

1

Reviewing number concepts

Key words

- Natural number
- Integer
- Prime number
- Symbol
- Multiple
- Factor
- Composite numbers
- Prime factor
- Square root
- Cube
- Directed numbers
- BODMAS

In this chapter you will learn how to:

- identify and classify different types of numbers
- find common factors and common multiples of numbers
- write numbers as products of their prime factors
- calculate squares, square roots, cubes and cube roots of numbers
- work with integers used in real-life situations
- revise the basic rules for operating with numbers
- perform basic calculations using mental methods and with a calculator.



This statue is a replica of a 22 000-year-old bone found in the Congo. The real bone is only 10 cm long and it is carved with groups of notches that represent numbers. One column lists the prime numbers from 10 to 20. It is one of the earliest examples of a number system using tallies.

Our modern number system is called the Hindu-Arabic system because it was developed by Hindus and spread by Arab traders who brought it with them when they moved to different places in the world. The Hindu-Arabic system is decimal. This means it uses place value based on powers of ten. Any number at all, including decimals and fractions, can be written using place value and the digits from 0 to 9.

1 Reviewing number concepts

1.1 Different types of numbers

Make sure you know the correct mathematical words for the types of numbers in the table.

You should already be familiar with most of the concepts in this chapter. It is included here so that you can revise the concepts and check that you remember them.

FAST FORWARD

You will learn about the difference between rational and irrational numbers in chapter 9. ▶

Find the 'product' means 'multiply'. So, the product of 3 and 4 is 12, i.e. $3 \times 4 = 12$.

Number	Definition	Example
Natural number	Any whole number from 1 to infinity, sometimes called 'counting numbers'. 0 is not included.	1, 2, 3, 4, 5, ...
Odd number	A whole number that cannot be divided exactly by 2.	1, 3, 5, 7, ...
Even number	A whole number that can be divided exactly by 2.	2, 4, 6, 8, ...
Integer	Any of the negative and positive whole numbers, including zero.	... -3, -2, -1, 0, 1, 2, 3, ...
Prime number	A whole number greater than 1 which has only two factors: the number itself and 1.	2, 3, 5, 7, 11, ...
Square number	The product obtained when an integer is multiplied by itself.	1, 4, 9, 16, ...
Fraction	A number representing parts of a whole number, can be written as a common (vulgar) fraction in the form of $\frac{a}{b}$ or as a decimal using the decimal point.	$\frac{1}{2}, \frac{1}{4}, \frac{1}{3}, \frac{1}{8}, \frac{13}{3}, 2\frac{1}{2}$ 0.5, 0.2, 0.08, 1.7

Exercise 1.1

FAST FORWARD

You will learn much more about sets in chapter 9. For now, just think of a set as a list of numbers or other items that are often placed inside curly brackets. ▶

Remember that a 'sum' is the result of an addition. The term is often used for *any* calculation in early mathematics but its meaning is very specific at this level.

1 Here is a set of numbers: $\{-4, -1, 0, \frac{1}{2}, 0.75, 3, 4, 6, 11, 16, 19, 25\}$

List the numbers from this set that are:

- (a) natural numbers (b) even numbers (c) odd numbers
- (d) integers (e) negative integers (f) fractions
- (g) square numbers (h) prime numbers (i) neither square nor prime.

2 List:

- (a) the next four odd numbers after 107
- (b) four consecutive even numbers between 2008 and 2030
- (c) all odd numbers between 993 and 1007
- (d) the first five square numbers
- (e) four decimal fractions that are smaller than 0.5
- (f) four vulgar fractions that are greater than $\frac{1}{2}$ but smaller than $\frac{3}{4}$.

3 State whether the following will be odd or even:

- (a) the sum of two odd numbers
- (b) the sum of two even numbers
- (c) the sum of an odd and an even number
- (d) the square of an odd number
- (e) the square of an even number
- (f) an odd number multiplied by an even number.

Living maths

- 4 There are many other types of numbers. Find out what these numbers are and give an example of each.
- Perfect numbers.
 - Palindromic numbers.
 - Narcissistic numbers. (In other words, numbers that love themselves!)

Using symbols to link numbers

Mathematicians use numbers and **symbols** to write mathematical information in the shortest, clearest way possible.

You have used the operation symbols $+$, $-$, \times and \div since you started school. Now you will also use the symbols given in the margin below to write mathematical statements.

