I

STATICS

1. Four forces of 3, 4, 5 and 2 lb. respectively act along the sides of a square, following each other in the order given. The sides of the square are 4 ft. Find the magnitude of the resultant force and determine its perpendicular distance from the centre of the square.

2. A body is being dragged along level ground with uniform velocity by a rope inclined at an angle θ to the horizontal. The coefficient of friction between the ground and the body being μ, shew by means of a triangle of forces or otherwise that the tension in the rope will be least when θ is such that $\tan\theta = \mu$. If the body weighs 5 cwt. and $\mu = 0{\cdot}3$, find the minimum tension in the rope.

3. A regular hexagon $ABCDEF$ is composed of six heavy rods, each of weight W, freely jointed together and suspended from the point A, two light stiff struts BF and CE being inserted to prevent change of shape. Shew that the force in CE is equal to $0{\cdot}866W$ and that the force in BF is five times as great.

4. In a geared crab the effort required to move a load of 4 tons is 45 lb., while 35 lb. effort is required to move 2 tons. The handle is at a radius of 18 in. and the drum for the load rope is 12 in. diameter. The driver pinions, starting from the one on the handle, have 15, 20 and 25 teeth respectively, and the followers in the same order have 80, 100 and 120 teeth respectively. Make a diagrammatic sketch of the arrangement and plot the load-efficiency graph. Would you expect the machine to overhaul and if so at what load?

5. A body is pulled up a plane, whose angle of inclination to the horizontal is α, by a force parallel to the slope. If the

angle of friction between the body and the plane is ϕ, shew that the efficiency of the system is given by the expression

$$\frac{\tan\alpha}{\tan\alpha + \tan\phi}.$$

What horizontal force is required to push a body weighing 100 lb. up the plane, if ϕ is 20° and α is 30°?

6. The figure shews a temporary luggage lift which is working up and down a vertical column. The weight of the

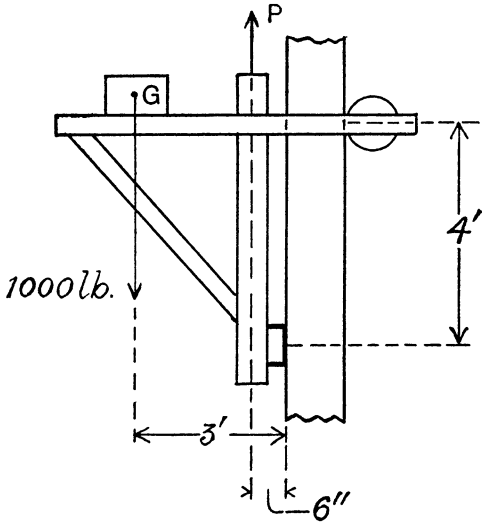

Fig. 1.

carriage and load is 1000 lb., its centre of gravity being at G as shewn. Friction at the wheel is negligible. The coefficient of friction between the slides and the post is 0·25. Find the force P necessary to raise the lift steadily. Make a sketch shewing in magnitude and direction all the forces acting on the lift.

7. Two equal uniform bars of length 16 cm. are freely hinged together. They are placed in a symmetrical position on a rough horizontal cylinder of radius 9 cm. so that each

bar makes an angle of 30° with the horizontal. Find the minimum coefficient of friction between the bars and the cylinder that will allow of equilibrium.

8. A motor-car weighs 2 tons, the distance between its axles being 10 ft. The weight on the back wheels is 25 cwt. when on the level and this increases to 26 cwt. when the front wheels are jacked up 10 in. vertically. Find the height of the centre of gravity of the car above the ground.

9. The figure represents a heavy block of metal A supported by a helical spring and guided to move vertically. The weight of A is 50 lb., the spring is such that a pull of 50 lb. elongates it 1 in. and the force of friction exerted by the guides resisting up-and-down motion is 10 lb. The weight A is pulled down until the total elongation of the spring is 2 in. and it is then released. Prove that it will rise through a distance of 1·6 in. and then drop 0·8 in. before coming permanently to rest.

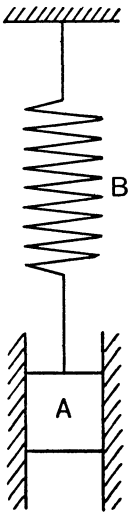

Fig. 2.

10. A non-uniform straight bar has a string 150 cm. long fixed to it at two points A and B which are 1 metre apart. This string is placed over a smooth peg and when in equilibrium the peg divides the string into two portions 60 cm. and 90 cm. Find the angle that the bar makes with the horizontal and the situation of its centre of gravity between A and B.

