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Groups, Graphs and Random Walks

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Contents

Preface

The current volume brings together several contributions from the invited speakers and guests of the workshop ‘Groups, Graphs and Random Walks’ held in Cortona (Italy) on June 2 to 6, 2014, on the occasion of the sixtieth anniversary of Wolfgang Woess.

Wolfgang was born in Vienna on July 23, 1954, to Friedrich and Elisabeth Woess, both professors at the University of Vienna. His father was also a gifted painter: when visiting Wolfgang, one immediately gets attracted to Friedrich Woess’s beautiful watercolor landscapes adorning the walls of his office at the university as well as of his cosy home.

Wolfgang studied mathematics at the Technical University of Vienna, where he obtained his diploma, at the University of Munich, and at the University of Salzburg, where he obtained his PhD under the supervision of Peter Gerl. After a period as an assistant professor at the Montanuniversität Leoben (1984–1989)—including a leave of absence at the University of Rome ‘La Sapienza’ (1984–1985), where he started a long and fruitful collaboration with the Italian Harmonic Analysis group led by Alessandro Figà-Talamanca—and eleven years as a professor at the University of Milan (1988–1999), he eventually became Professor at the Graz University of Technology in 1999, where he currently serves as the chair of the Institute of Discrete Mathematics.

Wolfgang has been and still is, for many of us, a great teacher, a colleague, and a dear friend. As a teacher, he had thirteen PhD students (essentially from the University of Milan and the Graz University of Technology) and several postdoctoral fellows who have obtained important recognition both at the scientific and the academic levels.

His publications (nearly a hundred) range among various mathematical subjects, including convolution powers of probability measures on groups and asymptotics of random walk transition probabilities (at the very beginning of Wolfgang’s research); recurrence, spectral radius and amenability, and spectral computations; boundary theory and harmonic functions; infinite electrical networks; context-free languages and their relations with groups and random walks; infinite graphs and groups; random walks on affine groups, buildings, horocyclic products, and lamplighter groups; finally and more recently, reflected random walks and stochastic dynamical systems; Brownian motion on strip ('quantum') complexes, treebolic spaces and SOL Geometry, and Markov processes on ultra-metric spaces. The long list of collaborators (more than thirty) includes, in order of multiplicity: Massimo Picardello, Laurent Saloff-Coste, Donald Cartwright, Vadim Kaimanovich and his former student Sara Brofferio.

One should also mention his beautiful and masterly written monographs *Random Walks on Infinite Graphs and Groups* (Cambridge University Press, 2000) and *Denumerable Markov Chains—Generating Functions, Boundary Theory, Random Walks on Trees* (European Mathematical Society Publishing House, 2009).

As mentioned, in the present volume we collect some papers contributed by participants to the Cortona conference: the themes are all intimately related to Wolfgang's research interests and scientific production. Here we overview, with a brief description, these contributions.

Growth of Groups and Wreath Products

Laurent Bartholdi (Georg-August University of Göttingen)

The central theme of this survey chapter is the Bartholdi–Erschler construction, via wreath products, of many groups of diverse types of growth: either intermediate, with many different growth functions, or of non-uniform exponential growth. On the way, Bartholdi discusses current hot topics of geometric group theory such as self-similar groups, branch groups, finite-automata groups, rooted trees, complete growth series and the like.

Random Walks on Some Countable Groups

Alexander Bendikov (Wroclaw University) and Laurent Saloff-Coste (Cornell University)

The chapter by Bendikov and Saloff-Coste studies decay of convolution powers of probability measures on non-finitely generated countable groups. Their methods are primarily based on explicit calculations of convolution powers of convex combinations of Haar measures and comparison techniques. It contains, in particular, an interesting collection of precise estimates for random walks on the infinite symmetric group $S^{(\infty)}$.

The Cost of Distinguishing Graphs

Debra Boutin (Hamilton College) and Wilfried Imrich (Leoben University)

Boutin and Imrich study the notion of distinguishing cost of a graph, recently introduced by the first author, which is defined as the smallest size of a vertex set whose set-wise stabilizer in the automorphism group is trivial, and therefore constitutes a measure of the symmetry of the given graph. Clearly, it exists if and only if the distinguishing number (minimal number of colors needed for a coloring, which is not preserved by any non-trivial automorphism) is at most two. Furthermore, it is always bounded from below by the minimal size of a base (set whose point-wise stabilizer is trivial). Thus, the distinguishing cost could serve as a finer measure of the degree of symmetry for graphs with equal distinguishing number.

