

INDEX

- Absolute, the, 67; circle (*see* Circle at infinity); lines, 64, 87
 Absolute invariants, 150, 315
 Affine transformation, 322
 Algebraic curves, 188, 277
 — surfaces, 364 ff.
 Anchor-ring, 112, 368; freedom-equations, 407; tangential equation, 377
 Angle between two lines, 4, 7; planes, 15; generating lines of cone, 99; of intersection of two spheres, 78; logarithmic expression, 64
 Apolar conics, 104; linear complexes, 351; pairs of numbers, 51; quadrics, 311
 Apparent cusp, 291; double-point, 281; inflexion, 287
 Associated points, 265
 Asymptotes of plane section of quadric, 203; of space-cubic, 294
 Asymptotic cone of sphere, 87; hyperboloid, 128
 — planes of cylinder, 105; of cubic curve, 294
 Axes of coordinates, 1
 — of quadric, 166; of plane section of quadric, 200
 — of rotation, 110; of symmetry, 121
 Axial coordinates, 337
 — pencil, 23
- BAKER, H. F., 311, 351
 Barycentric coordinates, 69
 Base-curve, 251
 Battaglini's complex, 330
 Bilinear symmetrical expression, 5, 7, 30, 51, 79, 126, 147, 313
 Binode = biplanar node, 374, 378
 Bisecants, 281; of cubic curve, 291; of quartic curve, 297, 300, 302
 Bitangent developable, 377; of ruled surface, 381
 Biunivocal = one-to-one, 25 (*see* Correspondence)
 Bundle of lines and planes, 23, 46
 Bundle-planes, 346
- Canonical equation of plane, 13; of quadric, 158; of two quadrics, 267 ff.
- Cartesian coordinates, 1, 69; general, 7
 CAYLEY, A., 306, 372, 386, 393, 398
 Cayley's ruled cubic, 384
 Centre of sphere, 74; of quadric, 163; of plane section of quadric, 132, 200
 — of symmetry, 121
 Circle, equations of, 77
 — at infinity, 68, 86
 Circular cone, 95, 113; cylinder, 106, 113
 — points at infinity, 67
 — sections, 204
 Class of space-curve, 286; of surface, 215, 372
 Classification of conics, 159; cubic curves (metrical), 294; cubic surfaces, 395, 396; quadrics (metrical), 164; (projective), 160, 221
 Coaxial spheres, 81
 Collinearity, 20
 Complex of lines, 218, 338; meeting a curve, 281
 —, linear, 339 ff.; relation to null system, 157; special or singular, 185, 347; containing the tangents of a cubic curve, 293; represented in S_3 , 346
 —, quadratic, 261, 353 ff.
 — of tangents to quadric, 330, 359
 —, harmonic, 329, 361
 —, tetrahedral, 263, 360
 Cone, 94 ff.; condition for, 149; as quadric with parabolic points, 148; as specialised quadric, 158; dual to curve, 216
 Configuration of lines on cubic surface, 393; of umbilics, 208
 Confocal quadrics, 235 ff., 254; cones and cylinders, 243
 Conformal transformation, 86
 Congruence of lines, 218, 338; of bisecants of curve, 281; of normals to quadric, 362
 —, focal, 228
 —, linear, 347, 349; singular, 347, 349
 Conical point, 373
 Conic at infinity on quadric, 163