11. A four-wheeled truck has a total distance between its axles of $a+b$. When on a level track its centre of gravity is a distance h above the rails and the centre of gravity is then a horizontal distance a from the front axle and b from the rear axle. If θ is the greatest slope upon which it can stand when both pairs of wheels are locked, find the greatest slope upon which it can stand when only the upper wheels are locked.

12. A camp stool consists of two light bars *ABC*, *DBE* pivoted at *B*, such that $BA = BD = BE = BC$, and at right angles to each other. A weight is placed in the middle of the canvas seat at *F*, the angle *AFD* being then 140°. State, giving reasons, whether the feet will tend to approach or recede from each other, and if in this position the state of limiting equilibrium has been reached, find the coefficient of friction between the feet and the ground.

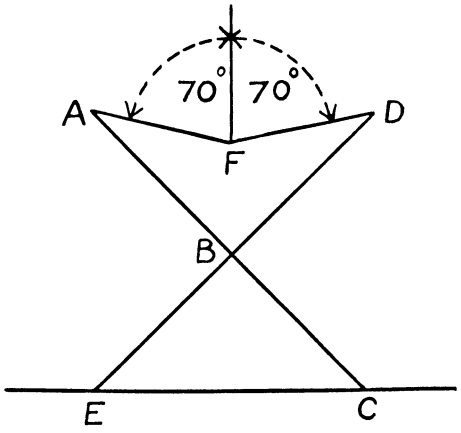

Fig. 3.

13. Two uniform planks of equal length weighing 40 lb. and 50 lb. respectively stand upon a smooth horizontal plane with their upper ends hinged together and are prevented from slipping by a rope joining their feet. If the angle between the planks at the hinge is 60°, what is the tension in the rope and the reaction at the hinge?

14. A simple bridge consists of two similar and uniform girders *AC* and *CB* hinged together at *C* and to their abutments at *A* and *B*. The span *AB* is 12 ft. and the rise of *C* above *AB* is 3 ft. Each girder weighs 1 cwt. and a vertical force of 8 cwt. is applied at *C*. Find

(i) The distance the central hinge will descend if the abutments move 1 in. apart horizontally.

(ii) The magnitude of the horizontal component of the reactions at A and B.

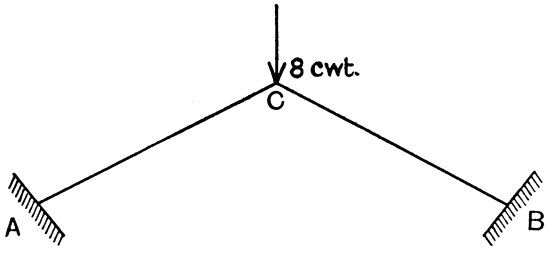

Fig. 4.

15. In the epicyclic train of wheels shewn in the diagram the arm D is capable of rotating about the centre of A. Shew that if A and C have the same number of teeth, C will not rotate on its spindle if A is held fixed and the arm D rotates. If A has 20 teeth and B has 10 teeth, find how many times B rotates for each revolution of D.

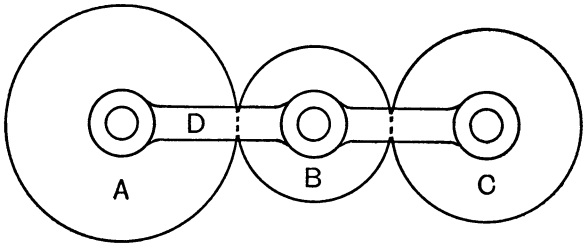

Fig. 5.

16. The countershaft driving a lathe runs at a speed of 180 revolutions per minute. The largest step on the speed cone is 10 in. diameter and the smallest is 4 in. diameter. Each pair of wheels in the back gear has teeth of number 15 and 45 respectively. If the belt is running on the smallest step on the countershaft cone and the back gear is "in", what is the surface speed in feet per min. of a piece of work 7 in. in diameter? What is the greatest possible speed?

17. The figure gives dimensioned side and front elevations of a pair of dockyard sheer-legs, *AB* and *AC*, the feet *B* and *C* being 20 ft. apart. They are supported by a guy-rope *AD* and are carrying a load of 15 tons suspended from *A*. Determine the forces in *AB*, *AC* and *AD*.

