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1
Banach manifolds and bundles

The geometry of Banach manifolds and bundles has been greatly developed since the 1960s and now there are many papers and a number of books covering a great variety of related topics. Here we intend to fix our notation and give a brief account of the basic results which will be used in the main part of the present work. Occasionally, some topics are dealt with in more detail. These refer to subjects either not easily found in the literature or their methods have a particular interest and cover explicit needs of the exposition.

1.1 Banach manifolds

The main references for this section are [AMR88], [AR67], [Bou67], and [Lan99], where the reader may find the necessary details.

1.1.1 Ordinary derivatives in Banach spaces

Let $\mathbb{E}$ and $\mathbb{F}$ be two Banach spaces. We denote by $\mathcal{L}(\mathbb{E}, \mathbb{F})$ the (Banach) space of continuous linear maps between $\mathbb{E}$ and $\mathbb{F}$. In particular, we set $\mathcal{L}(\mathbb{E}) := \mathcal{L}(\mathbb{E}, \mathbb{E})$, which is a Banach algebra. On the other hand, $\mathcal{L}is(\mathbb{E}, \mathbb{F})$ denotes the (open) set of invertible elements (viz. linear isomorphisms) of $\mathcal{L}(\mathbb{E}, \mathbb{F})$, while $\mathcal{L}is(\mathbb{E}) := \mathcal{L}is(\mathbb{E}, \mathbb{E})$. The latter space, viewed as a group under the composition of automorphisms, is denoted by $\mathrm{GL}(\mathbb{E})$ and is called the ***general linear group*** of $\mathbb{E}$.

A map $f : U \to \mathbb{F}$ ($U \subseteq \mathbb{E}$ open) is called ***differentiable at*** x if there exists a map $Df(x) \in \mathcal{L}(\mathbb{E}, \mathbb{F})$, the (Fréchet) ***derivative of*** f ***at*** x, such

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that

$$\lim_{h\to 0} \frac{\|f(x+h)-f(x)-[Df(x)](h)\|}{h} = 0 \qquad (h \neq 0).$$

The (***total***) ***derivative***, or ***differential***, of f is $Df\colon U \to \mathcal{L}(\mathbb{E},\mathbb{F})$. If Df is continuous, then we say that f is of class C^1. Inductively, we set

$$D^k f = D(D^{k-1}f)\colon U \longrightarrow \mathcal{L}^k(\mathbb{E},\mathbb{F}) \equiv \mathcal{L}(\mathbb{E},\mathcal{L}^{k-1}(\mathbb{E},\mathbb{F})),$$

if the latter derivative exists. The map f will be called ***smooth***, or (of class) C^∞, if the derivatives D^k exist for every k and are continuous.

For an excellent treatment of the differential calculus in Banach spaces we refer also to [Car67(a)].

1.1.2 Smooth structures

A ***Banach manifold*** M is a smooth manifold whose differential structure is determined by local charts of the form $(U,\phi,\mathbb{B})$, where the ***ambient space*** or ***model*** $\mathbb{B}$ is a Banach space. If all the charts have the same model $\mathbb{B}$ (a fact ensured in the case of a connected manifold), we say that B is ***modelled on*** $\mathbb{B}$ or it is a $\mathbb{B}$-***manifold***. If there is no ambiguity about the model, the charts will be simply denoted by (U,ϕ). The (maximal) ***atlas*** inducing the differential structure is denoted by $\mathcal{A}$.

A Banach space $\mathbb{B}$ is a Banach manifold whose differential structure is determined by the global chart $(\mathbb{B},\mathrm{id}_{\mathbb{B}})$.

For the sake of simplicity, unless otherwise stated, differentiability is assumed to be of class $\mathcal{C}^\infty$, a synonym of smoothness. Usually, a Banach manifold is assumed to be *Hausdorff*, equipped with smooth *partitions of unity*.

