CHAPTER I

INTRODUCTION. CORPUSCULAR THEORY. UNDULATORY THEORY

SECTION I. ANCIENT SPECULATIONS. DESCARTES. FERMAT

1. A complete theory of optics has to furnish an adequate account, not merely of the nature of light but also of the mode and mechanism of its propagation, as well as the nature of the medium in which the propagation takes place. And we shall see, as we pass in review, in historical order, the various theories that have been proposed, that our knowledge on these points is after all extremely limited.

2. This, indeed, is what is to be expected—illustrating, we may remark in passing, the limit and scope of scientific inquiry in general. For, although the phenomena with which, to confine ourselves to optical investigations alone, we have to deal are simple and well-known, and although we can formulate the laws governing them—like any other body of scientific laws—which state (as Karl Pearson has put it), "in conceptual short-hand, that routine of our perceptions, which forms for us the totality of the phenomena," to which they refer, when we come to inquire into the intimate nature of the processes, associated with these laws, we are confronted with insuperable difficulties. These are so subtle and deep-seated, that they will probably always elude our grasp. We are, therefore, reduced to preparing models, that shall approximate to the actual, as far as possible, and our task consists really in improving these models, more and more, so that they may more and more nearly approximate to the actual. Thus, of the mode of propagation of light and the nature of the medium which takes part in its propagation, we, with our limitations, can never have any direct knowledge.

We are, therefore, driven to reason by analogy and construct a picture which shall be as near a representation of the actual as we can make it. At first, only the outlines are noted; then, the details are gradually filled in, but the picture can never fully represent the actual, unless and until our powers of perception are so sharpened that we are able to take cognizance of the order of quantities which we now call infinitesimal.

3. Although speculations as to the nature of light are of remote antiquity, anything like a clear notion as to the nature of light belongs only to comparatively modern times.

4. Among the most ancient recorded speculations on the subject of light seem to have been those of the Hindus. In the *Nyaya Vasya**, there is a discussion, from the point of view of the Nyaya Philosophy, on the theory of the mirage:

"In the summer, the rays of the sun coming in contact with the heat issuing from the earth vibrate upwards and downwards and gradually reaching the distant observer's eye produce a false impression of water, by an incorrect association of sight and object."

It is clear that the writer of *Nyaya Vasya* was interested only in the metaphysics of the subject. For, with regard to this discussion, the commentary *Bartika* points out that in this case, although the solar rays are certainly there, as well as the vibrations, what is at fault is the impression of water—*the mistaken association of the phenomenon with water*, etc. All the same, it is evident from this, that the above was the generally accepted theory of the mirage at the time the *Vasya* was written and must have been treated of in formal books on Physics, which seem to have been lost.

* The Indian schools of Philosophy (called *Darsanas*) are classed under two main heads. One class, called the '*Astika*' (the believing), believes in the authority of the *Vedas*, and the other called the '*Nastika*' (non-believing) does not recognize this authority. Each of these is divided into six schools: *Nyaya*, *Vaisesika*, *Sankhya*, *Patanjal*, *Purva Mimansa* and *Uttara Mimansa* (or *Vedanta*) belong to the first or the Astika group, while *Charvak*, the Jain and four Buddhistic schools belong to the latter. *Nyaya Sutra*, of which the reputed author was *Gautoma*, the founder of the Nyaya school, deals with canons of correct reasoning. *Nyaya Vasya* is a commentary (by *Vatsayana*) on *Nyaya Sutra* and *Bartika* (by *Udyotaka*) is a commentary on the latter.

5. In the same work (*Nyaya Vasya*), the following explanation occurs of the formation of images by reflection:

"The 'eye rays,' striking against mirrors, return and come in contact with the face [to which the eye belongs, i.e., of the observer]. From this contact is derived the knowledge of the face. The '*rupa*,' i.e., form or colour of the mirror, helps in producing this knowledge."