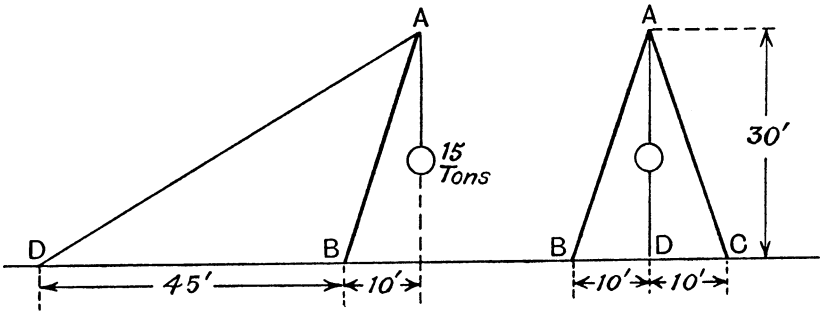

Fig. 6.

18. Six equal rigid bars, whose weight may be neglected, are freely joined at their ends so as to form a regular tetrahedron *ABCD*. This body is suspended from *A* and three equal weights *W* are hung from *B*, *C* and *D* respectively. Find the forces thereby produced in all the bars, stating which are in tension and which are in compression.

19. A trunk of pine-wood is 40 ft. long. Its cross-section is circular and it tapers uniformly, its diameter being 3 ft. at one end and 2 ft. at the other. Find the weight of the trunk and the distance of its centre of gravity from the heavier end. Pine weighs 28 lb. per cubic foot.

20. The roadway of a bridge consists of two light girders *AB* and *BC* hinged together at *B* and to the abutments at *A* and *C*. The roadway is supported by vertical tie rods *FD* and *GE* which are themselves supported by ties *HF*, *FG* and *GK* as shewn. Assuming that the reactions at *A*, *B* and *C* are

vertical, find the tension set up in *FD* and *GE* by a load of 1000 lb. at *P* 12 ft. from *A*.

Find also the tension in *FH*.

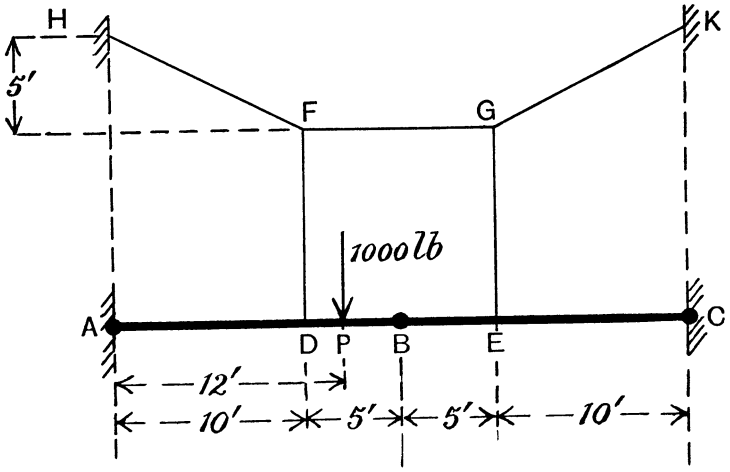

Fig. 7.

21. A bridge girder weighing 3 tons is being hauled across a river in the manner shewn, one winch on the far bank hauling the rope over a derrick tackle with a force *F*, while a winch on the near bank exerts a horizontal force *P* on the

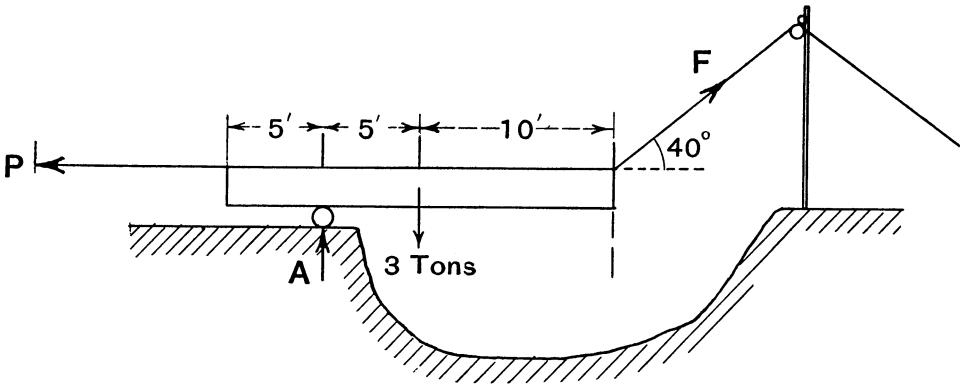

Fig. 8.

preventer tackle. The girder rests on a frictionless roller at A whose reaction is vertical. In the position shewn the girder is in equilibrium and horizontal. Find the values of F and P. Find also the force on the roller.

22. Find the approximate velocity ratio of a Single Spanish Burton consisting of two single blocks and a hook, the rope being rove as shewn. What would happen if the rope were continuous and passed through a pulley at A instead of having two ends, both connected to the hook?