- Conics on a quadric, 145; on a surface, 378
- Conjugate diameters, 132
- focal conics, 244
 - generators, 333
 - imaginary elements, 30
 - linear complexes, 351
 - lines, 130
 - planes, 126
 - points, 125, 147
 - tangents, 366
- Constant-number, 25; of algebraic surface, 372; of quadric, 144; of rational curve, 408
- Contact of quadrics, 317 ff.; along a line, 270, 318; along two lines, 270, 319; double, 267, 270, 318; quadruple, 268, 318; ring, 268, 318; simple, 268, 317; stationary, 269, 270, 318; triple, 270, 318
- Contravariants, 325
- Coordinates, 1; axial, 337; barycentric, 69; cartesian, 1, 69; homogeneous, 18; line, 26, 218, 337; plane, 25, 212; Plücker, 28, 337; projective, 48, 60; quadriplanar, 68; ray, 337; rectangular, 2; superabundant, 2, 28; volume, 69
- Coplanar points, condition for, 12, 20
- Correlations, 155
- Correspondence, one-to-one, 47; between points and coordinates, 1, 25, 47; between points and parameters, 278; geometrical, 53
- Corresponding points on two quadrics, 249
- Co-singular quadratic complexes, 357
- Covariants, 327
- Cross-ratio (metrical), 47; (projective), 53; of four numbers, 49
- Cubical ellipse, 294; hyperbola, 294; hyperbolic parabola, 294; parabola, 295
- Cubic curve, 189, 270, 271, 287 ff.; always rational, 190, 289; generated by three related axial pencils, 288; lying on a quadric, 291; metrical classification, 294
- surface, 388 ff.; always rational, 195, 390; generated by three related bundles of planes, 389; ruled, 194, 382 ff.; with four conic nodes, 91, 376; with three binodes, 376; classification, 395
- Curvature of a surface, 366; measure of, 368; mean, 368; positive and negative, 368
- , lines of, 139, 255 ff.
- Curve, algebraic, 188, 277 ff.
- , cubic (*see* Cubic curve)
 - , quartic (*see* Quartic curve)
 - , rational, 188, 277
- Curves on a quadric, 306
- Cuspidal edge, 286
- points, 383
- Cyclide, 408
- Cylinders, 105, 174
- Dandelin's theorem, 234
- Deficiency of a curve, 283 (*see* Genus)
- Deformable models, 205, 247
- Degenerate quadrics, 158, 221
- Degree of complex, 338; of ruled surface, 380
- Developable, 107, 284 ff.; has only parabolic points, 367, 372; Hessian of, 372; bitangent, 377; of cubic curve, 385
- circumscribing two quadrics, 253; point-equation, 327
 - of curve of intersection of two quadrics, 331; point-equation, 332
- Diameter, 131, 162
- Diametral planes, 131, 162
- Direction = point at infinity, 17
- Direction-angles, 2; cosines, 2; ratios, 7
- Director sphere, 128
- Directrix corresponding to focus, 225; of linear congruence, 347; of special linear complex, 158, 347
- curve, 382; developable, 382
- Discriminant of quadric, 149; of — surfaces, 374
- Discriminating cubic, 166
- Distance between two lines, 32; from point to plane, 31; from point to straight line, 32
- formula, 4, 7, 65
- Double contact, 267, 270, 318
- curve on surface, 378; on ruled surface, 381
- Double-points of homography, 56
- of plane curve, 281; of surface, 373; on plane section, 377
 - , apparent, 281
- Double-six, 395