1.1.3 Smooth maps

A map $f\colon M \to N$ is said to be ***smooth at*** $x \in M$ if there are charts (U,ϕ) and (V,ψ) of M and N, respectively, such that $x \in U$, $f(U) \subseteq V$, and the ***local representation*** or ***representative*** of f, with respect to the previous charts,

$$\psi \circ f \circ \phi^{-1}\colon \phi(U) \longrightarrow \psi(V) \tag{1.1.1}$$

is smooth at $f(x)$ in the sense of ordinary differentiability in Banach spaces. Short-hand notations for (1.1.1) are f_{VU} or $f_{\psi\phi}$. We also write $f_{\beta\alpha}$ for the local representation of f with respect to the charts (U_α,ϕ_α) and (U_β,ϕ_β), with $f(U_\alpha) \subseteq U_\beta$.

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1.1.4 The tangent space

A ***smooth curve at*** $x \in M$ is a smooth map $\alpha\colon J \to M$ with $\alpha(0) = x$, where J is an open interval of $\mathbb{R}$ containing 0. Two curves α and β at x are called ***equivalent*** or ***tangent*** if there is a chart (U, ϕ) at x such that

$$(\phi \circ \alpha)'(0) = (\phi \circ \beta)'(0) \tag{1.1.2}$$

Here we have that

$$(\phi \circ \alpha)'(t) = [D(\phi \circ \alpha)(t)](1), \tag{1.1.3}$$

for every $t \in J$ such that $\alpha(t) \in U$. Clearly, (1.1.2) is equivalent to

$$D(\phi \circ \alpha)(0) = D(\phi \circ \beta)(0). \tag{1.1.2$'$}$$

The equivalence classes of curves as above are denoted by $[(\alpha, x)]$ (or $[\alpha, x]$ for complicated expressions of curves) and are called ***tangent vectors at*** x. The set of all tangent vectors at x is the ***tangent space at*** x, denoted by T_xM.

Considering any chart $(U, \phi) \equiv (U, \phi, \mathbb{B})$ at x, we check that T_xM is in a bijective correspondence with $\mathbb{B}$ by means of the map

$$\overline{\phi}\colon T_xM \longrightarrow \mathbb{B}\colon [(\alpha, x)] \mapsto (\phi \circ \alpha)'(0). \tag{1.1.4}$$

Therefore, T_xM becomes a Banach space and $\overline{\phi}$ a continuous linear isomorphism. The Banach structure of T_xM is independent of the choice of the chart containing x. This is an immediate consequence of the following fact: If $(U, \phi, \mathbb{B})$ and $(U, \psi, \mathbb{B}')$ are two charts at x, then the following diagram is commutative:

$$\begin{tikzcd}
& T_xM \arrow[dl, "\overline{\phi}"'] \arrow[dr, "\overline{\psi}"] & \\
\mathbb{B} \arrow[rr, "D(\psi \circ \phi^{-1})(\phi(x))"'] & & \mathbb{B}'
\end{tikzcd}$$

Considering a Banach space $\mathbb{B}$ as a smooth manifold, the tangent space $T_b\mathbb{B}$, for every $b \in \mathbb{B}$, is identified with $\mathbb{B}$ by means of $\overline{\mathrm{id}}_{\mathbb{B}}$ (see § 1.1.2). In particular, $T_t\mathbb{R}$ is an 1-dimensional vector space, with the natural basis

$$\left.\frac{d}{dt}\right|_t := \overline{\mathrm{id}}_{\mathbb{R}}^{\,-1}(1). \tag{1.1.5}$$

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1.1.5 The tangent bundle

As usual, the ***tangent bundle*** of a (Banach) manifold M is determined by the triple (TM, M, τ_M), where

$$TM := \dot{\bigcup_{x\in M}} T_xM \equiv \bigsqcup_{x\in M} T_xM$$

(disjoint union) is the ***total space*** and $\tau_M : TM \to M$ the ***projection*** of the tangent bundle, with $\tau_M([(\alpha, x)]) := x$.