A Construction of the Measurable Poisson Boundary

Sara Brofferio (Paris-Sud University)

The chapter by Brofferio addresses an important problem about Poisson boundaries of random walks. Recall that, given a measure μ on a locally compact group G, the Poisson boundary is a measurable G-space (X, ν) with $\mu * \nu = \nu$ such that the Poisson transform $\phi \mapsto f_\phi(g) := \int_X \phi(gx)\,d\nu(x)$ defines an isometry of $L^\infty(X, \rho * \nu)$ onto the space $\mathcal{H}^\infty_\lambda(G)$ of bounded λ-a.e. μ-harmonic functions on G (here ρ is a probability measure on G equivalent to the Haar measure λ, and a function $f\colon G \to \mathbb{R}$ is termed λ-a.e. μ-harmonic if $f(g) = \int_G f(gh)\,d\mu(h)$ for λ-a.e. $g \in G$). When μ is supported on a dense countable subgroup Γ of G, there are two notions of Poisson boundary: one (as above) on G and one on Γ endowed with the discrete topology and the counting measure. In this chapter a kind of inductive construction is proposed to obtain the G-Poisson boundary from the Γ-Poisson boundary. Consider the action of Γ on $G \times X$ defined by $\gamma * (g, x) := (g\gamma^{-1}, \gamma x)$. Then Brofferio proves that the quotient space associated with the Γ-invariant sets is a kind of G-Poisson boundary. This is applied to describe the $\mathrm{Aff}(p, \mathbb{R})$ Poisson boundary of the Baumslag–Solitar group $\mathrm{BS}(1, p)$, where $\mathrm{Aff}(p, \mathbb{R})$ is the closure of the usual representation of $\mathrm{BS}(1, p)$ in the group $\mathrm{Aff}(\mathbb{R})$ of affine transformation of the real line.

Structure Trees, Networks and Almost Invariant Sets

Martin J. Dunwoody (University of Southampton)

Stallings' celebrated theorem (1968) about ends of groups states that a finitely generated group G has more than one end if and only if it admits a non-trivial decomposition as an amalgamated free product or an HNN extension over a finite subgroup. In the modern language of Bass–Serre theory, the theorem says that a finitely generated group G has more than one end if and only if it admits a non-trivial (that is, without a global fixed point) action on a simplicial tree with finite edge-stabilizers and without edge-inversions. This fundamental result, together with a question formulated by Wall, was a starting point for Dunwoody's accessibility theory: a finitely generated group G is said to be accessible if the process of iterated nontrivial splittings of G over finite subgroups always terminates in a finite number of steps. Dunwoody (1985) proved that every finitely presented group is accessible; he later showed that there do exist finitely generated groups that are not accessible. The notion of accessibility was later extended to the graph setting by Thomassen and Woess (1993): a graph is accessible if there is an integer n such that any two ends can be separated by removing at most n edges; in the same chapter, the authors obtain a number of results using structure trees. Dunwoody gives a self-contained account of the theory, initiated with the aforementioned works of Stallings and later developed by Dicks and Dunwoody, of structure trees for edge cuts in networks. Applications include a generalization of the Max-Flow Min-Cut theorem to infinite networks, a short proof of a conjecture of Kropholler, a relative version of Stallings theorem and a generalization of the Almost Stability theorem by Dicks and Dunwoody.

Amenability of Trees
Behrang Forghani (University of Connecticut) and Keivan Mallahi-Karai (Jacobs University of Bremen)
The chapter by Forghani and Mallahi-Karai gives a necessary and sufficient condition for a tree to be amenable. As an application of this result, it is proven that a Galton–Watson tree is, under some specific conditions, almost surely amenable.

Group Walk Random Groups
Agelos Georgakopoulos (University of Warwick)
The chapter by Georgakopoulos discusses a new class of random graphs that combines ideas from random graph theory as well as random walks. Take an infinite graph, and let G_n denote the intrinsic ball of radius n around a root vertex. Then construct the random graph on the boundary $\partial G_n = G_n \setminus G_{n-1}$ by letting random walks start in all the vertices in ∂G_n and connecting $x, y \in \partial G_n$ when the random walk starting in x leaves G_n in y, or vice versa. This is an entirely novel construction of a random graph, somehow interpolating between normal random graphs and random walk interlacements. Georgakopoulos studies various properties of the graph, such as the number of connections between macroscopic parts of ∂G_n as $n \to \infty$, relations to the Poisson boundary of graphs and effective conductances, and the Doob–Naïm's kernel.

Ends of Branching Random Walks on Planar Hyperbolic Cayley Graphs
Lorenz A. Gilch (Graz University of Technology) and Sebastian Müller (Aix-Marseille University)
Properties of branching random walks (BRWs) on a graph G are of interest when the trace of the process (the random subgraph formed of vertices visited by particles of BRW) is a proper subgraph of G. The authors study the trace of transient BRWs when G is a planar hyperbolic

Cayley graph: in this case it is shown that the trace of BRW has, almost surely, infinitely many ends.

