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AN INTRODUCTION TO THE THEORY OF CONTROL IN MECHANICAL ENGINEERING

AN INTRODUCTION TO THE THEORY OF CONTROL IN MECHANICAL ENGINEERING

BY

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To

PROFESSOR J. F. BAKER, O.B.E., M.A., SC.D., M.INST.C.E.

WITHOUT WHOSE INTEREST AND ENCOURAGEMENT
THIS BOOK WOULD NOT HAVE BEEN WRITTEN

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* The graph is available for download from www.cambridge.org/9781316611845

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PREFACE

To control and check disordered powers
WORDSWORTH, *The Prelude*

A control system is normally used for the control of relatively high power by means of low-power signals. During the past two decades what may conveniently be described as the *Theory of control* has come to embrace a considerable body of new facts and ideas concerning the design and performance of such systems.

Automatic controls are very extensively used to-day: to mention but a few applications, there are chemical process controls for temperature and flow, there are industrial regulators for purposes such as registration, synchronization, speed governing and voltage control, and there are aeronautical and military applications such as the automatic pilot and fire-control systems; the field is rapidly widening.

Every engineer must inevitably use automatic controls, and an intelligent appreciation of their design and operation would undoubtedly help him to treat them correctly and so obtain from them the best possible service. The theory of control can also be of great value to the student: by touching upon so many branches of applied engineering science it acts as a unifying influence, which is welcome in these days of excessive specialization; it also introduces to him important ideas which he might not otherwise meet.

There seems, nevertheless, to be a regrettable tendency amongst some practising engineers to regard the mathematical analysis of control systems with distrust; they prefer 'common sense and experience' to a careful examination, although practicable methods of making such a study have now been available for some years. This attitude is perfectly reasonable provided that only a low standard of performance is demanded of the controls used; but as soon as we wish to increase their efficiency and accuracy or go a stage further and attempt to find the best possible controller for a given job, exact analysis becomes essential. The apparent obscurity of control theory can be attributed largely to the early specialist control engineers who succeeded in surrounding much of their work with an aura of mystery, enhanced by an excessively illogical and confusing nomenclature: not only do different groups of control engineers call the same thing by different names, but they also use the same name (as, for example, 'reset') for two or more different things.*

This unsatisfactory state of affairs is partly due to the fact that until recently information on the subject has been limited to numerous articles in the technical press and papers read before various institutions; unfortunately, much of this literature has appeared in journals which the mechanical or chemical engineer in Britain would not normally see. Text-books on servo-mechanisms and feed-back amplifier theory have been published in America, but since no book devoted to the theory of control has yet appeared in this country, this one has been written as an attempt to fill the gap.

The book is an introductory account and intended to be perfectly intelligible to any engineer or physicist and not only to the specialist. The early chapters, which

* See *Nature*, **163**, 925 (1949).

are mainly descriptive, explain the principles which underlie the operation of all control systems; sufficient elementary mathematics is then introduced to estimate the performance of any simple system. In the last three chapters rather more advanced techniques are used to give an account of the methods universally employed by control engineers to-day; an attempt is thus made to enable the reader to understand the terminology, notation and ideas which appear in all modern contributions to the theory.

We shall be concerned rather with the methods, principles and philosophy of control than with specific applications. No attempt has been made to include those facts and figures which would be necessary if we were to provide a handbook for the technician or the designer. It is hoped, nevertheless, that even for them the presentation and organization of the material may assist in clarifying and co-ordinating their ideas; the extensive list of references in the bibliography should also prove of value to them. In order to make the book more useful to students, a number of examples,* including questions set in recent examination papers, have been given at the end of each chapter; some of these serve, in addition, to amplify the arguments without risk of overloading the book. A few of the minor points in the theory have been discussed in some detail but not with any intention of laying particular emphasis on the matters so treated; it is done deliberately, in the belief that if the treatment were confined to vague generalities and broad principles the book would have proved tedious both to read and to write; these diversions are intended therefore to leaven the whole by indicating some of the interesting sidelines into which the theory can lead and the methods which can appropriately be adopted to pursue them.

Although the field of application of controls is so wide, we shall show that when the basic principles have been grasped, the same methods of analysis can be applied to systems that are superficially very diverse. It can be shown, for example, that even economic controls and the control of infectious diseases are subject to the same laws and equations as are mechanical systems. With so great a choice, naturally no attempt can be made to study even a representative selection of practical controllers. The most we can do is to take as examples a few familiar types which illustrate the argument without introducing too many confusing side issues. If a particular control is not mentioned, let it not be supposed that the theoretical basis is too narrow to include it; rather is it hoped that the reader may be enabled to classify for himself any special control with which he is concerned, for he can then draw on the fund of information which has been accumulated during the past decade. In the majority of the examples chosen we have shown a preference for mechanical rather than electrical systems, largely because the latter have tended to monopolize the present literature.

