CHAPTER I

FUNDAMENTAL CONCEPTIONS IN MAGNETISM

Fundamental Definitions. It is well known that by artificial means we can cause certain substances, called *ferromagnetics*, to become more or less *permanently magnetised*, i.e. to possess the power of attracting or repelling other magnetised or magnetisable bodies, and we say that the region around a magnetised body constitutes a *magnetic field*. If a piece of unmagnetised iron be placed in such a region it becomes magnetised and is said to exhibit *induced magnetism*. If we place a piece of permanently magnetised material in a magnetic field, and find that it is not acted upon by translational forces but only by couples tending to set it in a particular direction, we say that it is in a *uniform field*; it is assumed that the field is not disturbed by the introduction of the material and that the latter's magnetism is not affected. It can be shown that a uniform field exists in the middle of a long solenoid or coil of wire wound uniformly upon a cylinder and carrying a steady current, and for most purposes the magnetic field due to the earth may be treated as uniform over large regions.

Now when a long rod of permanently magnetised material, e.g. a bar magnet, is suspended so that it may rotate freely in a horizontal plane, it is acted upon by the magnetic field of the earth, its axis comes to rest roughly along a north-south line, and we speak of *north-seeking* and *south-seeking* ends of the rod. However it is placed in a uniform field, two systems of parallel forces act upon the rod, and the resultants of these forces always appear to pass through two points or tiny regions, one near each end of the rod. These points are called *poles* and at them we may for purposes of calculation consider north- and south-seeking magnetism to be located. The line joining the poles is termed the *magnetic axis* of the magnet.

It is an experimental fact that like poles repel and unlike poles attract each other with forces which obey an inverse square law. This leads to the definition of a *unit pole* as one which will repel an equal and similar pole placed 1 cm. away *in vacuo* with a force of 1 dyne. The *pole strength* of the magnet is therefore measured by the number of unit poles to which each pole is equivalent. If we could place a unit pole at a fixed point in a magnetic field *in vacuo* it would be acted upon by a force which would be a measure of the *strength* or *magnetic intensity* of the field, and *unit magnetic intensity* exists at the point when the force on the unit pole is 1 dyne. The unit of magnetic intensity is now called the *oersted*, following the recommendation of the International Conference on Physics which met in London in 1934. It was previously termed the *gauss*, and although the reasons for the change are certainly not apparent, the term oersted will be used throughout this book in an attempt to carry out international obligations.

The work done in taking a unit north-seeking pole from a fixed point B to a fixed point A is known as the *potential difference* between the points A and B, and when B is located at infinity the work done in bringing the pole to A is known as the *potential* at the point A, for we may neglect magnetic forces at points an infinite distance from magnetised bodies. If A and B are separated by a small distance dx along the line AB, then the work done in taking unit pole from B to A, which must be independent of the path, is $-F_x dx = dV$, where F_x is the component of the magnetic intensity along AB at the point x; the minus sign is introduced because the field intensity decreases as the distance from a magnetic pole increases. Hence $F_x = -dV/dx$, or the field intensity is equal to the rate of decrease of potential with distance. The intensity at a point r from an isolated pole $+m$ is m/r^2, while the potential is m/r.

The direction in which a single north-seeking pole placed at a point in a magnetic field would be urged to move is known as the *direction of the line of force* at that point. In a uniform field such lines are parallel. By convention we may state that unit intensity exists at a point when one line of force passes through unit area placed perpendicular to the lines of force at that point, and thus we may express the magnetic intensity at a point in any field in terms of lines of force per sq. cm. Since at a distance of 1 cm. from a unit pole the intensity is 1 oersted over a surface of area 4π sq. cm., we see that the convention requires that 4π lines of force leave unit north-seeking pole. A bundle of lines of force leaving unit north-seeking pole and ending on a unit south-seeking pole is known as a *unit tube of force*, and, on the above convention, consists of 4π lines.

We have seen that when a permanent magnet is placed in a uniform field it is acted upon by a couple. Referring to Fig. 1, suppose that the magnet NS of pole strength m unit poles is placed in a uniform field of strength $\mathscr{H}$ oersteds in the direction shown in the plane of the paper. If the poles are located at the points N and S, then the couple acting on the magnet is $2ml\mathscr{H}\sin\theta$ in the sense indicated, where $2l$ is the distance between N and S in cm. The product $2ml = M$ is termed the *magnetic moment* of the magnet, and is clearly equal to the maximum couple which can be exerted on the magnet when it is placed in a uniform field of unit intensity; it is measured in dyne-cm. per oersted. It is perhaps unfortunate that no practical unit of magnetic moment has been named, for modern experiments show that there exists a fundamental unit of magnetic moment, the *Bohr magneton*, whose existence is just as real as that of the charge carried by an electron.

