

## An Introduction to Decision Theory

Now revised and updated, this introduction to decision theory is both accessible and comprehensive, covering topics including decision making under ignorance and risk, the foundations of utility theory, the debate over subjective and objective probability, Bayesianism, causal decision theory, game theory, and social choice theory. No mathematical skills are assumed, with all concepts and results explained in nontechnical and intuitive as well as more formal ways, and there are now over 140 exercises with solutions, along with a glossary of key terms and concepts. This second edition includes a new chapter on risk aversion as well as updated discussions of numerous central ideas, including Newcomb's problem, prisoner's dilemmas, and Arrow's impossibility theorem. The book will appeal particularly to philosophy students but also to readers in a range of disciplines, from computer science and psychology to economics and political science.

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Second Edition

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CAMBRIDGE  
UNIVERSITY PRESS

Cambridge University Press & Assessment  
978-1-316-60620-9 — An Introduction to Decision Theory  
Martin Peterson  
Frontmatter  
[More Information](#)



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Shaftesbury Road, Cambridge CB2 8EA, United Kingdom  
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477 Williamstown Road, Port Melbourne, VIC 3207, Australia  
314–321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi – 110025, India  
103 Penang Road, #05–06/07, Visioncrest Commercial, Singapore 238467

Cambridge University Press is part of Cambridge University Press & Assessment, a department of the University of Cambridge.

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Information on this title: [www.cambridge.org/9781316606209](http://www.cambridge.org/9781316606209)

DOI: 10.1017/9781316585061

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First edition published 2009

Second edition published 2017

*A catalogue record for this publication is available from the British Library*

ISBN 978-1-107-15159-8 Hardback

ISBN 978-1-316-60620-9 Paperback

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## Preface

This book is an introduction to decision theory. My ambition is to present the subject in a way that is accessible to readers with a background in a wide range of disciplines, such as philosophy, economics, psychology, political science and computer science. That said, I am myself a philosopher, so it is hardly surprising that I have chosen to discuss philosophical and foundational aspects of decision theory in some detail. In my experience, readers interested in specific applications of the subject may find it helpful to start with a thorough discussion of the basic principles before moving on to their chosen field of specialization.

My ambition is to explain everything in a way that is accessible to everyone, including readers with limited knowledge of mathematics. I therefore do my best to emphasize the intuitive ideas underlying the technical concepts and results before I state them in a more formal vocabulary. This means that some points are made twice, first in a nontechnical manner and thereafter in more rigorous ways. I think it is important that students of decision theory learn quite a bit about the technical results of the subject, but most of those results can no doubt be explained much better than what is usually offered in textbooks. I have tried to include only theorems and proofs that are absolutely essential, and I have made an effort to prove the theorems in ways I believe are accessible for beginners. In my experience, this sometimes comes into conflict with the ambition to present technical material in the minimalist style usually preferred by experts.

Most of the technical results are presented in twenty “boxes” clearly separated from the main body of the text. In principle, it should be possible to read the book without reading the boxes, although they hopefully deepen the student’s understanding of the subject. I have also included over 100 exercises (and solutions), most of which should be fairly straightforward. Unlike other textbooks, no exercise asks the reader to prove some theorem

I did not bother to prove myself. Finally, the Appendix contains a glossary in which I try to briefly explain some of the key terms and concepts. I believe the glossary might be particularly useful for readers wishing to study only a selected number of the chapters.

A large number of people deserve my sincere thanks. First of all, I would like to thank all the students who have contributed their invaluable input to this project. I have done my best to improve the manuscript in accordance with the advice I have received. I am also deeply indebted to a number of fellow teachers and colleagues: Barbro Björkman, Joanna Burch Brown, John Cantwell, Stephen John, Elselijn Kingma, Holger Rosencrantz, and Per Sandin. I am also very grateful for valuable comments on the original proposal and draft manuscript given by four anonymous readers. Finally, I wish to thank Hilary Gaskin at Cambridge University Press, who suggested I should write this textbook. Without her enthusiasm and encouragement, this book would never have been written. The project proved to be both challenging and time-consuming but always highly enjoyable.

## Preface to the Second Edition

In the eight years since the first edition was published I have received a large number of emails with mostly positive feedback, often accompanied by suggestions for improvements. Based on this helpful correspondence, I have decided to add a chapter on risk aversion (Chapter 10) to the second edition. I have also revised and updated the other chapters with new material. The chapters on pragmatic arguments and Bayesian decision theory have been merged into a single chapter (Chapter 8), and in the chapter on expected utility theory (Chapter 4) I have added about ten pages on the St. Petersburg and Pasadena paradoxes. The discussion of Newcomb's problem (in Chapter 9) has been updated with new examples, and the discussions of indefinitely iterated prisoner's dilemmas (in Chapter 12) and Arrow's impossibility theorem (in Chapter 13) have been somewhat extended. I have also added about forty new exercises.

I would like to thank my research assistant Robert Reed for helping me to 'translate' the text from British English to American English, and Dong An, Ben Eggleston, Jelle de Boer, Ismail Guler, Johan E. Gustafsson, Jack Hamilton, Karsten Klint Jensen, Luke Muehlhauser and Manel Pau for helpful feedback.