INDEX

413

- Double tangent plane**, 377; of cubic surface, 377
Dual (*see* Reciprocal)
EDGE, W. L., 386
Edge of regression, 285
Eight associated points, 265
Ellipsoid, 113; model, 206
Elliptic cylinder, 105; involution, 56; linear congruence, 347; paraboloid, 114; point, 148, 365; quartic curve, 298
Envelope, 114, 213
Enveloping cylinder 128
Equianharmonic tetrad, 50
Euler's equations of transformation, 39
Field, plane, 46
Field-plane, 346
Five-dimensional geometry, 343
Five points on a sphere, condition, 76
Flat space, 62, 341
Focal axes, 224, 227, 246, 301; congruence, 228, 247, 301; conics, 229; developable, 238, 301
Foci, 225, 229
Four-dimensional geometry, 340
Four-flat, 343
Four skew lines, transversal of, 186
Freedom-equations (*see* Parametric equations)
Fresnel's wave-surface, 361
Fundamental theorem of projective geometry, 54
Gaussian measure of curvature, 368
Generating lines of quadric, 181 ff.; cone, 98; hyperboloid, 115; paraboloid, 116; V_3^2 , 341
—, rectangular, 102
—, reality of, 117, 153
— of one quadric which touch another quadric, 353
— of ruled surface, 379
Genus of plane curve, 283; space-curve, 283; ruled surface, 381
Harmonic set of four numbers, 50; collinear points, 53
— complex, 329, 361
— conic-envelope of two conics, 104
HENDERSON, A., 393
HENRICI, O., 247
Heron's formula, extension to tetrahedra, 36
Hessian, 370
Homogeneous coordinates, 60; cartesian, 18, 69
Homography, 55, 70
HUDSON, R. W. H. T., 351
Hyperbolic cylinder, 105; rectangular, 106, 223
— involution, 56
— linear congruence, 347
— paraboloid, 114; rectangular, 323
— point, 148, 365
Hyperboloid of revolution, 111; of two sheets, 113; of one sheet, 113, 115; models, 115, 247; orthocyclic, 323; orthofocal, 325; orthogonal, 128, 267, 323; rectangular, 127, 266, 322
Hypercone, 342
Imaginary elements, 30
Indicatrix, 256
Infinity, points and lines at, 16, 65; plane at, 18, 21, 69
—, circle at, 68, 86
Inflexional curve, 368, 370
— tangents, 365; of quartic curve, 302
Inflexions, apparent, 287
Inpolar conics, 104; quadrics, 311
Intercepts, 13
Intersection of line and plane, 16; two planes, 14; three planes, 21; two lines, 23, 29; straight line and quadric, 122, 147; curves lying on quadric, 290, 291, 302, 303, 307
— of three complexes, 379
—, curve of, of two surfaces, 277; of confocal quadrics, 255; of two quadrics, 251, 267, 280, 295; tangential equation, 326, line-equation, 331
Invariance of angles under inversion, 85
Invariant-factors, 273
Invariants, 150 ff.; absolute, 150, 315; metrical, 172, 321; simultaneous, 309 ff.; of linear complex, 347
Inverse transformations, 37
Inversion, 84
Involution, 56, 70; linear complexes in, 351; on a cubic curve, 290
—, skew, 162