The total space TM is a Banach manifold, whose structure is induced as follows: Given a local chart $(U, \phi) \equiv (U, \phi, \mathbb{B})$, we define the map

$$\Phi\colon \pi^{-1}(U) \longrightarrow \phi(U) \times \mathbb{B} \tag{1.1.6}$$

by setting

$$\Phi(u) := \big(\tau_M(u), \overline{\phi}(u)\big) = \big(x, (\phi \circ \alpha)'(0)\big), \tag{1.1.7}$$

if $u = [(\alpha, x)] \in T_xM$ and $x \in U$. Then the collection of all pairs $(\pi^{-1}(U), \Phi)$, obtained by running (U, ϕ) in the maximal atlas of M, determines a smooth atlas on TM, whose maximal counterpart induces the desired smooth structure on TM.

1.1.6 The differential of a smooth map

The tangent spaces and the tangent bundle provide the appropriate framework for the development of a differential calculus on manifolds. Precisely: if $f : M \to N$ is a smooth map between two Banach manifolds, then the ***differential*** or ***tangent map of*** f ***at*** x is the map

$$T_xf : T_xM \longrightarrow T_{f(x)}N, \tag{1.1.8}$$

given by

$$T_xf([(\alpha, x)]) := [(f \circ \alpha, f(x))]. \tag{1.1.9}$$

This is a well-defined continuous linear map, independent of the choice of the representatives of the tangent vectors.

In various computations, the differential T_xf is handled by using local charts and the derivative of the corresponding local representation (1.1.1) of f. More precisely, if $(U, \phi, \mathbb{E})$ and $(V, \psi, \mathbb{F})$ are local charts of M and N, respectively, such that $x \in U$ and $f(U) \subseteq V$ (as ensured by

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the smoothness of f at x), then one proves that the next diagram is commutative.

$$\begin{tikzcd} T_xM \arrow[r, "T_xf"] \arrow[d, "\overline{\phi}"'] & T_{f(x)}N \arrow[d, "\overline{\psi}"] \\ \mathbb{E} \arrow[r, "D(\psi\circ f\circ\phi^{-1})(\phi(x))"'] & \mathbb{F} \end{tikzcd}$$

In particular, differentiating the map ϕ of a chart $(U, \phi, \mathbb{B})$, we obtain the following commutative diagram:

$$\begin{tikzcd} T_xU \equiv T_xM \arrow[r, "T_x\phi"] \arrow[dr, "\overline{\phi}"'] & T_{\phi(x)}\mathbb{B} \arrow[d, "\overline{\mathrm{id}}_{\mathbb{B}}"] \\ & \mathbb{B} \end{tikzcd}$$

Frequently, omitting $\overline{\mathrm{id}}_{\mathbb{B}}$, we simply write

$$\overline{\phi} \equiv T_x\phi. \tag{1.1.10}$$

1.1.7 Velocity vectors

Let $\alpha\colon J \to M$ be a smooth curve. The ***tangent*** or ***velocity vector at*** $\alpha(t)$ (or, simply, at t) is the vector

$$\dot{\alpha}(t) := T_t\alpha\Big(\frac{d}{dt}\Big|_t\Big) \in T_{\alpha(t)}M. \tag{1.1.11}$$

In particular, if α passes through x, i.e. $\alpha(0) = x$, then

$$\dot{\alpha}(t) = [(\alpha, x)]. \tag{1.1.12}$$

If the curve has a more complicated form, e.g. $f \circ \alpha$, then the corresponding velocity vector is denoted by $(f\circ\alpha)^{\boldsymbol{\cdot}}(t)$ instead of $\overset{\bullet}{\overbrace{(f\circ\alpha)}}(t)$.