The commentary *Bartika* thus amplifies the theory:

"The 'eye rays' rebound at mirrors, water, etc. On rebounding, they come in contact with the face. As the extremities (fore-part) of rays come into relation with the face, the face appears to be in front. This is the law relating to knowledge acquired by means of the eye, viz., the object which comes into relation with the extremities (fore-part) of the 'eye rays' is made out by this knowledge to be *in front*, e.g., the knowledge of the face of a man standing in front [of the observer]."

6. Before the days of *Nyaya Sutra*, a theory prevailed that every object emits rays*. The author of *Nyaya Sutra* points out that in that case, stones, etc. should be capable of being seen at night, and the author of *Nyaya Vasya* further argues that one could not imagine such a thing (viz., rays emitted by stones); whereas 'eye rays' are imaginable!

7 On the theory of transparency, we are told, in *Nyaya Sutra*, that "the 'eye rays' are not turned back by (i.e., are allowed to pass through) glass, etc. This is how objects placed beyond glass, etc. can come into relation with 'eye rays' and are seen. Opaque bodies like walls turn off 'eye rays' and therefore bodies cannot be seen through them."

8. As to the eye rays, it is stated in *Nyaya Kandali*, a treatise of the *Vaisesika* school, that "their form cannot be seen nor can they be touched, but they go to a distance and produce the knowledge of bodies, if nothing stands in the way." Moreover, the eye rays, like solar rays, are to be regarded (according to a commentary on the *Vedanta Parivasa* of the *Vedanta* school) as "*transparent bodies* [Art. 10] and may therefore *have rapid motion*."

* Cf. Pythagoras [Art. 9], who is said to have received his early training in India.

9. With regard to these extracts*, it should be noted that they only incidentally occur as illustrations of principles discussed in Hindu Philosophy. They have, therefore, no further interest from our present point of view, beyond the fact that they show that optical speculations in India dated beyond the days of the *Nyaya Sutra*. It is interesting, moreover, to note that similar speculations appear in the first systematic European work on light, that of Empedocles (444 B.C.). According to him, light consists of particles, projected from luminous bodies, and a vision is the effect of these bodies and a *visual influence*, emitted by the eye itself, although Pythagoras and his followers had previously maintained that vision was caused by particles continually projected from the surfaces of objects into the pupil of the eye. If, therefore, as is maintained by some, *Nyaya Sutra* was written between 500 B.C. and 200 B.C., it would follow that contemporary optical ideas in Greece and India proceeded on similar lines.

10. The fallacy of the theory of a 'visual influence' was discussed by Aristotle (350 B.C.), who argued that if a visual influence was emitted by the eye, we should be able to see in the dark. He considered it more probable that light consisted in an impulse, propagated through a continuous medium, rather than an emanation of distinct particles. Light, according to him, is the action of a transparent substance [Art. 8] and if there were absolutely no medium between the eye and any visible object, it would be absolutely impossible that we should see it. The meaning of the latter part of the argument seems to be that if, between the luminous object and the eye receiving the impression, there did not exist something endued with the physical property that makes it capable of transmitting the influence (whatever its nature may be) emitted by the luminous object, that influence could never reach the eye. This is, also, in effect the postulate of modern science.

11. From this time up to that of Descartes, optical discoveries related mainly to the two fundamental phenomena

* I am indebted to Mahamahopadhyaya Gurucharan Tarkadarsana-tirtha, Professor of Nyaya, Sanskrit College, Calcutta, for these extracts.

of reflection and refraction. Archimedes was evidently acquainted with the property of burning mirrors and seems to have made some experimental investigations on this subject, while Vitellio, a Pole, developed a mathematical theory of optics*. Roger Bacon is said to have invented the magic lantern, and is regarded by some as the first to have invented the telescope also. But the first person who is certainly known to have made a telescope was Janson, a Dutchman, whose son, by accidentally placing a concave and a convex spectacle glass at a short distance from each other, observed the increased apparent magnitude of an object seen through them. It was to Galileo, however, that we owe the first construction and use of such a telescope (the Galilean) for astronomical observations. And it was to him also that we owe its theory. Galileo states†, in his *Nuncius Sidereus*, that happening to hear that a Belgian had invented a perspective instrument by means of which distant objects appeared nearer and larger, he discovered its construction by considering the effects of refraction. Finally, Kepler worked out the true theory of the Astronomical telescope (a combination of convex lenses), made some experiments on the nature of coloured bodies and experimentally verified the formation of inverted images on the retina of the eye.

