61815 2	1186 4	40·77690	5050	102·02588	19455
·56	3·15040	7·15104 0	234 0	17·15953 9	1202 7	42·46315	5162	107·14500	20033
·57	3·19735	7·31973 2	235 6	17·71295 3	1219 0	44·20102	5270	112·46447	20621
·58	3·24460	7·49078 0	237 0	18·27855 7	1235 6	45·99159	5383	117·99018	21225
·59	3·29215	7·66419 8	238 4	18·85651 7	1252 3	47·83599	5497	123·72816	21840
1·60	3·34000	7·84000 0	240 0	19·44700 0	1269 0	49·73536	5611	129·68456	22466
·61	3·38815	8·01820 2	241 6	20·05017 3	1285 9	51·69084	5728	135·86565	23109
·62	3·43660	8·19882 0	243 0	20·66620 5	1302 8	53·70360	5845	142·27785	23760
·63	3·48535	8·38186 8	244 4	21·29526 5	1319 8	55·77481	5966	148·92768	24427
·64	3·53440	8·56736 0	246 0	21·93752 3	1337 1	57·90568	6086	155·82181	25109
1·65	3·58375	8·75531 2	247 6	22·59315 2	1354 4	60·09741	6209	162·96705	25800
·66	3·63340	8·94574 0	249 0	23·26232 5	1371 6	62·35123	6334	170·37032	26509
·67	3·68335	9·13865 8	250 4	23·94521 4	1389 2	64·66839	6458	178·03870	27229
·68	3·73360	9·33408 0	252 0	24·64199 5	1406 8	67·05013	6586	185·97940	27964
·69	3·78415	9·53202 2	253 6	25·35284 4	1424 5	69·49773	6716	194·19977	28715
1·70	3·83500	9·73250 0	255 0	26·07793 8	1442 2	72·01249	6845	202·70731	29476
·71	3·88615	9·93552 8	256 4	26·81745 4	1460 1	74·59570	6978	211·50964	30255
·72	3·93760	10·14112 0	258 0	27·57157 1	1478 3	77·24869	7111	220·61455	31050
·73	3·98935	10·34929 2	259 6	28·34047 1	1496 2	79·97279	7247	230·02998	31855
·74	4·04140	10·56006 0	261 0	29·12433 3	1514 5	82·76936	7384	239·76399	32678
1·75	4·09375	10·77343 8	262 4	29·92334 0	1532 8	85·63977	7522	249·82481	33519
·76	4·14640	10·98944 0	264 0	30·73767 5	1551 3	88·58540	7663	260·22084	34370
·77	4·19935	11·20808 2	265 6	31·56752 3	1569 8	91·60766	7805	270·96060	35239
·78	4·25260	11·42938 0	267 0	32·41306 9	1588 4	94·70797	7948	282·05278	36125
·79	4·30615	11·65334 8	268 4	33·27449 9	1607 1	97·88776	8093	293·50624	37025
1·80	4·36000	11·88000 0	270 0	34·15200 0	1626 1	101·14848	8241	305·32998	37945
·81	4·41415	12·10935 2	271 6	35·04576 2	1644 9	104·49161	8389	317·53319	38876
·82	4·46860	12·34142 0	273 0	35·95597 3	1664 0	107·91863	8540	330·12519	39826
·83	4·52335	12·57621 8	274 4	36·88282 4	1683 2	111·43105	8691	343·11548	40794
·84	4·57840	12·81376 0	276 0	37·82650 7	1702 5	115·03038	8847	356·51374	41777
1·85	4·63375	13·05406 2	277 6	38·78721 5	1721 8	118·71818	9000	370·32980	42778
·86	4·68940	13·29714 0	279 0	39·76514 1	1741 3	122·49598	9159	384·57367	43797
·87	4·74535	13·54300 8	280 4	40·76048 0	1760 8	126·36537	9317	399·25554	44832
·88	4·80160	13·79168 0	282 0	41·77342 7	1780 7	130·32793	9479	414·38576	45885
·89	4·85815	14·04317 2	283 6	42·80418 1	1800 3	134·38528	9642	429·97486	46957
1·90	4·91500	14·29750 0	285 0	43·85293 8	1820 2	138·53905	9804	446·03356	48047
·91	4·97215	14·55467 8	286 4	44·91989 7	1840 3	142·79086	9973	462·57276	49153
·92	5·02960	14·81472 0	288 0	46·00525 9	1860 4	147·14240	10139	479·60353	50281
·93	5·08735	15·07764 2	289 6	47·10922 5	1880 6	151·59533	10310	497·13714	51423
·94	5·14540	15·34346 0	291 0	48·23199 7	1900 8	156·15136	10482	515·18502	52590
1·95	5·20375	15·61218 8	292 4	49·37377 7	1921 4	160·81221	10654	533·75883	53771
·96	5·26240	15·88384 0	294 0	50·53477 1	1941 9	165·57960	10830	552·87039	54973
·97	5·32135	16·15843 2	295 6	51·71518 4	1962 4	170·45529	11008	572·53172	56196
·98	5·38060	16·43598 0	297 0	52·91522 1	1983 2	175·44106	11186	592·75504	57436
1·99	5·44015	16·71649 8	298 4	54·13509 0	2004 1	180·53869	11368	613·55276	58699
2·00	5·50000	17·00000 0	300 0	55·37500 0	2025 0	185·75000	11550	634·93750	59979