- Isotropic lines and planes**, 87
Ivory's theorem, 250
- Jacobian of four spheres**, 81
Joachimsthal's formulae, 6; ratio-equation, 125, 147
- KLEIN, F.**, 351, 352
Kummer surface, 358
- LAGUERRE, E.**, 65
Limiting points of two spheres, 82
Linear complex (*see* Complex)
 — congruence (*see* Congruence)
 — systems of spheres, 81; quadrics, 251 ff.; cones, 273
Line-equation of conic, 219; quadric, 219, 220, 313, 328; space-curve, 281
Line-series, 218, 338
Lines of curvature, 139, 255 ff.
Lines on a surface, 181, 378; cubic surface, 391 ff.; quartic surface, 397
Lüroth's theorem, 278
- Matrices**, 19, 27, 150, 159, 161, 165, 167, 221, 272, 280
Mean curvature, 368
 — point, 6
Measure of curvature, 368
Meridian curve, 110
Metrical geometry, 64
 — invariants, 172, 321
Meunier's theorem, 367
Möbius net, 60
Models, 115, 205, 247
Modulus of transformation, 39, 151
Moment of two lines, 33
Monoid, 398
- n dimensions, 62, 340, 400, 408
Net of rationality, 60
Node, 373
Non-euclidean geometry, 65, 186, 259, 335
Normal varieties, 404
Normals to quadric, 123, 137, 140, 362
Null-system, 156; connected with space-cubic, 293
- Oblate spheroid**, 110
Order of algebraic curve, 189, 277, 286; surface, 215, 364
- Orientation of plane**, 13; =line at infinity, 17
Origin, 1
Orthocentric tetrahedron, 43, 92; pentad, 43, 92
Orthocyclic quadric, 323
Orthofocal quadric, 325
Orthogonal cone, 103; hyperboloid, 128, 323
 — spheres, 78
 — transformation, 321
Orthoptic sphere, 128
Oscular line, 396
Osculating plane, 284; of cubic curve, 289, 293
Osculation, point of, 366
Outpolar conics, 104; quadrics, 311
- Painvin's complex**, 361
Parabolic cylinder, 106, 175; linear congruence, 347; point, 148, 365
Paraboloid, 173; elliptic, 114; circular sections, 205; hyperbolic, 114; generators, 183; models, 117, 206, 248
Parallelism, 5
Parametric equations of curve, 188, 278; surface, 191 ff.; anchor-ring, 407; cardioid, 282; conic, 189, 190; cubic curve, 189, 289, 295, 306; cubic surface, 391; ellipsoid, 193; elliptic quartic curve, 298; hyperbolic paraboloid, 188; hyperboloid of one sheet, 188; nodal or cuspidal quartic curve, 299; plane, 12, 20, 62; quartic curve of second species, 302; straight line, 11, 18, 61, 280; ruled cubic, 194; Steiner surface, 194, 398; curve of striction, 305; surface of Veronese, 401; second degree, 194, 400
Pencil of planes, 22; quadric loci, 251; quadric envelopes, 253; spheres, 81
 —, plane, 46, 346
Perpendicularity, 5
Perspective, 53
Pinch-points, 383
Plane, 12; lying on V_4^3 , 343
 — section of quadric, 145, 199 ff.; algebraic surface, 364
Plücker's coordinates, 28, 337; equations, 287

INDEX

415

- Point, double (*see* Double-points); elliptic, parabolic and hyperbolic, 148, 365; of osculation, 366
- Point-sphere, 74
- Polar congruence, 354
- lines w.r.t. quadric, 129, 149, 220; w.r.t. linear complex, 339, 350
- of line w.r.t. quadratic complex, 354
- plane w.r.t. algebraic surface, 369; cone, 100; quadric, 124, 147; sphere, 79; tetrahedron, 63
- quadric, 369; three-flat, 341
- tetrahedra, 129
- Polarising operator, 330, 369
- Polarity, 154
- Pole of given plane, 125
- Porisms, 102, 143, 146, 310, 312, 316
- Position-ratio, 6
- Power w.r.t. sphere, 75
- Prime, 62
- Principal diametral planes, 136, 166; directions on a quadric, 138; foci, 230; tangents, 365
- Prohessian, 372
- Projection of lines of curvature, 256; of surface of intersection of quadric varieties, 408; of surface of Veronese, 404
- , stereographic, 85, 192, 342
- Projective coordinates, 60; geometry, 46; invariants, 152, 309; ranges, 54
- Prolate spheroid, 110
- Ptolemy's theorem, 77
- Quadratic complex, 353
- Quadratic representing pairs of elements, 50, 70
- Quadric variety, 341
- Quadrics, 144; determined by three generators, 145; generated by related pencils of planes, 183; by transversals of three skew lines, 184; through nine points, 144; through a cubic curve, 289; of revolution, 175
- Quadriplanar coordinates, 68
- Quartic curve, 267, 295 ff.; cuspidal, 271, 299; elliptic, 298; nodal, 269, 298
- surfaces, 397 ff.; developable, 385; rational, 397; ruled, 385; with double-conic, 406; with double-line, 405; with twelve conic nodes, 376
- Radical axis, 78; centre, 78: plane, 78
- Radius-vector, 2, 7
- Rank of matrix, 20; space-curve, 287; surface, 380
- Rational algebraic curve, 188, 277; plane curve, 282; surface, 191, 381; quartic surfaces, 397
- Rationality, net of, 60
- Ray-coordinates, 337
- Reciprocal cones, 100; elements, 47; quadrics, 328; surfaces, 373
- of a cone, 101, 216; cylinder, 107
- Rectangular cone, 102; coordinates, 2; generators of cone, 102; hyperbolic cylinder, 106; hyperbolic paraboloid, 323; hyperboloid, 127, 266, 322; in non-euclidean geometry, 335
- Reducible curve, 280; surface, 364
- Regulus, 116; linear series, 347
- Representation, parametric (*see* Parametric)
- of surface on plane, 191
- Residual, 280
- Revolution, surface of, 110; quadrics, 175
- RICHMOND, H. W., 397
- Right-handed system, 2
- Ring-contact, 180, 234, 268
- Roman surface, 398
- Ruled surfaces, 114, 285, 379 ff.; cubics, 194, 382 ff.; quartics, 385
- SALMON, G., 260
- Scalar product of vectors, 15
- SCHLÄFLI, L., 393
- SCHRÖTER, H., 323
- Sections of quadric, 199
- Segre characteristics, 273
- Self-conjugate linear complex, 351; tetrahedra, 130
- Self-polar tetrahedron, 129; of two quadrics, 252
- Sheaf of planes, 23
- Signs of segments, 2
- Simplex, 62, 349
- Simultaneous invariants, 310
- Singular linear complex, 347; linear congruence, 347, 349; line, 355; points and planes of quadratic complex, 354; surface of quadratic complex, 354, 358; system of quadrics, 273
- Singularities of a surface, 373