12. Descartes published the law of refraction, originally discovered by Snell, and deduced the law from theory: or rather an analogy. "When rays meet ponderable bodies, they are liable to be deflected or stopped in the same way as the motion of a ball or stone impinging on another body."

Let a ball‡ thrown from A meet at B a cloth CBE, so weak that the ball is able to break through it and pass beyond, but with its resultant velocity reduced in some definite proportion, say $1 : k$.

Then, if BI = length, measured on the refracted ray $= AB$, the time to describe $BI = k \times$ the time to describe AB.

But the component velocity parallel to the cloth is unaffected.

* Lectures by Th. Young. † *Ency. Brit.*
‡ Whittaker, *A History of Theories of the Ether and Electricity.*

$\therefore$ BE = projection of BI, on the cloth,
$= k \,.\, BC$, where BC is the projection of AB.

$\therefore$ if $i = \angle CAB$,
$r = \angle BIE$,

$$\sin r = \frac{BE}{BI} = k \,.\, \frac{CB}{BA} = k \,.\, \sin i,$$

or the sines of the angles of incidence and refraction are in a constant ratio*.

13. He also propounded a theory of light. On this theory, light consists in pressure transmitted instantaneously through a medium, infinitely elastic, while colour, according to him, is due to a rotatory motion of the particles of the medium, the particles which rotate most rapidly giving the sensation of red, etc. But Descartes supposed [Art. 12] light to pass more quickly through a denser than a rarer medium, while Fermat, maintaining the contrary view, enunciated the principle of swiftest propagation of light.

14. Fermat's argument was metaphysical—"nature works by the shortest route." The result, however, is remarkably correct.

For, this law [Fermat's law of swiftest propagation of light] states that $\delta \int dt = 0$, where t is the time of propagation of light between two given points and δ is the operator of the calculus of variation. Now, since, on the wave theory, μ, the index of refraction, varies inversely as the velocity of propagation, the above is obviously the same as $\delta \int \mu ds = 0$, an equation which, as we shall see, analytically embodies a complete kinematical statement of all optical phenomena.

15. Thus, $$\delta \int \mu ds = \int \delta\mu ds + \int \mu d\delta s$$

$$= \int \left(\frac{\partial\mu}{\partial x}\delta x + \frac{\partial\mu}{\partial y}\delta y + \frac{\partial\mu}{\partial z}\delta z \right) ds + \int \mu \left(\frac{dx}{ds} d\delta x + \ldots + \ldots \right)$$

since $$ds^2 = dx^2 + dy^2 + dz^2$$

and therefore $$d\delta s = \left(\frac{dx}{ds} d\delta x + \ldots + \ldots \right).$$

* If v, v' be the velocities in the two media, $v'/v = 1/k$, and $v' \sin r = v \sin i$.

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Therefore, integrating the second term by parts, we have

$$\left[\mu \frac{dx}{ds}\right]_1^2 \delta x + \ldots + \int \left[\frac{\partial \mu}{\partial x} - \frac{d}{ds}\left(\mu \frac{dx}{ds}\right)\right] \delta x ds + \ldots + \ldots = 0$$

