CHAPTER I

INTRODUCTION

WE have upon the nearly spherical surface of our globe an arrangement of features whose relative positions, sizes, and shapes we desire to represent as well as possible upon a flat sheet, the map. A perfect representation is impossible, since a plane surface cannot be fitted to a spherical. But there are many different ways of obtaining an approximate representation, whose theory and properties constitute the subject of Map Projections.

Definition of a projection.

The positions of points upon the Earth are for convenience defined by reference to the meridians of longitude and parallels of latitude. Hence if we can find a way of representing the parallels and meridians upon our sheet, we can lay down the points in their positions relative to these lines, and make our map. Any such representation of meridians and parallels upon a plane is a map projection.

It is evident, from our definition, that we use the word Projection in a sense much wider than that which geometry gives it. The majority of map projections are not projections at all in the geometrical sense, and various attempts have been made to find a better word to describe the network of meridians and parallels. But no one has been successful. Map "construction" implies rather too much. The excellent word "graticule" has scarcely established itself, though it is perhaps less open to objection than any other. We shall, then, continue to use the word projection, with the warning that it is not to be interpreted as meaning a geometrical projection. Strictly geometrical projections are of very little use in map making, and it is a mistake to begin by considering the few that are used, and to proceed afterwards to the very many useful projections which are not derived from the sphere by any perspective construction.

The number of possible ways of constructing a projection is infinite, even if we restrict our definition to the statement that any *orderly* construction of meridians and parallels may be considered a projection. It is obvious, however, that all these constructions are not equally good; and to test the merits and defects of a projection we must consider what properties it should possess in order to be useful.

We shall find that map projections are to be judged by the following criteria:

(1) the accuracy with which they represent the scale along the meridians and parallels.

(2) the accuracy with which they represent areas.

(3) the accuracy with which they represent the shape of the features of the map.

(4) the ease with which they can be constructed. [B*]

We will consider these criteria in order.

The representation of scale.

The scale of a map in a given direction at any point is the ratio which a short distance measured on the map bears to the corresponding distance upon the surface of the Earth.

We must limit our definition to *short* distances because the scale of a map will generally vary from point to point; hence in defining scale we must confine ourselves to small elements of distance in the way which is familiar to every beginner in the differential calculus.

We must also be careful to see that we are comparing distances in directions which really correspond, the one to the other, upon the Earth and upon the map. The meridians and parallels all over the Earth cut one another at right angles. But there are many map projections in which they do not cut one another at right angles, and in consequence two directions at right angles upon the Earth do not necessarily correspond to two directions at right angles upon the map. We shall avoid confusion if we confine ourselves as much as possible to the consideration of scale along the meridians and the parallels of the map, which necessarily correspond to the meridians and parallels of the Earth.

It would of course be desirable that the scale of the map should be correct in every direction at every point. If it were, the plane map would be a perfect representation of the spherical surface, and could therefore be fitted to it. But this is impossible. Hence the scale of a map cannot be correct all over the map.

We can, however, choose a projection in which the scale in a certain direction, say along the meridian, or along the parallel, is correct at every point of the map. But in this case the scale in any other direction will be wrong at most points. And one of our objects will be to keep this necessary error as small as possible.

The representation of areas.

For some purposes, especially political and statistical, it is important that areas should be represented in their correct proportions. A projection which does this is called an equal area projection, or an equivalent projection. We shall use the former name in this book.

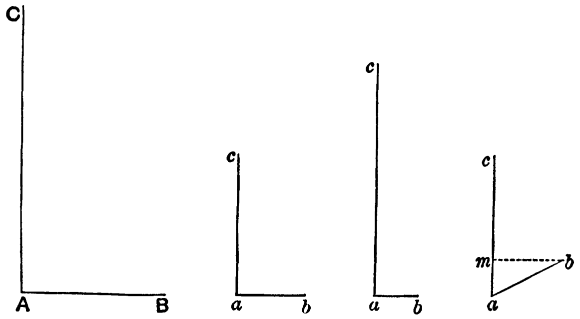


Fig. 1.

Suppose that *AB*, *AC* are two short distances at right angles to one another at any point on the Earth. If the corresponding distances *ab*, *ac* upon the map were always in the same proportion and also at right angles to one another, the projection would clearly be an equal area projection. But these conditions cannot be fulfilled, for if they were fulfilled at every point, the map would be a perfect map, which is impossible.

There are however two distinct ways in which the equality of areas may be preserved upon the map.

(α) *ab*, *ac* may still be at right angles, but with the scale of one increased and of the other decreased, in inverse proportion.

(β) or *ab*, *ac* may be no longer at right angles; but while the scale of *ac* is maintained correct, that of *ab* is increased in such a proportion that the perpendicular distance of *b* from *ac* is correct.

It is clear that in either case the projection is equal area.

The representation of shape.