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1.1.8 The tangent map

Let $f\colon M \to N$ be a smooth map. The ***tangent map*** or ***(total) differential*** of f is obtained by gluing together the differentials $T_x f$, for all $x \in M$; that is,

$$Tf\colon TM \longrightarrow TN\colon Tf\big|_{T_xM} = T_x f. \tag{1.1.13}$$

The following diagram is also commutative:

$$\begin{tikzcd} TM \arrow[r, "Tf"] \arrow[d, "\tau_M"'] & TN \arrow[d, "\tau_N"] \\ M \arrow[r, "f"'] & N \end{tikzcd}$$

Note: For the differentials of maps on manifolds we prefer to use the *functorial* T instead of d, the latter been reserved for the exterior differential of differential forms.

1.1.9 Vector fields

A ***vector field*** on M is a ***section*** of the tangent bundle; that is, a map of the form $X\colon M \to TM$ such that $\pi \circ X = \mathrm{id}_M$. The set of *smooth* vector fields on M is denoted by $\mathcal{X}(M)$. The bracket of vector fields determines the structure of a Lie algebra on $\mathcal{X}(M)$.

A vector field X induces a derivation of the algebra of smooth functions on M by $X(f)(x) = T_x f(X_x)$, for every $x \in M$. For the correspondence between vector fields and derivations of smooth functions or Banach space valued maps on M see, for instance, [AMR88].

Given a chart $(U, \phi, \mathbb{B})$ of M and the corresponding chart $(\pi^{-1}(U), \Phi)$ of the tangent bundle (see § 1.1.5), the local representation of $X \in \mathcal{X}(M)$, with respect to the previous charts, is the map $\Phi \circ X \circ \phi^{-1}$ (see § 1.1.3) shown also in the following diagram:

$$\begin{tikzcd} M \arrow[r, "X"] \arrow[d, "\phi"'] & \pi^{-1}(U) \arrow[d, "\Phi"] \\ \phi(U) \arrow[r, "\Phi \circ X \circ \phi^{-1}"'] & \phi(U) \times \mathbb{B} \end{tikzcd}$$

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Then, the ***(local) principal part*** of X (with respect to the above representation) is the map

$$X_\phi := \mathrm{pr}_2 \circ \Phi \circ X \circ \phi^{-1} \colon \phi(U) \longrightarrow \mathbb{B}. \tag{1.1.14}$$

If we consider an indexed chart $(U_\alpha, \phi_\alpha, \mathbb{B})$, then we set

$$X_\alpha := X_{\phi_\alpha}. \tag{1.1.14$'$}$$

1.1.10 Related vector fields

Let $f \colon M \to N$ be a smooth map. Two vector fields $X \in \mathcal{X}(M)$ and $Y \in \mathcal{X}(N)$ are ***f-related***, if $Tf \circ X = Y \circ f$. Equivalently,

$$T_x f(X_x) = Y_{f(x)}, \qquad x \in M.$$

1.1.11 Integral curves

A smooth curve $\alpha \colon J_\alpha \to M$ (J_α: open interval containing 0) such that $\alpha(0) = x$ and

$$X(\alpha(t)) = \dot{\alpha}(t), \qquad t \in J_\alpha$$

is called an ***integral curve*** of $X \in \mathcal{X}(M)$ with ***initial condition*** x. Locally, the problem of finding α reduces to the determination of a smooth curve $\beta \colon J_\beta \to \mathbb{B}$ such that $\beta(0) = \phi(x)$ and

$$\beta'(t) = X_\phi(\beta(t)), \qquad t \in J_\beta \tag{1.1.15}$$

[recall also (1.1.14)]. The theory of differential equations in Banach spaces ensures the existence and uniqueness of such a β. Thus $\alpha = \phi^{-1} \circ \beta$ is an integral curve of X with initial condition $\alpha(0) = x$.

If M is a Hausdorff manifold, then there is a unique integral curve α with $\alpha(0) = x$, defined on a maximal interval of $\mathbb{R}$ containing 0.