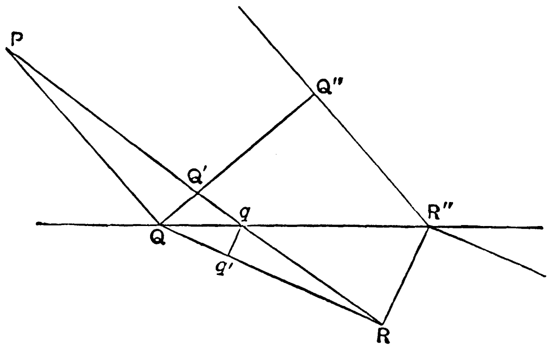
or, since δx, δy, δz are independent,

$$\frac{\partial \mu}{\partial x} = \frac{d}{ds}\left(\mu \frac{dx}{ds}\right), \text{ etc.,}$$

while, at a surface bounding media (1, 2), $\mu \frac{dx}{ds}$, etc., are continuous, giving the ordinary laws of reflection and refraction and the path of a ray in a heterogeneous singly-refracting medium.

16. Again, let PQ, QR be the incident and refracted rays at a surface of separation between the media of refractive indices μ, μ' and let $Q''R''$ be a ray parallel to PQ, and PqR a ray consecutive to PQR.

Then, drawing $QQ'Q''$ perpendicular to PQ and qq', RR'' perpendicular to QR, we have



since $$\mu PQ + \mu' QR = \mu Pq + \mu' qR,$$

$$\mu Q'q = \mu' Qq',$$

or $$\mu Q''R'' = \mu' QR.$$

This gives Huyghens' construction for singly-refracting media.

17. If the second medium is doubly-refracting, QQ', RR' are the traces of the wave fronts (in the plane of the paper) and thus, since the wave surface is the envelope of the wave front, the wave surface can be determined, in the usual way [Art. 54], as the envelope of

$$lx + my + nz = v,$$

where l, m, n are the direction-cosines of the normal to the wave front, and v the velocity of propagation, while l, m, n and v must be connected by a relation which, however, requires to be determined on some theory independent of Fermat's principle (such as Fresnel's). Thus, Fermat's principle is seen to be capable of giving a complete kinematical account of double refraction, also—with the help of a subsidiary hypothesis, regarding the law of variation of μ with direction.

18. Again, since $\delta \int \mu ds = 0$ we may take $\mu ds = dV$, and we have $\delta V = 0$ at each reflection or refraction.

And, since

$$\frac{\partial V}{\partial s} = \mu, \text{ or } \frac{\partial V}{\partial x} : \frac{\partial V}{\partial y} : \frac{\partial V}{\partial z} :: \alpha \,.\, \beta \,.\, \gamma,$$

where α, β, γ are the direction-cosines of the ray, we conclude that $V =$ constant is a surface orthogonal to a system of rays.

Accordingly, if a given surface can, at any stage, be made to coincide with a surface $V =$ constant, a surface can always be drawn to coincide with any other member of the family $V =$ constant: Or

Any system of rays originally orthogonal to a surface will always be orthogonal to a surface, after any number of reflections and refractions.

19. Since, then, the rays of light (which are orthogonal to a surface) may be regarded as normals to a family of surfaces, and, by Sturm's theorem, all the normals to a surface in the neighbourhood of a point converge to or diverge from two focal lines at right angles to one another, each of which passes through the centre of curvature of one of the principal normal sections and is perpendicular to the plane of that section, we conclude that all the rays of a thin pencil which can be cut at right angles by a surface pass through two lines, such that the planes containing either of them and the principal ray are perpendicular to each other.

20. Again, since the equation of a surface near the origin with the axis of z along the normal at the origin is, to the second order,

$$2z = \frac{x^2}{\rho_1} + \frac{y^2}{\rho_2},$$

the *characteristic function* (V = constant) for a thin pencil in a medium μ, with the axial ray proceeding along the axis of z, is

$$V = \mu\left(2z - \frac{x^2}{\rho_1} - \frac{y^2}{\rho_2}\right),$$

approximately, if aberration is neglected. With the help of this equation, the problem of reflection and refraction of direct and oblique pencils (aberration being neglected) can be treated in the usual way.