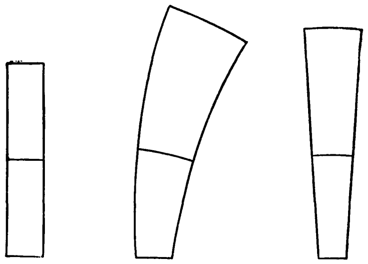
The representation of shape as nearly correctly as possible is perhaps the most important function of a map. It is evidently not possible to represent the shape of a large country correctly upon a map, for if it were, the map would be perfect, which is impossible.

But if at any point the scale along the meridian and the parallel is the same (not correct, but the same in the two directions) and the parallels and meridians of the map are at right angles to one another, then the shape of any very small area on the map is the same as the shape of the corresponding small area upon the Earth. The projection is then called *orthomorphic* (right shape).

But it is important to notice the restriction to *very small areas*. Since the scale necessarily varies from point to point, big areas are not correctly represented. Hence it is clear that the term orthomorphic must be used in a carefully limited sense. It has, in fact, a mathematical significance and interest which is apt to be of little use in practice. [A*]

For example, suppose we had a strip of country a mile wide, along a meridian, and divided into two equal parts by parallels of latitude (Fig. 2 *a*). A projection which is orthomorphic in the mathematical sense might represent the figure thus (Fig. 2 *b*). It will be noticed that all the angles are preserved in their true magnitudes as right angles, but the strip on the map is no longer of uniform width, it is no longer divided into equal parts, and it is unsymmetrical.

Another orthomorphic projection might represent the same strip thus (Fig. 2*c*): the angles are preserved as before, the areas are modified so that the strip is no longer bisected; but there is symmetry and no general bending.



(*a*) (*b*) (*c*)

Fig. 2.

It is clear that the latter projection may be much superior to the former in representing the shapes of considerable areas, especially for countries having a great extent in latitude. The essential difference is, that in the former case the projections of the meridians are not straight lines, while in the latter case they are.

We shall find that an orthomorphic projection is generally not much use for map making unless the meridians are straight lines on the map. [C*]

We shall also notice that projections which are not, mathematically speaking, orthomorphic may often represent the shape of large areas better than is done by projections that are orthomorphic.

We shall therefore be prepared to find that the mathematical property of orthomorphism does not always, or indeed usually, give the map any considerable practical advantages; and that orthomorphism may generally be sacrificed to other less theoretically elegant, but more useful properties.

We may define orthomorphic projections by either of two properties:

(α) At any point the scale in all directions is the same. (This is the definition used above.)

Or (β) The angles in any small figure on the Earth are preserved unaltered on the map. (It is geometrically obvious that this definition is equivalent to the preceding.)

It follows from either of these definitions that orthomorphic projections preserve the shapes of small areas unaltered, though the scale on which they are represented varies from point to point upon the map. At first sight this preservation of shape appears to be an important property. It should however be remembered that, so long as we do not attempt to represent too large a fraction of the whole Earth upon one map, a great many of the usual projections are pretty nearly orthomorphic for small areas; while if we remove the restriction to small areas the general shape is often better preserved in projections which are not orthomorphic than in those which are.

The representation of true bearings and distances.

We have just seen that the definition of orthomorphism restricts the property to very small areas; and that the existence of perfect orthomorphism, according to definition, is no guarantee even for the approximate preservation of the shape of large configurations.

We want some criterion for the degree of success of a given projection in preserving the shape of a country from gross distortion, and we shall find it useful to consider how far the true bearings and distances from point to point are preserved. For example, we have a map of Europe on a given projection, and we enquire: What is the percentage error of the representation of the distance from Hanover to St Petersburg; or what is the error in the azimuth of this line. Such questions cannot be answered by considering very small areas.

There is a class of projections sometimes named *azimuthal*, from the fact that the azimuths, or true bearings, *from the centre of the map*, of all points, are shown correctly. One of these azimuthal projections also shows distances from the centre of the map correct, and is called the azimuthal equidistant projection. We shall find it useful to ask of each projection, how nearly does it approximate to the azimuthal equidistant, and further, how well does it preserve azimuths and distances, not only from the centre, but from any other point?

The objection to the term *azimuthal* is that it is hard to pronounce, and several writers have followed Germain in calling always this class of projection *zenithal*. Since their most prominent and valuable property is the preservation unaltered of azimuths, or true bearings, from the centre, it appears to the writer that the former name is preferable to the latter, and that it is unfortunate that *zenithal*, which has no very clear meaning, should replace *azimuthal*, whose meaning is precise. We shall not, however, try to return in this book to the older fashion.

Ease of drawing.

This is a property which is theoretically uninteresting, but which is, in practice, of extreme importance. As a general rule, projections which are not built up of straight lines and circles are hard to draw. This rule excludes at once all the strictly geometrical projections, except the stereographic, which is built entirely of circles. [D*]

Further, arcs of circles of very large radius are hard to draw; and for this reason graphical constructions often break down somewhere or other, requiring circles too large for the drawing table.