* These differences have been modified; see page A 4.

† For $P_2(x)$, δ^2 is $+30$ throughout.

x	$P_2(x)$†	$P_3(x)$	δ^2	$P_4(x)$	δ^2	$P_5(x)$	δ^2	$P_6(x)$	δ^2
2·00	5·50000	17·00000 0	300 0	55·37500	2025	185·75000	11550	634·9375	5998
·01	5·56015	17·28650 2	301 6	56·63516	2046	191·07681	11734	656·9221	6128
·02	5·62060	17·57602 0	303 0	57·91578	2067	196·52096	11922	679·5195	6261
·03	5·68135	17·86856 8	304 4	59·21707	2089	202·08433	12110	702·7430	6394
·04	5·74240	18·16416 0	306 0	60·53925	2110	207·76880	12301	726·6059	6533
2·05	5·80375	18·46281 2	307 6	61·88253	2131	213·57628	12492	751·1221	6669
·06	5·86540	18·76454 0	309 0	63·24712	2153	219·50868	12687	776·3052	6810
·07	5·92735	19·06935 8	310 4	64·63324	2174	225·56795	12883	802·1693	6954
·08	5·98960	19·37728 0	312 0	66·04110	2197	231·75605	13082	828·7288	7099
·09	6·05215	19·68832 2	313 6	67·47093	2218	238·07497	13281	855·9982	7244
2·10	6·11500	20·00250 0	315 0	68·92294	2240	244·52670	13485	883·9920	7396
·11	6·17815	20·31982 8	316 4	70·39735	2263	251·11328	13687	912·7254	7546
·12	6·24160	20·64032 0	318 0	71·89439	2284	257·83673	13894	942·2134	7701
·13	6·30535	20·96399 2	319 6	73·41427	2307	264·69912	14102	972·4715	7856
·14	6·36940	21·29086 0	321 0	74·95722	2329	271·70253	14312	1003·5152	8016
2·15	6·43375	21·62093 8	322 4	76·52346	2353	278·84906	14525	1035·3605	8176
·16	6·49840	21·95424 0	324 0	78·11323	2373	286·14084	14738	1068·0234	8338
·17	6·56335	22·29078 2	325 6	79·72673	2398	293·58000	14955	1101·5201	8506
·18	6·62860	22·63058 0	327 0	81·36421	2420	301·16871	15173	1135·8674	8673
·19	6·69415	22·97364 8	328 4	83·02589	2443	308·90915	15393	1171·0820	8843
2·20	6·76000	23·32000 0	330 0	84·71200	2466	316·80352	15616	1207·1809	9016
·21	6·82615	23·66965 2	331 6	86·42277	2489	324·85405	15841	1244·1814	9191
·22	6·89260	24·02262 0	333 0	88·15843	2512	333·06299	16066	1282·1010	9369
·23	6·95935	24·37891 8	334 4	89·91921	2537	341·43259	16295	1320·9575	9551
·24	7·02640	24·73856 0	336 0	91·70536	2558	349·96514	16527	1360·7691	9731
2·25	7·09375	25·10156 2	337 6	93·51709	2583	358·66296	16760	1401·5538	9919
·26	7·16140	25·46794 0	339 0	95·35465	2607	367·52838	16993	1443·3304	1·0105
·27	7·22935	25·83770 8	340 4	97·21828	2630	376·56373	17231	1486·1175	1·0297
·28	7·29760	26·21088 0	342 0	99·10821	2654	385·77139	17472	1529·9343	1·0489
·29	7·36615	26·58747 2	343 6	101·02468	2679	395·15377	17711	1574·8000	1·0686
2·30	7·43500	26·96750 0	345 0	102·96794	2701	404·71326	17956	1620·7343	1·0884
·31	7·50415	27·35097 8	346 4	104·93821	2728	414·45231	18202	1667·7570	1·1086
·32	7·57360	27·73792 0	348 0	106·93576	2749	424·37338	18449	1715·8883	1·1289
·33	7·64335	28·12834 2	349 6	108·96080	2776	434·47894	18699	1765·1485	1·1497
·34	7·71340	28·52226 0	351 0	111·01360	2800	444·77149	18953	1815·5584	1·1705
2·35	7·78375	28·91968 8	352 4	113·09440	2824	455·25357	19206	1867·1388	1·1918
·36	7·85440	29·32064 0	354 0	115·20344	2849	465·92771	19464	1919·9110	1·2134
·37	7·92535	29·72513 2	355 6	117·34097	2874	476·79649	19722	1973·8966	1·2351
·38	7·99660	30·13318 0	357 0	119·50724	2899	487·86249	19984	2029·1173	1·2572
·39	8·06815	30·54479 8	358 4	121·70250	2924	499·12833	20247	2085·5952	1·2796
2·40	8·14000	30·96000 0	360 0	123·92700	2949	510·59664	20513	2143·3527	1·3023
·41	8·21215	31·37880 2	361 6	126·18099	2974	522·27008	20781	2202·4125	1·3252
·42	8·28460	31·80122 0	363 0	128·46472	2999	534·15133	21051	2262·7975	1·3484
·43	8·35735	32·22726 8	364 4	130·77844	3026	546·24309	21325	2324·5309	1·3722
·44	8·43040	32·65696 0	366 0	133·12242	3050	558·54810	21598	2387·6365	1·3958
2·45	8·50375	33·09031 2	367 6	135·49690	3077	571·06909	21876	2452·1379	1·4201
·46	8·57740	33·52734 0	369 0	137·90215	3102	583·80884	22156	2518·0594	1·4446
·47	8·65135	33·96805 8	370 4	140·33842	3127	596·77015	22438	2585·4255	1·4693
·48	8·72560	34·41248 0	372 0	142·80596	3155	609·95584	22721	2654·2609	1·4944
·49	8·80015	34·86062 2	373 6	145·30505	3180	623·36874	23008	2724·5907	1·5199
2·50	8·87500	35·31250 0	375 0	147·83594	3206	637·01172	23297	2796·4404	1·5456

