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Josef Lauri and Raffaele Scapellato

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Preface to the Second Edition

In this second edition of our book we have tried to maintain the same structure as the first edition, namely a text which, although not providing an exhaustive coverage of graph symmetries and reconstruction, provides a detailed coverage of some particular areas (generally motivated by our own research interest), which is not a haphazard collection of results but which presents a clear pathway through this thick forest. And our aim remains that of producing a text which can relatively quickly guide the reader to the point of being able to understand and carry out research in the topics which we cover.

Among the additions in this edition we point out the use of the free computer programs GAP, GRAPE and *Sage* to construct and investigate some well-known graphs, including examples with properties like being semisymmetric, a topic which was treated in the first edition but for which examples are not easy to construct ‘by hand’. We have also updated some chapters with new results, improved the presentation and proofs of others, and introduced short treatments of topics such as character theory of abelian groups and their Cayley graphs to emphasise the connection between graph theory and other areas of mathematics.

We have corrected a number of errors which we found in the first edition, and for this we would like to thank colleagues who have pointed out several of them, particularly Bill Kocay, Virgilio Pannone and Alex Scott.

A special thanks goes to Russell Mizzi for help with overhauling Chapter 6, where we also introduce the new idea of two-fold isomorphisms, and to Leonard Soicher and Matan Zif-Av for several helpful tips regarding the use of GAP and GRAPE.

The second author would like to thank the Politecnico di Milano for giving him the opportunity, by means of a sabbatical, to focus on the work needed

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Preface to the Second Edition

to complete the current edition of this book. He also thanks the University of Malta for its kind hospitality during this sabbatical.

The authors will maintain a list of corrections and addenda at <http://staff.um.edu.mt/josef.lauri>.

Josef Lauri
Raffaele Scapellato

Preface to the First Edition

This book arose out of lectures given by the first author to Masters students at the University of Malta and by the second author at the Università Cattolica di Brescia.

This book is not intended to be an exhaustive coverage of graph theory. There are many excellent texts that do this, some of which are mentioned in the References. Rather, the intention is to provide the reader with a more in-depth coverage of some particular areas of graph theory. The choice of these areas has been largely governed by the research interests of the authors, and the flavour of the topics covered is predominantly algebraic, with emphasis on symmetry properties of graphs. Thus, standard topics such as the automorphism group of a graph, Frucht's Theorem, Cayley graphs and coset graphs, and orbital graphs are presented early on because they provide the background for most of the work presented in later chapters. Here, more specialised topics are tackled, such as graphical regular representations, pseudosimilarity, graph products, Hamiltonicity of Cayley graphs and special types of vertex-transitive graphs, including a brief treatment of the difficult topic of classifying vertex-transitive graphs. The last four chapters are devoted to the Reconstruction Problem, and even here greater emphasis is given to those results that are of a more algebraic character and involve the symmetry of graphs. A special chapter is devoted to graph products. Such operations are often used to provide new examples from existing ones but are seldom studied for their intrinsic value.

Throughout we have tried to present results and proofs, many of which are not usually found in textbooks but have to be looked for in journal papers. Also, we have tried, where possible, to give a treatment of some of these topics that is different from the standard published material (for example, the chapter on graph products and much of the work on reconstruction).

Although the prerequisites for reading this book are quite modest (exposure to a first course in graph theory and some discrete mathematics, and elementary knowledge about permutation groups and some linear algebra), it was our intention when preparing this book that a student who has mastered its contents would be in a good position to understand the current state of research in most of the specialised topics covered, would be able to read with profit journal papers in these areas, and would hopefully have his or her interest sufficiently aroused to consider carrying out research in one of these areas of graph theory.

We would finally like to thank Professor Caroline Series for showing an interest in this book when it was still in an early draft form and the staff at Cambridge University Press for their help and encouragement, especially Roger Astley, Senior Editor, Mathematical Sciences, and, for technical help with L^AT_EX, Alison Woollatt, who, with a short style file, solved problems that would have baffled us for ages. Thanks are also due to Elise Oranges, who edited this book thoroughly and pointed out several corrections.

The first author would also like to thank the Academic Work Resources Fund Committee and the Computing Services Centre of the University of Malta, the first for some financial help while writing this book and the second for technical assistance. He also thanks his M.Sc. students at the University of Malta, who worked through draft chapters of this book and whose comments and criticism helped to improve the final product.

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Raffaele Scapellato