DIFFERENTIAL GEOMETRY

OF THREE DIMENSIONS

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PREFACE TO THE FOURTH IMPRESSION

THE present impression is substantially a reprint of the original work. Since the book was first published a few errors have been corrected, and one or two paragraphs rewritten. Among the friends and correspondents who kindly drew my attention to desirable changes were Mr A. S. Ramsey of Magdalene College, Cambridge, who suggested the revision of § 5, and the late R. J. A. Barnard of Melbourne University, whose influence was partly responsible for my initial interest in the subject.

The demand for the book, since its first appearance twenty years ago, has justified the writer's belief in the need for such a vectorial treatment. By the use of vector methods the presentation of the subject is both simplified and condensed, and students are encouraged to reason geometrically rather than analytically. At a later stage some of these students will proceed to the study of multidimensional differential geometry and the tensor calculus. It is highly desirable that the study of the geometry of Euclidean 3-space should thus come first; and this can be undertaken with most students at an earlier stage by vector methods than by the Ricci calculus. A student's appreciation of the more general case will undoubtedly be enhanced by an earlier acquaintance with differential geometry of three dimensions.

The more elementary parts of the subject are discussed in Chapters I–XI. The remainder of the book is devoted to differential invariants for a surface and their applications. It will be apparent to the reader that these constitute a powerful weapon for analysing the geometrical properties of surfaces, and of systems of curves on a surface. The unit vector, $\mathbf{n}$, normal to a surface at the current point, plays a prominent part in this discussion. The first curvature of the surface is the negative of the divergence of $\mathbf{n}$; while the second curvature is expressible simply in terms of the divergence and the Laplacian of $\mathbf{n}$ with respect to the surface.

Extensive applications of these invariants to the geometry of surfaces are given in the second volume of this book. Applications to physical problems connected with curved surfaces have been given elsewhere* by the author.

* 1. On differential invariants in geometry of surfaces, with some applications to mathematical physics. *Quarterly Journal of Mathematics*, Vol. 50, pp. 230–69 (Cambridge, 1925).

2. On small deformation of surfaces and of thin elastic shells. *Ibid.*, Vol. 50, pp. 272–96 (1925).

3. On the motion of an extensible membrane in a given curved surface. *Phil. Mag.*, Vol. 23, pp. 573–80 (1937).

4. On transverse vibrations of curved membranes. *Phil. Mag.*, Vol. 28, pp. 632–34 (1939).

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