† For $P_2(x)$, δ^2 is $+30$ throughout.

x	$P_2(x)$†	$P_3(x)$	δ^2	$P_4(x)$	δ^2	$P_5(x)$	δ^2	$P_6(x)$	δ^2
2·50	8·87500	35·31250 0	375 0	147·83594	3206	637·0117	2330	2796·440	1·546
·51	8·95015	35·76812 8	376 4	150·39889	3232	650·8877	2358	2869·836	1·571
·52	9·02560	36·22752 0	378 0	152·99416	3260	664·9995	2389	2944·803	1·597
·53	9·10135	36·69069 2	379 6	155·62203	3285	679·3502	2417	3021·367	1·626
·54	9·17740	37·15766 0	381 0	158·28275	3312	693·9426	2448	3099·557	1·651
2·55	9·25375	37·62843 8	382 4	160·97659	3339	708·7798	2478	3179·398	1·680
·56	9·33040	38·10304 0	384 0	163·70382	3366	723·8648	2507	3260·919	1·706
·57	9·40735	38·58148 2	385 6	166·46471	3392	739·2005	2540	3344·146	1·735
·58	9·48460	39·06378 0	387 0	169·25952	3419	754·7902	2568	3429·108	1·763
·59	9·56215	39·54994 8	388 4	172·08852	3448	770·6367	2602	3515·833	1·792
2·60	9·64000	40·04000 0	390 0	174·95200	3474	786·7434	2631	3604·350	1·821
·61	9·71815	40·53395 2	391 6	177·85022	3500	803·1132	2663	3694·688	1·850
·62	9·79660	41·03182 0	393 0	180·78344	3530	819·7493	2695	3786·876	1·880
·63	9·87535	41·53361 8	394 4	183·75196	3556	836·6549	2727	3880·944	1·911
·64	9·95440	42·03936 0	396 0	186·75604	3584	853·8332	2760	3976·923	1·940
2·65	10·03375	42·54906 2	397 6	189·79596	3612	871·2875	2792	4074·842	1·971
·66	10·11340	43·06274 0	399 0	192·87200	3640	889·0210	2824	4174·732	2·003
·67	10·19335	43·58040 8	400 4	195·98444	3668	907·0369	2858	4276·625	2·034
·68	10·27360	44·10208 0	402 0	199·13356	3695	925·3386	2891	4380·552	2·067
·69	10·35415	44·62777 2	403 6	202·31963	3724	943·9294	2925	4486·546	2·097
2·70	10·43500	45·15750 0	405 0	205·54294	3752	962·8127	2958	4594·637	2·131
·71	10·51615	45·69127 8	406 4	208·80377	3781	981·9918	2992	4704·859	2·165
·72	10·59760	46·22912 0	408 0	212·10241	3809	1001·4701	3027	4817·246	2·196
·73	10·67935	46·77104 2	409 6	215·43914	3838	1021·2511	3061	4931·829	2·232
·74	10·76140	47·31706 0	411 0	218·81425	3867	1041·3382	3097	5048·644	2·265