In such a case a series of points on the circle must be computed or constructed graphically. Hence the formulae of computation become very important.

We shall ask ourselves at the end of the section on each projection: Is it easy to draw, or can tables for it be constructed easily?

The choice of projection for a map.

It is clear that we cannot say anything in detail upon this subject until we have examined the properties of the principal projections in use. But we shall do well to bear in mind from the beginning that there are three broad classes of maps:

(α) Maps of the whole world or of a hemisphere, on one sheet. We may call any map that represents a hemisphere, or more, a World Map. These will always be on small scales.

(β) Maps which show a considerable portion of the Earth, such as a continent, but not a whole hemisphere. These will also be on a small scale. We shall find it convenient to call them Atlas Maps.

(γ) Maps on a comparatively large scale, each representing in detail a fairly small area of country. We may call these Survey Maps.

The projections for maps in sections (α) and (β) will generally be constructed independently for each map, and there will be no question of adjacent maps fitting. But it may be thought desirable that the sheets of a Survey map should fit together, so that they may be combined to form larger maps if necessary. This will require that the projection for the whole survey should be determined, and that each sheet should not be plotted independently but should be a definite part of this projection. We shall see that this is practicable only for a small country like Great Britain, and has in any case considerable disadvantages.

It is evident, however, that in the *use* of Survey maps, the question of projections does not often arise. Each sheet covers so small a portion of the whole surface of the Earth that it is practically a perfect representation, if the original choice of a projection for the Survey has been well made, and more particularly, if the perpendicularity of meridians and parallels is preserved. This reservation is of prime importance. [E*]

Atlas maps cannot be treated as practically errorless. In these the errors in scale, area, and shape become considerable, and are unavoidable. We cannot make measurements upon the map until we know the errors which are due to the projection. We shall therefore arrange our detailed consideration of projections so as to give the common Atlas projections as much prominence and priority as is consistent with an orderly development of the subject.

CHAPTER II

THE PRINCIPAL SYSTEMS OF PROJECTIONS: CONICAL PROJECTIONS

THE greater number of useful projections for Atlas maps belong to one or the other of two great classes, the Conical (including the cylindrical) and the Zenithal projections. These names describe the method of construction. It is useful to give each projection a second name which describes its principal property, such as equal area, orthomorphic, and so on. We shall therefore describe projections as the Conical equal area, the Zenithal orthomorphic, the first, or generic name, describing its construction, the second, or specific name, its most important property. When the name of the inventor, Lambert or Gauss, Sanson or Delisle, is usually associated with the projection, we may give it in brackets, thus: Conical orthomorphic with two standard parallels (Lambert's second, or Gauss').

We shall find, however, that this principle of nomenclature cannot be made to cover all cases without some appearance of pedantry, and that there are well known projections, such as Mercator's or the Stereographic, which will be treated in their systematic places but referred to generally by their simple names. Thus the Stereographic projection is a zenithal orthomorphic; but as it is one of several which can be thus named, it is convenient to call it simply the Stereographic.

We shall find, also, that this way of naming projections is convenient, rather than consistently logical. For zenithal projections are, from one point of view, only special cases of conical. Moreover, all zenithal projections are azimuthal, so that one of the principal properties of this class of projection is implied in its generic name. Thus a zenithal equal area projection has two important properties: it is both azimuthal and equal area. But as we become familiar with the subject this want of strict consistency in the nomenclature will not be a serious difficulty.

We shall find it best to defer the consideration of the classification of map projections until we have become acquainted with the more obvious properties of each projection. It will then be evident that a logical classification requires in the first place a re-consideration of the names which are given to the projections. Inasmuch however as a disturbance of the accepted names, even though they are unsystematic, would certainly create more confusion than it would remove, we shall find it best to retain the generally accepted names, taking Germain's *Traité des projections* as our standard authority, and shall make suggestions for a more logical nomenclature only in our discussion of classification, without attempting to introduce any reform of such doubtful advantage into the body of the work.

Conical projections.

In all the usual conical projections the meridians are straight lines converging to a point, the vertex, and the parallels are concentric circles described about that point.

The meridians are equally spaced, and make with one another angles which are a certain fraction n of the angles which the corresponding terrestrial meridians make with one another at the poles. The quantity n we will call the constant of the cone. It must lie between the values 0 and 1.

The spacing of the parallels depends upon the particular property which we wish the projection to fulfil.

One parallel, and sometimes a second, is made of the true length; that is to say, if the map is to be on the scale of one-millionth, the length of the complete parallel on the map will be one-millionth of the corresponding terrestrial parallel. This is called a Standard parallel.

Simple conical projection with one standard parallel.

Suppose that we begin by constructing the Conical projection with one standard parallel. If R is the radius of the Earth (supposed spherical) and ϕ the latitude of the selected parallel, the length of the parallel is $2\pi R \cos \phi$.