MATHEMATICS
FOR
ACTUARIAL STUDENTS

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by

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PART II

Finite Differences, Probability &
Elementary Statistics

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MEMORANDUM

If the modern student, who aspires to be an Actuary, is faced with a wider range of actuarial subjects and a more searching test of his understanding of actuarial principles than was the student of an earlier generation, he has this compensation—that the means to acquire such understanding are more readily available and put into a form which may more readily be assimilated.

This book is such an example of the help afforded to the actuarial student, and he is fortunate to have the opportunity to study a work of this merit at the very beginning of his Course. A complete grasp of its principles will not only assist him to satisfy his examiners as to his proficiency in the mathematical problems by which he will be tested, but when he has so satisfied them he will have laid such foundation for his future work that he will ever have cause to be grateful to the author of this treatise.

H. J. P. O.

NOTATION

$P_n(x)$ a polynomial (i.e., a rational integral function) of degree n in x.

$n_{(r)}$ $\dfrac{n(n-1)(n-2)\dots(n-r+1)}{r(r-1)(r-2)\dots 3.2.1} = \dfrac{n^{(r)}}{r!}$, where n may be positive or negative, integral or fractional.

This is represented in other works by the symbols $\binom{n}{r}$ or n_r; see *J.I.A.* vol. LXIII, p. 58.

$n^{(r)}$ $n(n-1)(n-2)\dots(n-r+1)$.

There seems to be no recognized symbol for the more general factorial $n(n-h)(n-2h)\dots(n-\overline{r-1}h)$. It may sometimes be convenient to represent this by the same symbol $n^{(r)}$, but in that case the symbol must be specially defined and consistently used. Cf. *post*, p. 19.

$n^{(-r)}$ $\dfrac{1}{(n+1)(n+2)\dots(n+r)} = \dfrac{1}{(n+r)^{(r)}}$.

A different notation is employed in certain other works—see *post*, p. 19.

$\underset{bc\dots}{\Delta^t u_a}$ the divided difference of order t. This notation is explained and reference is made to other notations in Chapter III.

Paragraphs and examples marked with an asterisk, thus *, are intended for the advanced student only, and need not be read by students preparing for Part I of the Institute Examinations.

REFERENCES

J.I.A. *Journal of the Institute of Actuaries.*
T.F.A. *Transactions of the Faculty of Actuaries.*
J.S.S. *Journal of the Institute of Actuaries Students' Society.*

AUTHOR'S PREFACE

As explained in the Preface to Part I of this book, the revision of the syllabus for the Examinations of the Institute of Actuaries rendered it necessary to divide *An Elementary Treatise on Actuarial Mathematics* into two parts and to make additions to that part of the book intended for students who were reading for the first Part of the Examinations. The opportunity has been taken to revise thoroughly the Chapters on Finite Differences and Probability and to bring the book up to date. While little alteration has been made in a few Chapters of *Actuarial Mathematics* the majority have been entirely rewritten, and it is therefore advisable to refer to each Chapter of the present book in detail.

Chapters I and II do not differ markedly from the first two Chapters on Finite Differences in *Actuarial Mathematics*. Chapter III, on Interpolation with Unequal Intervals, has been rewritten. The introduction of Dr Aitken's notation has simplified the general problem of divided differences, and as a result a more logical exposition of the theorems connected with these differences has been given. The earlier parts of Chapter IV, on Central Differences, have been remodelled, and much new matter inserted. With regard to Chapter V as given in *Actuarial Mathematics* it was thought that the treatment of the principles of Inverse Interpolation was to a certain extent incomplete. Dr Aitken suggested lines on which improvement might be made and the initial paragraphs have therefore been redrafted. Considerable alterations have been made in the Chapter on Summation: the first few paragraphs have been recast, a new section dealing with the application of operators to the summation of algebraic series has been introduced and paragraphs on the operator $[n]$ have been inserted. Chapter VII, on Miscellaneous Theorems, has been enlarged by the introduction of new material and a fuller treatment has been given of the matter in the previous book. Chapter VIII is entirely new. It was considered that there was now an opportunity to collect in one Chapter a number of modern methods and special devices which have recently appeared from the pens of Mr Lidstone and Dr Aitken, but which are scattered over the pages of various Journals. The Chapter on Approxi-

mate Integration has been amended in order to clarify certain proofs and theorems, but is otherwise unaltered.

In view of the introduction of Elementary Statistics into the syllabus for Part I of the Examinations it was considered advisable to approach the study of Probability from a slightly different angle. As a consequence, Chapter X has been largely rewritten. New Chapters on Elementary Statistics have been introduced: these Chapters do not deal exhaustively with the subject, but are, it is hoped, sufficient to enable the student to solve simple types of problem met with in practical work. Little alteration has been made in the Chapter dealing with Mean Value and the application of the Calculus to Probability.

The Examples are mainly those which have already appeared in *Actuarial Mathematics*. Examples 11 and 12 contain exercises in Statistics and the Miscellaneous Examples contain many new questions.

In the preparation of this Part of the book I have received much help from various sources. I have had access to the notes written for the Actuarial Tuition Service and some of the Lessons on Probability and Statistics (prepared originally by P. M. Marples) have proved of value. My thanks are due to Mr H. Tetley and Mr O. C. J. Klagge for helpful suggestions on certain points.

It is, however, to Dr A. C. Aitken and Mr G. J. Lidstone to whom I am most indebted. Dr Aitken generously put at my disposal his notes on various subjects connected with Finite Differences. In particular, his suggestions on Inverse Interpolation, the divided difference notation and the methods of cross-means have been such that I was able to adopt them with little alteration. Throughout the preparation of the book, from the earliest stages to the final proof-reading, Mr Lidstone has been my constant guide and mentor. A great deal of the work has been inspired by him and his comments and criticisms on all points have been invaluable. He has read through and annotated the whole of the book with that thoroughness for which he is justly famous, and I am deeply grateful to him for his help.

H. F.

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