2·75	10·84375	47·86718 8	412 4	222·22803	3895	1061·7350	3130	5167·724	2·300
·76	10·92640	48·42144 0	414 0	225·68076	3924	1082·4448	3167	5289·104	2·334
·77	11·00935	48·97983 2	415 6	229·17273	3953	1103·4713	3203	5412·818	2·371
·78	11·09260	49·54238 0	417 0	232·70423	3983	1124·8181	3237	5538·903	2·405
·79	11·17615	50·10909 8	418 4	236·27556	4011	1146·4886	3274	5667·393	2·442
2·80	11·26000	50·68000 0	420 0	239·88700	4041	1168·4865	3310	5798·325	2·478
·81	11·34415	51·25510 2	421 6	243·53885	4071	1190·8154	3348	5931·735	2·516
·82	11·42860	51·83442 0	423 0	247·23141	4100	1213·4791	3384	6067·661	2·552
·83	11·51335	52·41796 8	424 4	250·96497	4130	1236·4812	3421	6206·139	2·591
·84	11·59840	53·00576 0	426 0	254·73983	4159	1259·8254	3459	6347·208	2·628
2·85	11·68375	53·59781 2	427 6	258·55628	4189	1283·5155	3496	6490·905	2·667
·86	11·76940	54·19414 0	429 0	262·41462	4220	1307·5552	3534	6637·269	2·706
·87	11·85535	54·79475 8	430 4	266·31516	4249	1331·9483	3573	6786·339	2·745
·88	11·94160	55·39968 0	432 0	270·25819	4279	1356·6987	3611	6938·154	2·785
·89	12·02815	56·00892 2	433 6	274·24401	4311	1381·8102	3650	7092·754	2·826
2·90	12·11500	56·62250 0	435 0	278·27294	4339	1407·2867	3690	7250·180	2·866
·91	12·20215	57·24042 8	436 4	282·34526	4372	1433·1322	3727	7410·472	2·907
·92	12·28960	57·86272 0	438 0	286·46130	4401	1459·3504	3769	7573·671	2·949
·93	12·37735	58·48939 2	439 6	290·62135	4432	1485·9455	3807	7739·819	2·991
·94	12·46540	59·12046 0	441 0	294·82572	4462	1512·9213	3849	7908·958	3·032
2·95	12·55375	59·75593 8	442 4	299·07471	4495	1540·2820	3888	8081·129	3·077
·96	12·64240	60·39584 0	444 0	303·36865	4525	1568·0315	3929	8256·377	3·119
·97	12·73135	61·04018 2	445 6	307·70784	4555	1596·1739	3971	8434·744	3·163
·98	12·82060	61·68898 0	447 0	312·09258	4588	1624·7134	4012	8616·274	3·207
2·99	12·91015	62·34224 8	448 4	316·52320	4618	1653·6541	4052	8801·011	3·252
3·00	13·00000	63·00000 0	450 0	321·00000	4650	1683·0000	4096	8989·000	3·297

† For $P_2(x)$, δ^2 is +30 throughout.