Amenability and Ergodic Properties of Topological Groups: From Bogolyubov Onwards

Rostislav Grigorchuk (Texas A&M University) and Pierre de la Harpe (University of Geneva)

The theory of amenable groups emerged from the study of the axiomatic properties of the Lebesgue integral and the discovery of the Hausdorff–Banach–Tarski paradox at the beginning of the last century. The first definition of an amenable group, by the existence of an invariant finitely additive probability measure, is due to von Neumann in his 1929 seminal paper. The term *amenable* was introduced in the 1950s by M.M. Day, who played a central role in the development of the modern theory of amenability by using means and applying techniques from functional analysis. In these references, groups do not have topology; in other words, they are just 'discrete groups'. Amenability is considered explicitly for topological groups in later articles by Dixmier, Fomin and Rickart and (for locally compact groups) by Greenleaf in his influential book. The chapter by Grigorchuk and de la Harpe is a survey on amenability and ergodicity for topological groups (as opposed to locally compact groups), as in Bogolyubov's 1939 paper, emphasizing the characterizations true in general and those true only in the locally compact case. This is of particular importance in view of the recent renewed interest in 'large' groups, for example, topological full groups of Cantor minimal dynamical systems (these groups are sources of infinite simple non-elementary amenable groups).

Schreier Graphs of Grigorchuk's Group and a Subshift Associated to a Non-Primitive Substitution

Rostislav Grigorchuk (Texas A&M University), Daniel Lenz (Friedrich Schiller University Jena) and Tatiana Nagnibeda (University of Geneva)

The authors describe a remarkable connection between a class of Laplace-type operators on Schreier graphs associated with the Grigorchuk group of intermediate growth and a class of Schrödinger operators with potentials defined by sampling over a strictly ergodic aperiodic subshift defined by a non-primitive substitution. This beautifully elucidated connection provides an example of a reduction of a hard (read: not strictly one-dimensional) problem to an easy (read: one-dimensional) problem. From this point of view, it is reminiscent of work on the XY spin chain, which reduces the hard (many-body) problem to an easy (one-particle) effective Hamiltonian via the Jordan–Wigner transformation. This chapter also includes a substantial and clear discussion of the spectral characteristics of aperiodic one-dimensional Schröödinger operators, the definition and properties of the Grigorchuk group, the structure of the substitution τ

and its subshift, and the relationship between this subshift and the graphs associated with the action of Grigorchuk's group on the infinite binary tree.

Thompson's Group F Is Not Liouville

Vadim Kaimanovich (University of Ottawa)

Thompson's group F, introduced by Richard Thompson in 1965, is the (finitely presented, infinite) group consisting of all orientation preserving piecewise-linear dyadic self-homeomorphisms of the closed unit interval $[0,1]$. One of the most important open questions in geometric group theory is whether or not Thompson's group F is amenable. Note that Brin and Squier proved that F does not contain non-abelian free subgroups: as a consequence, if F is not amenable, then it would constitute another counterexample to the so-called 'von Neumann conjecture', which stated that a finitely generated group is amenable if and only if it does not contain non-abelian free subgroups. The first finitely generated (resp. finitely presented) counterexamples to this conjecture were found by Olshanskii in 1980 using his Tarski monsters (resp. by Olshanskii and Sapir in 2002).

The classical Liouville theorem asserts that the only bounded harmonic functions on Euclidean space are the constants. Now, the notion of a harmonic function (based on the mean value property) can in fact be defined for an arbitrary Markov chain and, in particular, for random walks on groups. Given a probability measure μ on a group G, denote by $H^\infty(G,\mu) = \{f \in \ell^\infty(G) : f = f * \mu\}$ the space of all bounded μ-harmonic functions on G. One then says that the random walk (G,μ) is Liouville if $H^\infty(G,\mu)$ consists only of the constant functions. Furstenberg (1973) proved that any group carrying a non-degenerate Liouville random walk is amenable. One calls a group G Liouville if (G,μ) is Liouville for any symmetric and finitely supported probability measure μ on G. In his chapter, Kaimanovich shows that Richard Thompson's group F is not Liouville. More precisely, he proves that the random walk on F driven by any strictly non-degenerate finitely supported probability measure μ has a non-trivial Poisson boundary.