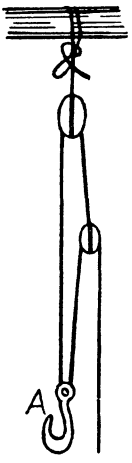

Fig. 9.

The efficiency of a tackle when lifting 3 tons is 70 per cent. and the velocity ratio is 80. Calculate

(1) The effort required to move this load.

(2) The part of the effort that is then being used in overcoming friction.

(3) The effort that would have to be exerted to prevent the 3 tons from "taking charge".

23. A retaining wall has the form indicated (Fig. 10), the necessary dimensions being given. The wall weighs 40 tons per foot run. A continuous column of water has got between the wall and its backing, shewn shaded from A to D. Find the magnitude of the resultant thrust on the base DE estimated per foot run of wall, and its direction with the vertical.

24. Fig. 11 shews two strings passing over two smooth pulleys carrying weights of 3 lb. and 4 lb. A third weight W is suspended from the point joining the two strings and a horizontal force F is applied at this point. If the two strings each make an angle of 50° with the vertical, calculate the values of F and W and check by graphical means.

25. A heavy cylinder of weight W lb. rests on a rough plane and against a rough wall, the coefficient of friction between the cylinder and both ground and wall being 0·2.

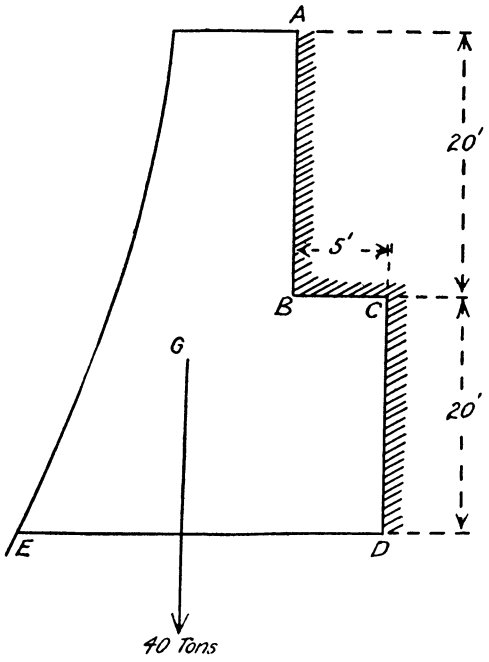

Fig. 10.

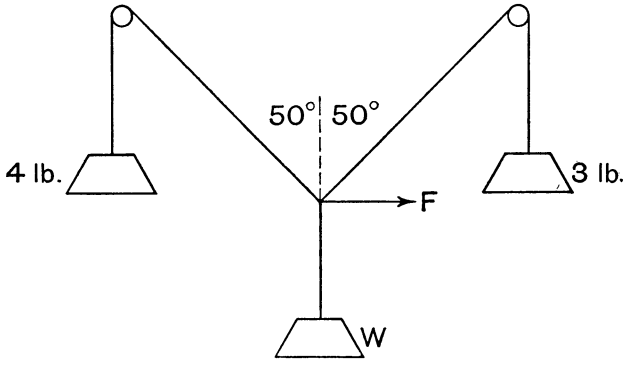

Fig. 11.

It is pushed horizontally at its highest point with a certain force, F lb., until movement just occurs. Shew that this is the case when $F = \frac{3}{11} W$.

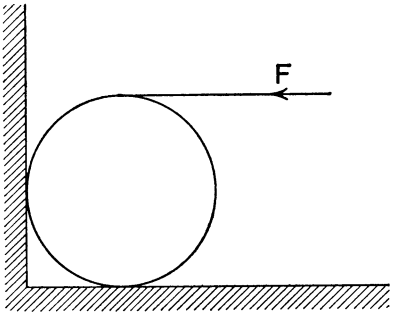

Fig. 12.

26. A tripod is formed of three light bars, each of length l, and from the apex of the tripod a weight W is suspended. The feet of the tripod rest on the ground at the corners of an equilateral triangle of side a. Find the thrust in each rod.

If the feet are just about to slip, what is the coefficient of friction between the feet and the ground?

27. Four equal uniform rods each of weight 5 kilogrammes are hinged together to form a rhombus $ABCD$. The system is suspended from A and is prevented from collapsing by a light rod interposed between B and D and of such a length that the angle at A is 60°. Find the thrust in this rod.

28. A horse has to draw a sledge weighing 2000 lb. up an incline of 1 in 12. The coefficient of friction between snow and runners is 0·05 and the horse can exert a pull of 212 lb. wt. Find the least possible inclination of its path to the line of greatest slope.

29. A wire of uniform section and weight w per unit length is stretched between two points distant l apart at the same