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More information

x	$P_2(x)$†	$P_3(x)$	δ^2	$P_4(x)$	δ^2	$P_5(x)$	δ^2	$P_6(x)$	δ^2
3·00	13·00000	63·00000	450	321·00000	4650	1683·0000	4096	8989·000	3·297
·01	13·09015	63·66225	452	325·52330	4682	1712·7555	4136	9180·286	3·343
·02	13·18060	64·32902	453	330·09342	4713	1742·9246	4180	9374·915	3·388
·03	13·27135	65·00032	454	334·71067	4745	1773·5117	4223	9572·932	3·436
·04	13·36240	65·67616	456	339·37537	4777	1804·5211	4265	9774·385	3·482
3·05	13·45375	66·35656	458	344·08784	4809	1835·9570	4308	9979·320	3·529
·06	13·54540	67·04154	459	348·84840	4840	1867·8237	4353	10187·784	3·578
·07	13·63735	67·73111	460	353·65736	4874	1900·1257	4396	10399·826	3·627
·08	13·72960	68·42528	462	358·51506	4905	1932·8673	4439	10615·495	3·674
·09	13·82215	69·12407	464	363·42181	4938	1966·0528	4486	10834·838	3·725
3·10	13·91500	69·82750	465	368·37794	4970	1999·6869	4529	11057·906	3·774
·11	14·00815	70·53558	466	373·38377	5003	2033·7739	4574	11284·748	3·824
·12	14·10160	71·24832	468	378·43963	5035	2068·3183	4620	11515·414	3·877
·13	14·19535	71·96574	470	383·54584	5069	2103·3247	4665	11749·957	3·926
·14	14·28940	72·68786	471	388·70274	5101	2138·7976	4711	11988·426	3·979
3·15	14·38375	73·41469	472	393·91065	5135	2174·7416	4758	12230·874	4·032
·16	14·47840	74·14624	474	399·16991	5167	2211·1614	4804	12477·354	4·083
·17	14·57335	74·88253	476	404·48084	5200	2248·0616	4851	12727·917	4·139
·18	14·66860	75·62358	477	409·84377	5235	2285·4469	4897	12982·619	4·191
·19	14·76415	76·36940	478	415·25905	5267	2323·3219	4946	13241·512	4·246
3·20	14·86000	77·12000	480	420·72700	5301	2361·6915	4993	13504·651	4·301
·21	14·95615	77·87540	482	426·24796	5335	2400·5604	5041	13772·091	4·357
·22	15·05260	78·63562	483	431·82227	5368	2439·9334	5089	14043·888	4·413
·23	15·14935	79·40067	484	437·45026	5403	2479·8153	5138	14320·098	4·468
·24	15·24640	80·17056	486	443·13228	5435	2520·2110	5187	14600·776	4·527
3·25	15·34375	80·94531	488	448·86865	5471	2561·1254	5236	14885·981	4·585
·26	15·44140	81·72494	489	454·65973	5505	2602·5634	5285	15175·771	4·641
·27	15·53935	82·50946	490	460·50586	5538	2644·5299	5336	15470·202	4·701
·28	15·63760	83·29888	492	466·40737	5574	2687·0300	5386	15769·334	4·761
·29	15·73615	84·09322	494	472·36462	5607	2730·0687	5435	16073·227	4·820
3·30	15·83500	84·89250	495	478·37794	5642	2773·6509	5488	16381·940	4·880
·31	15·93415	85·69673	496	484·44768	5678	2817·7819	5538	16695·533	4·943
·32	16·03360	86·50592	498	490·57420	5711	2862·4667	5588	17014·069	5·002
·33	16·13335	87·32009	500	496·75783	5746	2907·7103	5642	17337·607	5·065
·34	16·23340	88·13926	501	502·99892	5783	2953·5181	5693	17666·210	5·129
3·35	16·33375	88·96344	502	509·29784	5816	2999·8952	5746	17999·942	5·190
·36	16·43440	89·79264	504	515·65492	5853	3046·8469	5797	18338·864	5·256
·37	16·53535	90·62688	506	522·07053	5886	3094·3783	5852	18683·042	5·319
·38	16·63660	91·46618	507	528·54500	5924	3142·4949	5904	19032·539	5·385
·39	16·73815	92·31055	508	535·07871	5958	3191·2019	5957	19387·421	5·449
3·40	16·84000	93·16000	510	541·67200	5994	3240·5046	6013	19747·752	5·517
·41	16·94215	94·01455	512	548·32523	6030	3290·4086	6066	20113·600	5·583
·42	17·04460	94·87422	513	555·03876	6065	3340·9192	6121	20485·031	5·651
·43	17·14735	95·73902	514	561·81294	6102	3392·0419	6175	20862·113	5·717
·44	17·25040	96·60896	516	568·64814	6137	3443·7821	6231	21244·912	5·788
3·45	17·35375	97·48406	518	575·54471	6175	3496·1454	6287	21633·499	5·856
·46	17·45740	98·36434	519	582·50303	6209	3549·1374	6342	22027·942	5·926
·47	17·56135	99·24981	520	589·52344	6247	3602·7636	6398	22428·311	5·997
·48	17·66560	100·14048	522	596·60632	6283	3657·0296	6456	22834·677	6·067
·49	17·77015	101·03637	524	603·75203	6320	3711·9412	6511	23247·110	6·140
3·50	17·87500	101·93750	525	610·96094	6356	3767·5039	6570	23665·683	6·211

† For $P_2(x)$, δ^2 is $+30$ throughout.