Exercise 1.2

$=$ is equal to
 \neq is not equal to
 \approx is approximately equal to
 $<$ is less than
 \leq is less than or equal to
 $>$ is greater than
 \geq is greater than or equal to
 \therefore therefore
 $\sqrt{\quad}$ the square root of

Remember that the 'difference' between two numbers is the result of a subtraction. The order of the subtraction matters.

- 1 Rewrite each of these statements using mathematical symbols.

- 19 is less than 45
- 12 plus 18 is equal to 30
- 0.5 is equal to $\frac{1}{2}$
- 0.8 is not equal to 8.0
- -34 is less than 2 times -16
- therefore the number x equals the square root of 72
- a number (x) is less than or equal to negative 45
- π is approximately equal to 3.14
- 5.1 is greater than 5.01
- the sum of 3 and 4 is not equal to the product of 3 and 4
- the difference between 12 and -12 is greater than 12
- the sum of -12 and -24 is less than 0
- the product of 12 and a number (x) is approximately -40

- 2 Say whether these mathematical statements are true or false.

- | | |
|-------------------------------------|-------------------------------------|
| (a) $0.599 > 6.0$ | (b) $5 \times 1999 \approx 10\,000$ |
| (c) $8.1 = 8\frac{1}{10}$ | (d) $6.2 + 4.3 = 4.3 + 6.2$ |
| (e) $20 \times 9 \geq 21 \times 8$ | (f) $6.0 = 6$ |
| (g) $-12 > -4$ | (h) $19.9 \leq 20$ |
| (i) $1000 > 199 \times 5$ | (j) $\sqrt{16} = 4$ |
| (k) $35 \times 5 \times 2 \neq 350$ | (l) $20 \div 4 = 5 \div 20$ |
| (m) $20 - 4 \neq 4 - 20$ | (n) $20 \times 4 \neq 4 \times 20$ |

- 3 Work with a partner.

- Look at the symbols used on the keys of your calculator. Say what each one means in words.
- List any symbols that you do not know. Try to find out what each one means.

1.2 Multiples and factors

You can think of the multiples of a number as the 'times table' for that number. For example, the multiples of 3 are $3 \times 1 = 3$, $3 \times 2 = 6$, $3 \times 3 = 9$ and so on.

Multiples

A **multiple** of a number is found when you multiply that number by a positive integer. The first multiple of any number is the number itself (the number multiplied by 1).

1 Reviewing number concepts

Worked example 1

- (a) What are the first three multiples of 12?
 (b) Is 300 a multiple of 12?

(a) 12, 24, 36

To find these multiply 12 by 1, 2 and then 3.

$$12 \times 1 = 12$$

$$12 \times 2 = 24$$

$$12 \times 3 = 36$$

(b) Yes, 300 is a multiple of 12.

To find out, divide 300 by 12. If it goes exactly, then 300 is a multiple of 12.

$$300 \div 12 = 25$$

Exercise 1.3

- 1 List the first five multiples of:

(a) 2 (b) 3 (c) 5 (d) 8
 (e) 9 (f) 10 (g) 12 (h) 100

- 2 Use a calculator to find and list the first ten multiples of:

(a) 29 (b) 44 (c) 75 (d) 114
 (e) 299 (f) 350 (g) 1012 (h) 9123

- 3 List:

(a) the multiples of 4 between 29 and 53
 (b) the multiples of 50 less than 400
 (c) the multiples of 100 between 4000 and 5000.

- 4 Here are five numbers: 576, 396, 354, 792, 1164. Which of these are multiples of 12?

- 5 Which of the following numbers are not multiples of 27?

(a) 324 (b) 783 (c) 816 (d) 837 (e) 1116

The lowest common multiple (LCM)

The lowest common multiple of two or more numbers is the smallest number that is a multiple of all the given numbers.

Worked example 2

Find the lowest common multiple of 4 and 7.

$$M_4 = 4, 8, 12, 16, 20, 24, \mathbf{28}, 32$$

$$M_7 = 7, 14, 21, \mathbf{28}, 35, 42$$

$$\text{LCM} = 28$$

List several multiples of 4. (Note: M_4 means multiples of 4.)

List several multiples of 7.

Find the lowest number that appears in both sets. This is the LCM.