21. To illustrate this, consider the following example:

A pencil of rays is refracted through a prism and the axis of the pencil is constantly in a principal plane of the prism: also the angular position of the focal lines at incidence and after the first and second refraction relative to the edge of the prism is defined by α, β, γ. If, now, the distances of the initial and final foci from the first and second surfaces are u_1, u_2 and v_1, v_2, respectively, to obtain the equations of refraction:—

Let us take U_1, U_2 as the distances of the foci after the first refraction from the first surface, and V_1, V_2, those from the second surface.

Then $V_1 - U_1 = V_2 - U_2$ = length of the axial ray in the prism.

Let, finally, ϕ, ϕ' be the angles of incidence and refraction at the first surface, and ψ, ψ', those at the second surface.

The edge of the prism being taken as the axis of y and the normal to the face of incidence as the axis of z, the direction-cosines of one of the focal lines before incidence are

$$\sin\alpha\cos\phi, \quad \cos\alpha, \quad \sin\alpha\sin\phi.$$

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Thus, the characteristic function, before incidence, becomes

$$V=\mu\left\{x\sin\phi+z\cos\phi-\frac{1}{2u_1}(x\sin\alpha\cos\phi+y\cos\alpha+z\sin\alpha\sin\phi)^2\right.$$
$$\left.-\frac{1}{2u_2}(x\cos\alpha\cos\phi-y\sin\alpha+z\cos\alpha\sin\phi)^2\right\}.$$

From the continuity of the function, at $z=0$, we get, by equating coefficients of x, x^2, xy, and y^2,

$$\mu\sin\phi=\mu'\sin\phi',$$
$$\mu\left(\frac{\sin^2\alpha}{u_1}+\frac{\cos^2\alpha}{u_2}\right)\cos^2\phi=\mu'\left(\frac{\sin^2\beta}{U_1}+\frac{\cos^2\beta}{U_2}\right),$$
$$\mu\left(\frac{\cos^2\alpha}{u_1}+\frac{\sin^2\alpha}{u_2}\right)=\mu'\left(\frac{\cos^2\beta}{U_1}+\frac{\sin^2\beta}{U_2}\right),$$
$$\mu\sin\alpha\cos\alpha\left(\frac{1}{u_1}-\frac{1}{u_2}\right)\cos\phi=\mu'\sin\beta\cos\beta\left(\frac{1}{U_1}-\frac{1}{U_2}\right)\cos\phi',$$

and similar equations for the second refraction.

22. In order to take account of aberration, we must obtain the characteristic function up to the order zx^2.

Let the equation of the characteristic surface ($V=0$) be

$$2z=\frac{x^2}{\rho_1}+\frac{y^2}{\rho_2}+2z(\alpha x^2+\beta xy+\gamma y^2)+\text{etc.}$$

Now the perpendicular from the origin on the tangent plane at x', y', z' to the surface $2z=\frac{x^2}{\rho_1}+\frac{y^2}{\rho_2}$ is

$$\frac{z'}{\sqrt{1+\frac{x'^2}{\rho_1^2}+\frac{y'^2}{\rho_2^2}}}=z'\left(1-\frac{1}{2}\frac{x'^2}{\rho_1^2}-\frac{1}{2}\frac{y'^2}{\rho_2^2}\right),$$

if terms of the order zx^2 are retained.

Therefore, in $\quad 2z=\frac{x^2}{\rho_1}+\frac{y^2}{\rho_2}$

we must write $\quad z\left(1-\frac{1}{2}\frac{x^2}{\rho_1^2}-\frac{1}{2}\frac{y^2}{\rho_2^2}\right)$ instead of z,

so that the result may be correct up to this order.

Hence the equation of the characteristic surface ($V=$ constant) is of the form

$$V+z=\frac{1}{2}\frac{x^2}{\rho_1}+\frac{1}{2}\frac{y^2}{\rho_2}+z\left(\frac{1}{2}\frac{x^2}{\rho_1^2}+\frac{1}{2}\frac{y^2}{\rho_2^2}\right)$$
$$+ax^3+3bx^2y+3cxy^2+dy^3 \text{ (say)},$$