1.2 Banach-Lie groups

Beside the references given in the begining of § 1.1, here we add [Bou72] and [Mai62].

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1.2.1 Basic notations

A ***Banach-Lie group*** G is a Banach manifold with a compatible group structure, i.e. the ***multiplication*** or ***product***

$$\boldsymbol{\gamma}\colon G \times G \longrightarrow G\colon (x,y) \mapsto \boldsymbol{\gamma}(x,y) := xy \equiv x \cdot y,$$

and the ***inversion***

$$\boldsymbol{\alpha}\colon G \longrightarrow G\colon x \mapsto \boldsymbol{\alpha}(x) := x^{-1}$$

are smooth maps. $\boldsymbol{\gamma}$ comes from the Greek word γινόμενο meaning product. Observe the use of the bold typeface $\boldsymbol{\gamma}$ to distinguish the product from the normal γ usually denoting a curve. $\boldsymbol{\alpha}$ (bold typeface, again) is the first letter of αντιστροφή, the Greek word for inversion. The unit (element) of G is denoted by e.

The ***left translation*** by $g \in G$ is the diffeomorphism

$$\lambda_g\colon G \longrightarrow G\colon x \mapsto \lambda_g(x) := gx.$$

Similarly, the ***right translation*** by $g \in G$ is

$$\rho_g\colon G \longrightarrow G\colon x \mapsto \rho_g(x) := xg.$$

The differentials of $\boldsymbol{\gamma}$ and $\boldsymbol{\alpha}$, in terms of the translations, are given, respectively, by

$$T_{(x,y)}\boldsymbol{\gamma}(u,v) = T_x\rho_y(u) + T_y\lambda_x(v), \tag{1.2.1}$$

$$T_x\boldsymbol{\alpha}(u) = -T_e\lambda_{x^{-1}} \circ T_x\rho_{x^{-1}}(u) = -T_x(\lambda_{x^{-1}} \circ \rho_{x^{-1}})(u), \tag{1.2.2}$$

for every $x, y \in G$ and every $u \in T_xG$, $v \in T_yG$.

In the following subsections G will denote a Banach-Lie group.

1.2.2 Invariant vector fields

A vector field $X \in \mathcal{X}(G)$ is said to be ***left invariant*** if it is λ_g-related with itself, for every $g \in G$; that is,

$$T\lambda_g \circ X = X \circ \lambda_g, \qquad g \in G;$$

equivalently,

$$T_e\lambda_g(X_e) = X_g, \qquad g \in G.$$

The set of all left invariant vector fields on G forms a Lie subalgebra of $\mathcal{X}(G)$, denoted by $\mathcal{L}(G)$ and called the ***Lie algebra of*** G.

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$\mathcal{L}(G)$ is in bijective correspondence with T_eG by means of the linear isomorphism

$$\boldsymbol{h}\colon \mathcal{L}(G) \ni X \longmapsto X_e \in T_eG \tag{1.2.3}$$

whose inverse is given by

$$\boldsymbol{h}^{-1}(v) = X^v; \qquad v \in T_eG, \tag{1.2.4}$$

where $X^v \in \mathcal{L}(G)$ is defined by

$$X^v(x) = T_eL_x(v), \qquad x \in G. \tag{1.2.5}$$

Therefore, T_eG becomes a Lie algebra by setting (same symbol of bracket !)

$$[u, v] := \boldsymbol{h}\left(\left[\boldsymbol{h}^{-1}(u), \boldsymbol{h}^{-1}(v)\right]\right).$$

Equivalently, if $u = X_e$ and $v = Y_e$, for $X, Y \in \mathcal{L}(G)$, then

$$[X_e, Y_e] = [X, Y]_e,$$

under the appropriate interpretation of the bracket in each side.

