# ANALYTICAL GEOMETRY
# OF
# THREE DIMENSIONS

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BY

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# PREFACE

Until recent years there has been a tendency, in England at least, to regard Geometry as if it were a mine which had been worked out and exhausted. Mathematical interest was largely transferred to analysis. The great stimulus given by Cayley, Salmon and Clifford in the 'sixties and 'seventies of last century had dissipated, and no great successor to these pioneers had appeared. But if their influence in Britain had become weakened, it grew upon the Continent, especially in Italy, and it is from Italy, largely through the medium of Professor H. F. Baker, that once more a renewed interest in geometry has arisen and is flourishing in England.

It is seventy-one years since Salmon's *Treatise on the Analytic Geometry of Three Dimensions* was first published. It has been translated into German, French and Italian, and has been expanded into two volumes in later English editions.* In its first form it embodied the results of many very recent researches, and, brought up to date and including several new topics, it is still recognised as the standard work in the English language. There seems, however, to be room for a text-book written more in accordance with the tendencies of the present "cosmic epoch", to apply a suggestive term of Whitehead's. Fashions in mathematics, as in other things, alter. The facts remain but their values change. Rather, perhaps, new principles, wider and more unifying, are discovered, leading to different treatment and a different emphasis being put on the various developments.

In some ways the present text-book should be regarded as an introduction to Professor Baker's inspiring volumes on the *Principles of Geometry*. This work, especially in the two recent volumes, shows strongly the Italian influence, and the same must be acknowledged in the case of the present text-book. It is natural that the Italian school, which has been responsible for a great part of modern geometrical research, should have produced also some of the finest text-books, such as Bianchi's *Lezioni di geometria analitica* (Pisa, 1920), Castelnuovo's book with the same title (6th ed. Milan, 1924), Berzolari's two Hoepli manuals entitled *Geometria analitica* (Milan: I, 3rd ed. 1925; II, 2nd ed. 1922), and Comessati's *Lezioni di geometria analitica e proiettiva* (Milan, 1930). To these, as well as to Salmon, Baker, the Collected Mathematical Papers of Cayley and of Klein,

* Vol. 1, 7th ed. revised by R. A. P. Rogers and edited by C. H. Rowe, 1928 Vol. 2, 5th ed. revised by Rogers, 1915.

Hudson's classic on *Kummer's Quartic Surface*, Pascal's *Repertorium*, the *Enzyklopädie der mathematischen Wissenschaften*, and other sources difficult to particularise, I have to express my indebtedness.

Being an elementary text-book it may be used by beginners. For such it may be useful to indicate a first course of reading: Chap. I (omitting 1·8*), Chap. II (omitting 2·34, 2·35, 2·36, 2·41, 2·5, 2·731, 2·8, 2·91), Chap. IV (omitting 4·2, 4·31, 4·51, 4·81), Chap. V (omitting 5·122, 5·123, 5·23, 5·6), Chap. VI, Chap. VII (omitting 7·38, 7·73), Chap. VIII (omitting 8·11–8·13, 8·24, 8·41–8·432, 8·53–8·54, 8·64, 8·72, 8·74, 8·9), Chap. IX (omitting 9·3, 9·4, 9·6, 9·7), Chap. X (omitting 10·71).

Except for a few insignificant references the subject-matter of differential geometry has been excluded from this book. On the other hand free use has been made of homogeneous coordinates, tangential coordinates and line-coordinates. In the case of metrical geometry the circle at infinity is used wherever it is applicable; this is especially the case in the treatment of foci, which follows somewhat closely on the lines of Berzolari. There are several illustrative references to Non-Euclidean Geometry, and much use has been made, as in Baker's volumes, of geometry of higher dimensions, especially in the exposition of line-geometry. In the enumeration of types of linear systems of quadrics opportunity has been taken to explain the notation of invariant-factors. No exhaustive treatment of the theory of algebraic curves and surfaces has been attempted; the two chapters which have been devoted to these are intended rather to be suggestive, and are confined practically to curves of the third and fourth orders, and to ruled and rational cubic and quartic surfaces.

I have to express my grateful thanks to Mr F. P. White for much encouragement in the preparation of the book; to Professor W. Saddler, D.Sc., Christchurch, who read the entire manuscript, for many helpful and valuable suggestions; and to Mr F. F. Miles, M.A., Lecturer in Mathematics at Victoria University College, for great assistance in reading the proof-sheets and in checking the examples.

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* This is to be understood as including all further subdivisions, as 1·81, 1·82, etc.

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