An Alternative Proof of the Subadditive Ergodic Theorem

Anders Karlsson (University of Geneva and Uppsala universitet)

Karlsson's chapter contains a self-contained, clean and short proof of Kingman's celebrated subadditive ergodic theorem. The scheme of the proof follows the classical proof of Birkhoff's ergodic theorem, via the maximal ergodic lemma. There are, however, two important differences: first, the maximal ergodic lemma is replaced by a result by Derriennic (1975), which yields non-positivity of a limit integral $\lim \int_B \frac{a(n,x)}{n}\, d\mu(x)$ (as opposed to the usual $\int_B a(1,x)\, d\mu(x)$, which is the same in the case of additive cocycles), and, second, there is a clever argument at the end of the proof to derive everywhere convergence in the subadditive case, from the same statement for additive cocycles.

Boundaries of $\mathbb{Z}^n$-Free Groups

Andrei Malyutin (St. Petersburg Department of V.A. Steklov Mathematical Institute), Tatiana Nagnibeda (University of Geneva) and Denis Serbin (Stevens Institute of Technology)

Given an arbitrary ordered Abelian group Λ (for instance, $\mathbb{Z}$ or $\mathbb{R}$), a Λ-tree is a metric space whose metric takes values in Λ and is subject to certain tree axioms. This notion goes back to the early 1960s, when Lyndon introduced the notion of abstract length functions on groups, and, a few years later, to Chiswell, who related such length functions with group actions on $\mathbb{Z}$- and $\mathbb{R}$-trees – providing a construction of the tree on which the group acts – and to Tits, who explicitly gave the first formal definition of an $\mathbb{R}$-tree. This theory has significantly developed since then, the main problem being addressed is to find the group theoretic information carried by a Λ-tree action, in particular, the structure of Λ-free groups. Malyutin, Nagnibeda, and Serbin study the Poisson boundary of groups acting on $\mathbb{Z}^n$-trees (here $\mathbb{Z}^n$ is equipped with the right lexicographic order). The groups considered in this chapter constitute a natural generalization of free groups; the class of $\mathbb{Z}^n$-free groups includes, in particular, all limit groups, and is closed under taking subgroups, free products and amalgamated free products along maximal cyclic subgroups. The authors provide a construction of the Poisson boundary for these groups directly in terms of the action of the group on the $\mathbb{Z}^n$-tree.

Buildings, Groups of Lie Type, and Random Walks

James Parkinson (University of Sydney)

Parkinson masterly surveys the fascinating theory of random walks on buildings and associated groups of Lie type and Kac–Moody groups. The author does a beautiful job of explaining ideas that are potentially quite technical and provides a comprehensive update of recent results dealing with probability theory on groups of Lie type defined over other p-adic fields, and extensions of these results into the setting of Kac–Moody groups. The unifying feature is the combinatorial-geometrical notion of a building, introduced by Tits in the 1950s in his successful attempt to give a uniform geometric interpretation of semi-simple Lie groups. Here the author focuses on the classes of buildings on which random walks have been studied, including spherical buildings, affine buildings (playing an important role in the study of Lie groups over p-adic fields), and Fuchsian and twin buildings (extensively used in the theory of Kac–Moody groups) and shows how the theory of random walks on buildings leads to limit theorems for random walks on the associated groups.

On Some Random Walks Driven by Spread-out Measures

Laurent Saloff-Coste (Cornell University) and Tianyi Zheng (Stanford University)

The chapter by Saloff-Coste and Zheng concerns heat kernel estimates for random walks on finitely generated groups and focuses on the case when the groups have polynomial volume growth and the initial distribution of the random walk has a suitable (slow) decay. The techniques used allow for the treatment of a variety of examples. The authors also give an application of the method to wreath products.

Topics in Mathematical Cristallography

Toshikazu Sunada (Meiji University)

The volume ends with the survey by Sunada. It is based on his book *Topological Crystallography* and, in connection with tight frames, sheds light, with a new geometric insight, on the relationship of his study on the standard realization of crystal lattices with several different topics such as tight frames in Euclidean space, crystallography, algebraic geometry (rational points in Grassmannians, the Abel–Jacobi map), number theory (quadratic Diophantine equations) and combinatorics (spherical designs).

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Rome, Milano and Graz TCS, MS and ESH

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Sono davvero commosso.

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Conference Photographs

The conference participants at the Palazzone (photo courtesy of W. Woess).

The conference participants at Piazza Garibaldi in Cortona
(photo courtesy of L. Bartholdi).

Wolfgang Woess with some of his (former) PhD students
(photo courtesy of J. Kloas).

Near the Palazzone (photo courtesy of W. Woess).

Typical math discussions during a coffee break inside the Palazzone (photo courtesy of A. Georgakopoulos).

At the conference dinner (photo courtesy of L. Bartholdi).