- Skew involution**, 162
 — lines, 23; shortest distance, 32
 S_n = space of n dimensions, 340
Special linear complex, 185, 347
Sphere, 74 ff.
Sphero-conic, 259
Spheroid, 110
Stationary plane, 366
Steiner surface, 194, 379, 398 ff.
Steinerian, 371
Stereographic projection, 85, 192, 342
Straight line, 11 (*see* Lines)
Striction, line of, 304
Superabundant coordinates, 2, 28
Surfaces, algebraic, 364 ff.; ruled, 114, 285, 379 ff.; with ∞^2 conics, 378
Sylvester's law of inertia, 353
Symmetry, 121

Tact-invariant, 317
Tangent-cone, 127
 — lines of space-curve, 284; cubic curve, 292
 — linear congruence, 355
 — plane of surface, 212, 364; quadric, 123, 147; sphere, 80
 — three-flat, 341
Tangential equations, 212 ff.; cone, 98, 217; cylinder, 106; plane curve, 216; space-curve, 284; quadric, 124, 214, 328; surface, 213, 377
Tangents, conjugate, 366; inflexional or principal, 365
Tetrahedra, mutually inscribed, 71, 72, 306; polar, 129
Tetrahedral complex, 263, 360
Tetrahroid, 361

Tetrahedron, orthocentric, 43; of reference, 62; self-conjugate, 130; self-polar, 129; of two quadrics, 252; volume, 34
Three-flat, 341
Tore (*see* Anchor-ring)
Torsal line, 396
Transformation, affine, 322; birational, 86; conformal, 86; of coordinates, 3, 37, 48; inverse, 37; linear, 47, 150, 321; orthogonal, 321; quadratic, 86; spherical, 86
Triangle of reference, 61
Triple orthogonal system, 237
Triple points, 378
Trisecants, 283, 288, 297, 302
Tritangent planes, 377; of cubic surface, 391
Trope, 376

Umbilics, 207, 256
Unicursal curve, 283
Uniplanar node, 374
Unit-point, 48
Unode, 374

Vector-product, 15
Vectors, 14
Veronese, surface of, 400
Vertex of paraboloid, 140, 174
Virtual conic, 160; quadric, 121, 161; sphere, 74
 V_n^2 = quadric variety in S_{n+1} , 341, 343
Volume of tetrahedron, 34
Volume-coordinates, 69

Wave-surface, 361
Weddle surface, 397
Weight of invariant, 152