

Contents

Preface *page ix*

PART ONE INTRODUCTION	1
1 Group theory	3
1.1 Introduction and basic notation	3
1.2 Generators, congruences and presentations	5
1.3 Decision problems	7
1.4 Subgroups and Schreier generators	8
1.5 Combining groups	10
1.6 Cayley graphs	20
1.7 Quasi-isometries	23
1.8 Ends of graphs and groups	25
1.9 Small cancellation	26
1.10 Some interesting families of groups	28
2 Formal languages and automata theory	36
2.1 Languages, automata and grammars	36
2.2 Types of automata	38
2.3 More on grammars	50
2.4 Syntactic monoids	51
2.5 Finite state automata, regular languages and grammars	52
2.6 Pushdown automata, context-free languages and grammars	60
2.7 Turing machines, recursively enumerable languages and grammars	81
2.8 Linearly bounded automata, context-sensitive languages and grammars	85

2.9	Turing machines and decidability	87
2.10	Automata with more than one input word	91
3	Introduction to the word problem	97
3.1	Definition of the word problem	97
3.2	Van Kampen diagrams	98
3.3	The Dehn function	102
3.4	The word problem as a formal language	105
3.5	Dehn presentations and Dehn algorithms	110
3.6	Filling functions	114
PART TWO FINITE STATE AUTOMATA AND GROUPS		115
4	Rewriting systems	117
4.1	Rewriting systems in monoids and groups	117
4.2	The use of <i>fsa</i> in the reduction process	123
5	Automatic groups	125
5.1	Definition of automatic groups	126
5.2	Properties of automatic groups	129
5.3	Shortlex and geodesic structures	135
5.4	The construction of shortlex automatic structures	135
5.5	Examples of automatic groups	139
5.6	Closure properties	142
5.7	The falsification by fellow traveller property	143
5.8	Strongly geodesically automatic groups	145
5.9	Generalisations of automaticity	146
6	Hyperbolic groups	150
6.1	Hyperbolicity conditions	150
6.2	Hyperbolicity for geodesic metric spaces and groups	153
6.3	Thin bigons, biautomaticity and divergence	154
6.4	Hyperbolic groups have Dehn presentations	158
6.5	Groups with linear Dehn functions are hyperbolic	161
6.6	Equivalent definitions of hyperbolicity	164
6.7	Quasigeodesics	165
6.8	Further properties of hyperbolic groups	166
7	Geodesics	169
7.1	Introduction	169
7.2	Virtually abelian groups and relatively hyperbolic groups	170
7.3	Coxeter groups	173

<i>Contents</i>	vii
7.4 Garside groups	176
7.5 Groups with geodesics lying in some subclass of $\mathcal{R}eg$	182
7.6 Conjugacy geodesics	182
8 Subgroups and coset systems	184
8.1 Rational and quasiconvex subsets of groups	184
8.2 Automatic coset systems	189
9 Automata groups	194
9.1 Introducing permutational transducers	194
9.2 Automata groups	199
9.3 Groups of tree automorphisms	203
9.4 Dual automata	210
9.5 Free automata groups	212
9.6 Decision problems	213
9.7 Resolving famous problems	215
PART THREE THE WORD PROBLEM	219
10 Solubility of the word problem	221
10.1 The Novikov–Boone theorem	221
10.2 Related results	226
11 Context-free and one-counter word problems	228
11.1 Groups with context-free word problem	228
11.2 Groups with one-counter word problem	232
12 Context-sensitive word problems	236
12.1 Lakin’s example	237
12.2 Some further examples	239
12.3 Filling length	240
12.4 Groups with linear filling length	242
13 Word problems in other language classes	249
13.1 Real-time word problems	249
13.2 Indexed word problems	254
13.3 Poly-context-free word problems	254
14 The co-word problem and the conjugacy problem	256
14.1 The co-word problem	256
14.2 Co-context-free groups	257
14.3 Indexed co-word problems	267
14.4 The conjugacy problem	268

<i>References</i>	270
<i>Index of Notation</i>	283
<i>Index of Names</i>	284
<i>Index of Topics and Terminology</i>	287