In practice, the designer of a control system will often have only a very approximate knowledge of the various time constants and other parameters involved in it. He cannot, therefore, expect to lay out a complete new system from theoretical considerations alone. The analytical approach is none the less important for two

* Several are based, by kind permission, on those in the book by G. S. Brown and D. P. Campbell (**17**).

main reasons. First, his equations will enable the designer to decide which parameters it is most profitable to adjust in order to improve the performance and, secondly, he can obtain from the theoretical treatment some indication of the best performance which is available from any given type of control; the analysis thus provides him with a goal at which to aim and helps him to estimate what degree of refinement is economically desirable.

It follows that our aims in studying the theory of control are threefold: the first is to gain a clear understanding of the mode of operation of such controls; the second is to enable us to estimate the performance of a proposed control system; and the third is to gain some insight into the design of a control which will attain a specified performance. We shall find that if, in general, one attempts to improve the performance of a system by increasing the sensitivity of the controller or by other means, such action will normally increase the tendency towards continuous oscillation of the system as a whole; one of the main problems, therefore, is to determine the best methods of improving performance while maintaining an acceptable standard of stability.

The arrangement of this book is influenced by the considerations above. In the first chapter we introduce the basic concepts of control and explain the various systems of classification which can be adopted, indicating the value of each; this chapter serves also to introduce some of the special terms and nomenclature now in general use. Before giving quantitative expression to these ideas it is necessary to consider in detail some of the essential components of control systems; among the simplest of these are hydraulic components, measuring elements, data transmission links and mechanical power amplifiers. The second chapter is devoted to a study of such components; the concept of a transfer function, which characterizes their behaviour, is introduced and its significance explained. We shall also derive the transfer functions of more complex components, such as continuously variable speed drives, which constitute complete open-chain control systems in themselves.

It is now possible, in the third chapter, to find equations of motion for some typical control systems; these are called their governing equations. The characteristics of the human operator and certain non-linear effects are also examined qualitatively at this stage. We have thus thoroughly prepared the ground for the quantitative analysis of performance which is given for some simple systems in Chapter IV. To make such a study it is necessary to solve the governing equation for some specified type of input or disturbance. Two general types of input are important: the first is a sudden change in the control setting and the second is a continuous simple-harmonic variation. The solution for the first kind of input describes the behaviour of the system immediately after any disturbance; it is called the transient response. When the input varies harmonically, on the other hand, the output also oscillates but with an amplitude and phase which are, in general, different from those of the input; the harmonic response, as it is called, though apparently of less practical significance than the transient response, is of great importance in design. We also introduce in this chapter the new ideas of feed-back and order of control.

The next step is to widen the scope of our investigations, which have been confined, hitherto, to systems having relatively simple governing equations; in

Chapter V we make a more general approach to servo theory. We investigate there all the possible kinds of transfer function and show how they can be used to write down the governing equations of complex systems with ease; the loop and overall transfer functions are also discussed. The importance of this chapter lies in the broadening of outlook and the adoption of a notation appropriate to the new approach.

For the first five chapters no more mathematics is required than the ability to solve simple linear differential equations with constant coefficients; even this is unnecessary if the reader is willing merely to check the correctness of the solutions which are given. In the remaining chapters some further mathematical methods are introduced: the first of these is the use of complex numbers. They are such a convenience in control theory that we would be unduly handicapped if we were to attempt to avoid them. Since some readers may be unfamiliar with the properties of complex numbers or possibly a little distrustful of their own powers to apply them, we have collected together in Appendix A all that it is necessary to know about them for our purposes.

Another mathematical tool employed in the final chapters is the use of the Laplace transformation for solving more complicated governing equations. We use the method reluctantly, for some readers may find it difficult, although its main object is, in fact, to simplify the solution of differential equations by reducing it to a problem in pure algebra. Its introduction is justified, however, by the fact that, as it greatly assists the solution of equations of high order, it is almost invariably used by writers on control theory, and an appreciation of the method is thus indispensable to anybody wishing to follow the current literature. In Appendix B we briefly explain the method.

Analytical methods are used in Chapter VI to determine the transient solution to a general type of governing equation and also to determine a criterion of stability for more complex equations than could be treated by the methods of the earlier chapters. For the purposes of design the analytical approach is generally too cumbersome; instead, various graphical procedures depending on a plot of the harmonic response locus (called the Nyquist diagram) and its developments are now widely employed. These are examined in Chapter VII. The transient response and the harmonic response approaches are complementary; we shall emphasize the close relationship which exists between the two by deriving the Nyquist stability criterion from a transient approach. In conclusion, we give a short historical summary of the main landmarks in the development of control theory.