For convenience, sometimes, we shall denote by $\mathfrak{g}$ the Lie algebra T_eG with the previous structure. As is the custom, we shall denote the Lie algebra of G by $\mathfrak{g}$ and $\mathcal{L}(G)$ interchangeably, as a result of the identification (1.2.3).

1.2.3 The exponential map

The ***exponential map*** of G is the map

$$\exp \equiv \exp_G\colon T_eG \longmapsto G\colon v \mapsto \exp(v) := \alpha(1),$$

where α is the integral curve of $X = \boldsymbol{h}^{-1}(v) \in \mathcal{L}(G)$ with initial condition $\alpha(0) = e$. Recall that the left invariant vector fields are *complete*, thus the domain of α is $\mathbb{R}$.

1.2.4 The adjoint representation

The ***adjoint representation*** of G is the map $\mathrm{Ad}\colon G \to \mathrm{Aut}(\mathfrak{g})$, with

$$\mathrm{Ad}(g) := T_e(\rho_{g^{-1}} \circ \lambda_g) = T_e(\lambda_g \circ \rho_{g^{-1}}).$$

It is a smooth map whose differential at $e \in G$,

$$T_e\mathrm{Ad}(g)\colon T_eG \equiv \mathfrak{g} \longrightarrow \mathcal{L}(\mathfrak{g}),$$

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is given by

$$\big(T_e\mathrm{Ad}(g)(X)\big)(Y) = [X,Y]; \qquad X,Y \in \mathfrak{g}.$$

1.2.5 Lie algebra-valued differential forms

Let B be a Banach manifold and let G be a Banach-Lie group with Lie algebra $\mathfrak{g}$.

Heuristically, a $\mathfrak{g}$-***valued differential form of degree*** k ($\mathfrak{g}$-***valued*** k-***form***, for short) on B is a smooth map ω assigning a k-alternating (antisymmetric) map $\omega_x \in \mathcal{A}_k(T_xB,\mathfrak{g})$ to each $x \in B$. Formally, ω is a smooth section of the vector bundle of k-alternating maps

$$A_k(TB,\mathfrak{g}) := \bigcup_{x\in B} \mathcal{A}_k(T_xB,\mathfrak{g}),$$

described in detail in § 1.4.4(e) (see also § 1.4.1). The set of $\mathfrak{g}$-valued k-forms on B is denoted by $\Lambda^k(B,\mathfrak{g})$.

Important examples of $\mathfrak{g}$-valued forms are the Maurer-Cartan forms on a Lie group defined below, and the Maurer-Cartan differentials defined in the next subsection. More specifically, the ***left Maurer-Cartan*** (or ***left canonical***) ***form*** on G is the 1-form $\omega^l \in \Lambda^1(G,\mathfrak{g})$ given by

$$\omega^l_g(v) := T_g\lambda_{g^{-1}}(v); \qquad g \in G, v \in T_gG.$$

Analogously, the ***right Maurer-Cartan form*** on G is the differential form $\omega^r \in \Lambda^1(G,\mathfrak{g})$ defined by

$$\omega^r_g(v) := T_g\rho_{g^{-1}}(v); \qquad g \in G, v \in T_gG.$$

The form ω^l is ***left invariant***, i.e. $\lambda^*\omega^l = \omega^l$. Likewise, ω^r is ***right invariant***, i.e. $\rho^*\omega^r = \omega^r$. The two forms satisfy the respective ***Maurer-Cartan equations***:

$$\begin{aligned} d\omega^l &= -\frac{1}{2}\big[\omega^l,\omega^l\big] = -\omega^l \wedge \omega^l, \\ d\omega^r &= \frac{1}{2}\big[\omega^r,\omega^r\big] = \omega^r \wedge \omega^r. \end{aligned}$$

For the exterior product, the bracket and the exterior differentiation of $\mathfrak{g}$-valued forms, we refer to the general theory of [Bou71, § 8.3], [Car67(b)] and [Nab00, § 4.2].

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