Exercise 1.4

- 1 Find the LCM of:

(a) 2 and 5 (b) 8 and 10 (c) 6 and 4
 (d) 3 and 9 (e) 35 and 55 (f) 6 and 11
 (g) 2, 4 and 8 (h) 4, 5 and 6 (i) 6, 8 and 9
 (j) 1, 3 and 7 (k) 4, 5 and 8 (l) 3, 4 and 18

FAST FORWARD

Later in this chapter you will see how prime factors can be used to find LCMs. ▶

- 2 Is it possible to find the highest common multiple of two or more numbers? Give a reason for your answer.

Factors

A **factor** is a number that divides exactly into another number with no remainder. For example, 2 is a factor of 16 because it goes into 16 exactly 8 times. 1 is a factor of every number. The largest factor of any number is the number itself.

Worked example 3

Find the factors of:

- (a) 12 (b) 25 (c) 110

(a) $F_{12} = 1, 2, 3, 4, 6, 12$

Find pairs of numbers that multiply to give 12:

1×12

2×6

3×4

Write the factors in numerical order.

(b) $F_{25} = 1, 5, 25$

1×25

5×5

Do not repeat the 5.

(c) $F_{110} = 1, 2, 5, 10, 11, 22, 55, 110$

1×110

2×55

5×22

10×11

F_{12} means the factors of 12.

To list the factors in numerical order go down the left side and then up the right side of the factor pairs. Remember not to repeat factors.

Exercise 1.5

- 1 List all the factors of:
- (a) 4 (b) 5 (c) 8 (d) 11 (e) 18
 (f) 12 (g) 35 (h) 40 (i) 57 (j) 90
 (k) 100 (l) 132 (m) 160 (n) 153 (o) 360
- 2 Which number in each set is not a factor of the given number?
- (a) 14 {1, 2, 4, 7, 14}
 (b) 15 {1, 3, 5, 15, 45}
 (c) 21 {1, 3, 7, 14, 21}
 (d) 33 {1, 3, 11, 22, 33}
 (e) 42 {3, 6, 7, 8, 14}
- 3 State true or false in each case.
- (a) 3 is a factor of 313 (b) 9 is a factor of 99
 (c) 3 is a factor of 300 (d) 2 is a factor of 300
 (e) 2 is a factor of 122 488 (f) 12 is a factor of 60
 (g) 210 is a factor of 210 (h) 8 is a factor of 420
- 4 What is the smallest factor and the largest factor of any number?

FAST FORWARD

Later in this chapter you will learn more about divisibility tests and how to use these to decide whether or not one number is a factor of another. ▶

1 Reviewing number concepts

The highest common factor (HCF)

The highest common factor of two or more numbers is the highest number that is a factor of all the given numbers.

Worked example 4

Find the HCF of 8 and 24.

$$F_8 = \underline{1}, \underline{2}, \underline{4}, \underline{8}$$

$$F_{24} = \underline{1}, \underline{2}, \underline{3}, \underline{4}, \underline{6}, \underline{8}, 12, 24$$

$$\text{HCF} = 8$$

List the factors of each number.

Underline factors that appear in both sets.

Pick out the highest underlined factor (HCF).

Exercise 1.6

FAST FORWARD

You will learn how to find HCFs by using prime factors later in the chapter. ▶

- Find the HCF of each pair of numbers.
 (a) 3 and 6 (b) 24 and 16 (c) 15 and 40 (d) 42 and 70
 (e) 32 and 36 (f) 26 and 36 (g) 22 and 44 (h) 42 and 48
- Find the HCF of each group of numbers.
 (a) 3, 9 and 15 (b) 36, 63 and 84 (c) 22, 33 and 121
- Not including the factor provided, find two numbers that have:
 (a) an HCF of 2 (b) an HCF of 6
- What is the HCF of two different prime numbers? Give a reason for your answer.

Word problems involving HCF usually involve splitting things into smaller pieces or arranging things in equal groups or rows.

Living maths

- Simeon has two lengths of rope. One piece is 72 metres long and the other is 90 metres long. He wants to cut both lengths of rope into the longest pieces of equal length possible. How long should the pieces be?
- Ms Sanchez has 40 canvases and 100 tubes of paint to give to the students in her art group. What is the largest number of students she can have if she gives each student an equal number of canvases and an equal number of tubes of paint?
- Indira has 300 blue beads, 750 red beads and 900 silver beads. She threads these beads to make wire bracelets. Each bracelet must have the same number and colour of beads. What is the maximum number of bracelets she can make with these beads?