The principal symbols and notation which we employ are listed, but to prevent any possible confusion one or two points are worth special mention here. We use the symbol D as an abbreviation for the differential operator d/dt, so that Dx means dx/dt; when manipulating equations D can be treated exactly like an algebraic quantity. The symbol p is reserved for the parameter in the Laplace transformation; it is not an operator but a complex number.* Another notation we shall employ may be unfamiliar to some readers: $\theta(t)$ signifies 'the variable θ, expressed as

* In America s is commonly used for the Laplace variable, as $1/p$ was used by Heaviside to signify the operation of integration.

a function of time'. For example, we might have $\theta(t) = A \sin \omega t$, if θ varies harmonically with time. $\theta(p)$ means the Laplace transform of $\theta(t)$. It is frequently convenient to have a brief notation for 'a function Q of the operator D' which we shall denote by $Q(D)$; $Q(i\omega)$ or $Q(p)$ then signify the same function Q of $i\omega$ or of p (i.e. D is replaced by $i\omega$ or p). Thus if $Q(D)$ is $1/(1+TD)$, then $Q(p) = 1/(1+Tp)$ and $Q(i\omega) = 1/(1+i\omega T)$.

A word of warning is advisable concerning some of the graphs; these have been drawn for illustrative purposes only and may not always be exactly accurate; the reader desiring to obtain *precise* information should refer to the original references given.

Any originality which this book possesses must lie in the presentation rather than in any specific contributions by the author to the field which he surveys; methods and notation have been adopted throughout from the published literature, but reference is made to all the major sources. A debt should also be acknowledged to the course of lectures on Automatic Control given in 1946 at the Northampton Polytechnic Institute by Professor A. Porter and Dr L. Jofeh. The author is most grateful to Professor Porter, who read the first draft of the manuscript and made many helpful comments, and to Dr G. D. S. MacLellan, who read the first proofs and checked many of the examples, but neither of these must be held responsible for any errors or heresies which may remain. He is also greatly indebted to two graduate students, J. H. Horlock and F. N. Kirby, whose suggestions have enabled him to clarify a number of obscurities in the exposition. Thanks are due, finally, to the Syndics of the Cambridge University Press for permission to use blocks and problems from questions set for the Mechanical Sciences Tripos and for their care and courtesy in all matters connected with the publication of this book.

R. H. M.

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SYMBOLS AND NOTATION

		Defined in Section
	$D \equiv \frac{d}{dt}, \quad D^n \equiv \frac{d^n}{dt^n}$	*Preface*
θ_i, θ_o	Input and output signals for complete system	2
$\theta = \theta_i - \theta_o$	Error signal	2
θ_1, θ_2	Input and output signals for a single element	6
θ_s, θ_{st}	Steady state and static errors	18
K, k, m, etc.	Loop and element gain factors and sensitivities	6
T_1, T_2, etc.	Time constants	6
$\omega/2\pi$	Applied frequency in cycles per sec.	6
$\omega_n/2\pi$	Undamped natural frequency	19
$\omega_r/2\pi$	Resonant frequency (for maximum magnification)	19
c	Damping ratio (actual damping/critical damping)	6
$d = \omega/\omega_n$	Frequency ratio	19
$L(D), S(D)$, etc.	Element transfer functions	22
$\theta_o = Y(D)\theta$	The governing equation	23
$Y(D) = KG(D)$	Loop transfer function	23
$Y(i\omega) = Ae^{-i\psi}$	The harmonic response function	24
A	Loop magnification	24
ψ	Loop phase lag	24
$Y_o(D) = \theta_o/\theta_i$	Overall transfer function	23
$M = \lvert Y_o(i\omega) \rvert$	Overall magnification	24
$\phi = \text{Arg}\, Y_o(i\omega)$	Overall phase lag	24
$Z(D) = 1 + Y(D) = 0$	The characteristic equation	25
$a = \alpha + i\omega$	Principal root of characteristic equation	25
s	Order of control	18
$\mathscr{L}$	The Laplace operator	*Appendix B*
p	Laplace variable	*Appendix B*
$\theta(t)$	Error expressed as a function of time	25
$\mathscr{L}\{\theta(t)\} = \theta(p)$	Laplace transform of error	*Appendix B*
$W(t)$	The weighting function (the unit impulse response)	25
$w = \omega T$	Dimensionless frequency parameter	28
λ	Attenuation in decibels	30
μ	Frequency change in octaves	30