1.3 Prime numbers

Prime numbers have exactly two factors: one and the number itself.

Composite numbers have more than two factors.

The number 1 has only one factor so it is not prime and it is not composite.

Finding prime numbers

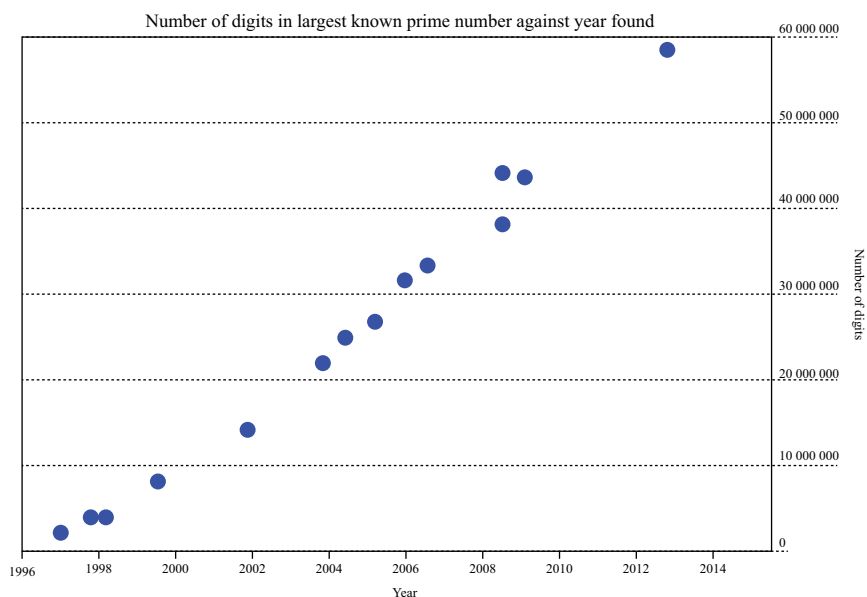
Over 2000 years ago, a Greek mathematician called Eratosthenes made a simple tool for sorting out prime numbers. This tool is called the 'Sieve of Eratosthenes' and the figure on page 7 shows how it works for prime numbers up to 100.

1 Reviewing number concepts

1	②	③	4	⑤	6	⑦	8	9	10	Cross out 1, it is not prime. Circle 2, then cross out other multiples of 2. Circle 3, then cross out other multiples of 3. Circle the next available number then cross out all its multiples. Repeat until all the numbers in the table are either circled or crossed out. The circled numbers are the primes.
⑪	12	⑬	14	15	16	⑰	18	⑱	20	
21	22	⑳	24	25	26	27	28	㉑	30	
⑳	32	33	34	35	36	㉗	38	39	40	
④①	42	④③	44	45	46	④⑦	48	49	50	
51	52	⑤③	54	55	56	57	58	⑤⑨	60	
⑥①	62	63	64	65	66	⑥⑦	68	69	70	
⑦①	72	⑦③	74	75	76	77	78	⑦⑨	80	
81	82	⑧③	84	85	86	87	88	⑧⑨	90	
91	92	93	94	95	96	⑨⑦	98	99	100	

You should try to memorise which numbers between 1 and 100 are prime.

Other mathematicians over the years have developed ways of finding larger and larger prime numbers. Until 1955, the largest known prime number had less than 1000 digits. Since the 1970s and the invention of more and more powerful computers, more and more prime numbers have been found. The graph below shows the number of digits in the largest known primes since 1996.



Today anyone can join the Great Internet Mersenne Prime Search. This project links thousands of home computers to search continuously for larger and larger prime numbers while the computer processors have spare capacity.

Exercise 1.7

FAST FORWARD

A good knowledge of primes can help when factorising quadratics in chapter 10. ▶

- Which is the only even prime number?
- How many odd prime numbers are there less than 50?
- List the composite numbers greater than four, but less than 30.
 - Try to write each composite number on your list as the sum of two prime numbers. For example: $6 = 3 + 3$ and $8 = 3 + 5$.
- Twin primes are pairs of prime numbers that differ by two. List the twin prime pairs up to 100.

1 Reviewing number concepts

Tip

Whilst super-prime numbers are interesting, they are not on the syllabus.

Remember a product is the answer to a multiplication. So if you write a number as the product of its prime factors you are writing it using multiplication signs like this:
 $12 = 2 \times 2 \times 3$.

Prime numbers only have two factors: 1 and the number itself. As 1 is not a prime number, do not include it when expressing a number as a product of prime factors.

Choose the method that works best for you and stick to it. Always show your method when using prime factors.

- 5 Is 149 a prime number? Explain how you decided.
- 6 Super-prime numbers are prime numbers that stay prime each time you remove a digit (starting with the units). So, 59 is a super-prime because when you remove 9 you are left with 5, which is also prime. 239 is also a super-prime because when you remove 9 you are left with 23 which is prime, and when you remove 3 you are left with 2 which is prime.
- (a) Find two three-digit super-prime numbers less than 400.
 (b) Can you find a four-digit super-prime number less than 3000?
 (c) Sondra's telephone number is the prime number 987-6413. Is her phone number a super-prime?

Prime factors

Prime factors are the factors of a number that are also prime numbers.

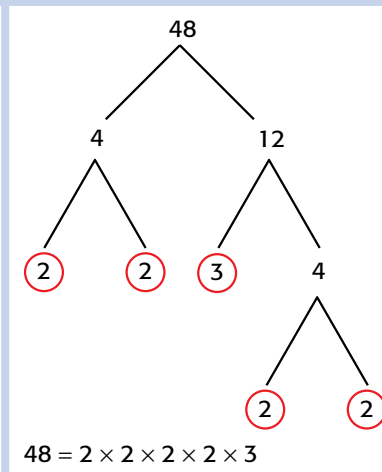
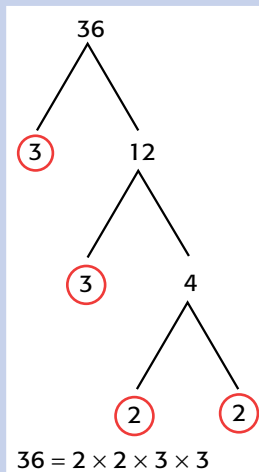
Every composite whole number can be broken down and written as the product of its prime factors. You can do this using tree diagrams or using division. Both methods are shown in worked example 5.

Worked example 5

Write the following numbers as the product of prime factors.

- (a) 36 (b) 48

Using a factor tree



Write the number as two factors.

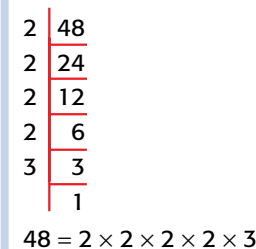
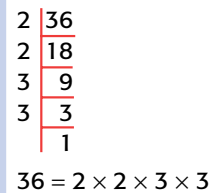
If a factor is a prime number, circle it.

If a factor is a composite number, split it into two factors.

Keep splitting until you end up with two primes.

Write the primes in ascending order with \times signs.

Using division



Divide by the smallest prime number that will go into the number exactly. Continue dividing, using the smallest prime number that will go into your new answer each time. Stop when you reach 1. Write the prime factors in ascending order with \times signs.

Exercise 1.8

When you write your number as a product of primes, group all occurrences of the same prime number together.

FAST FORWARD

You can also use prime factors to find the square and cube roots of numbers if you don't have a calculator. You will deal with this in more detail later in this chapter. ▶

1 Express the following numbers as the product of prime factors.

- (a) 30 (b) 24 (c) 100 (d) 225 (e) 360
 (f) 504 (g) 650 (h) 1125 (i) 756 (j) 9240

Using prime factors to find the HCF and LCM

When you are working with larger numbers you can determine the HCF or LCM by expressing each number as a product of its prime factors.

Worked example 6

Find the HCF of 168 and 180.

$$\begin{aligned} 168 &= \underline{2} \times \underline{2} \times 2 \times \underline{3} \times 7 \\ 180 &= \underline{2} \times \underline{2} \times \underline{3} \times 3 \times 5 \\ 2 \times 2 \times 3 &= 12 \\ \text{HCF} &= 12 \end{aligned}$$

First express each number as a product of prime factors. Use tree diagrams or division to do this. Underline the factors *common* to both numbers. Multiply these out to find the HCF.

Worked example 7

Find the LCM of 72 and 120.

$$\begin{aligned} 72 &= \underline{2} \times \underline{2} \times 2 \times \underline{3} \times \underline{3} \\ 120 &= 2 \times 2 \times 2 \times 3 \times 5 \\ 2 \times 2 \times 2 \times 3 \times 3 \times 5 &= 360 \\ \text{LCM} &= 360 \end{aligned}$$

First express each number as a product of prime factors. Use tree diagrams or division to do this. Underline the *largest* set of multiples of each factor. List these and multiply them out to find the LCM.

Exercise 1.9

1 Find the HCF of these numbers by means of prime factors.

- (a) 48 and 108 (b) 120 and 216 (c) 72 and 90 (d) 52 and 78
 (e) 100 and 125 (f) 154 and 88 (g) 546 and 624 (h) 95 and 120

2 Use prime factorisation to determine the LCM of:

- (a) 54 and 60 (b) 54 and 72 (c) 60 and 72 (d) 48 and 60
 (e) 120 and 180 (f) 95 and 150 (g) 54 and 90 (h) 90 and 120

3 Determine both the HCF and LCM of the following numbers.

- (a) 72 and 108 (b) 25 and 200 (c) 95 and 120 (d) 84 and 60

Living maths

Word problems involving LCM usually include repeating events. You may be asked how many items you need to 'have enough' or when something will happen again at the same time.

- 4 A radio station runs a phone-in competition for listeners. Every 30th caller gets a free airtime voucher and every 120th caller gets a free mobile phone. How many listeners must phone in before one receives both an airtime voucher *and* a free phone?
- 5 Lee runs round a track in 12 minutes. James runs round the same track in 18 minutes. If they start in the same place, at the same time, how many minutes will pass before they both cross the start line together again?

1 Reviewing number concepts

Tip

Divisibility tests are not part of the syllabus. They are just useful to know when you work with factors and prime numbers.

Divisibility tests to find factors easily

Sometimes you want to know if a smaller number will divide into a larger one with no remainder. In other words, is the larger number divisible by the smaller one?

These simple divisibility tests are useful for working this out:

A number is exactly divisible by:

- 2 if it ends with 0, 2, 4, 6 or 8 (in other words is even)
- 3 if the sum of its digits is a multiple of 3 (can be divided by 3)
- 4 if the last two digits can be divided by 4
- 5 if it ends with 0 or 5
- 6 if it is divisible by both 2 and 3
- 8 if the last three digits are divisible by 8
- 9 if the sum of the digits is a multiple of 9 (can be divided by 9)
- 10 if the number ends in 0.

There is no simple test for divisibility by 7, although multiples of 7 do have some interesting properties that you can investigate on the internet.

Exercise 1.10

23	65	92	10	104	70	500	21	64	798	1223
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- 1 Look at the box of numbers above. Which of these numbers are:
 - (a) divisible by 5?
 - (b) divisible by 8?
 - (c) divisible by 3?
- 2 Say whether the following are true or false.

(a) 625 is divisible by 5	(b) 88 is divisible by 3
(c) 640 is divisible by 6	(d) 346 is divisible by 4
(e) 476 is divisible by 8	(f) 2340 is divisible by 9
(g) 2890 is divisible by 6	(h) 4562 is divisible by 3
(i) 40090 is divisible by 5	(j) 123 456 is divisible by 9
- 3 Can \$34.07 be divided equally among:
 - (a) two people?
 - (b) three people?
 - (c) nine people?
- 4 A stadium has 202 008 seats. Can these be divided equally into:
 - (a) five blocks?
 - (b) six blocks?
 - (c) nine blocks?
- 5
 - (a) If a number is divisible by 12, what other numbers must it be divisible by?
 - (b) If a number is divisible by 36, what other numbers must it be divisible by?
 - (c) How could you test if a number is divisible by 12, 15 or 24?

1.4 Powers and roots

REWIND

In section 1.1 you learned that the product obtained when an integer is multiplied by itself is a square number. ◀

Square numbers and square roots

A number is squared when it is multiplied by itself. For example, the **square** of 5 is $5 \times 5 = 25$. The symbol for squared is 2 . So, 5×5 can also be written as 5^2 .

The **square root** of a number is the number that was multiplied by itself to get the square number. The symbol for square root is $\sqrt{\quad}$. You know that $25 = 5^2$, so $\sqrt{25} = 5$.

Cube numbers and cube roots

A number is cubed when it is multiplied by itself and then multiplied by itself again. For example, the **cube** of 2 is $2 \times 2 \times 2 = 8$. The symbol for cubed is 3 . So $2 \times 2 \times 2$ can also be written as 2^3 .