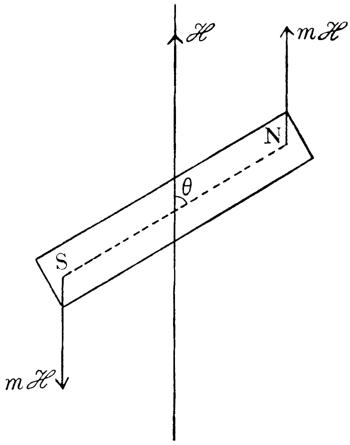

Fig. 1. Magnet in uniform field.

The extent to which a body is magnetised is measured by its magnetic moment per unit volume or *intensity of magnetisation*. This is a vector quantity, since magnetic moment has both magnitude and direction. If we could take a very long

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uniformly magnetised rod, we could imagine the cross-section at one end to be covered with m north-seeking or positive unit poles and at the other end with m south-seeking or negative unit poles. The intensity of magnetisation is, by definition, equal to $2ml$ divided by $2l\alpha$, where α is the area of cross-section of the rod; hence the intensity of magnetisation of a uniformly magnetised rod is equal to its pole strength per unit area of cross-section. In practice, however, the poles are never located at the ends of a rod, but always at a short distance from each end; in other words, the *magnetic length* or *equivalent length of a magnet* is always less than its geometrical length.

Magnetic Shell. For many purposes it is helpful to imagine a thin sheet of material uniformly magnetised in a direction normal to its surface, one side being north-seeking and the other south-seeking. Such an arrangement is known as a

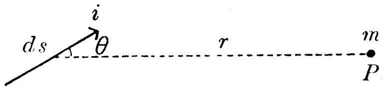


Fig. 2

magnetic shell, and the *strength of the shell* at any point on its surface is defined as the product of the intensity of magnetisation and the thickness of the shell at that point; a *uniform shell* is one in which this product is the same all over the shell. The conception is important in discussions of magnetic problems, as it may be shown that a closed circuit carrying a current of i e.m. units is equivalent to a uniform shell of strength i whose boundary is the circuit. It is sometimes convenient to picture an electron moving in a closed orbit or a spinning electron as a magnetic shell. Moreover, the potential at any point near a uniform shell is equal to the product of its strength and the solid angle subtended by the boundary of the shell at that point, the angle being taken as positive when lines radiating from the point fall first upon the north-seeking surface of the shell; the shape of the shell has no influence on the value for potential, the boundary remaining constant.

If we consider an element ds of a wire carrying a current i, it is deduced from experiment that the force on a pole of strength m placed at a point P (Fig. 2) is proportional to $mi\frac{ds}{r^2}\sin\theta$, an expression due to Laplace. The force acts at right angles to the line joining the mid-point of the element to P and to the plane containing the element and this line. Hence if a wire is bent into a circle of radius r, the force on unit pole at the centre is proportional to $2\pi i/r$, and the *electromagnetic unit of current* is accordingly defined as that which, when flowing in a single complete turn of 1 cm. radius, causes a force of 2π dynes to act upon unit pole placed at the centre. When the current circulates in a clockwise direction, as viewed by an observer, the circuit behaves as a magnetic shell with its south-seeking face towards the observer.

It is easy to show, either by using Laplace's rule or the conception of a magnetic shell, that the work done in taking a unit pole once round a closed path surrounding a conductor carrying a current i e.m. units in opposition to the field is $4\pi i$ ergs. This result may be extended by taking a unit pole along the axis of a solenoid, completing the closed path by returning outside the windings, when the work done is $4\pi Ni$ ergs, where N is the total number of turns on the solenoid carrying the current i. If the solenoid is uniformly wound with n turns per cm. length, then it follows that the field along its axis is $4\pi ni$ oersteds.

Permeability. From the definition of unit pole it follows that the force between two poles of strength m_1 and m_2 a distance d apart *in vacuo* is $m_1 m_2/d^2$ dynes. If the poles are embedded in a medium, the force becomes equal to $m_1 m_2/\mu d^2$, where μ is a constant known as the *permeability* of the medium. We have at present no means of determining by direct experiment the value of the quantity μ used in this particular expression, and its dimensions are generally considered unknown. Let us now suppose that we have a long straight solenoid uniformly wound with n turns per cm. length and carrying a current i measured in electromagnetic units (1 ampere $= \frac{1}{10}$ e.m. unit). The magnetic field inside the middle portions of the solenoid and along its axis is uniform and equal to $4\pi ni$.

If now a small number of turns N' of wire are wound upon the solenoid and joined to a ballistic galvanometer, the quantity of electricity discharged through the latter when the current i is reversed is $2\,\dfrac{N'A\,.\,4\pi ni}{R\,.\,10^8}$ coulombs, where A is the mean area of cross-section of the solenoid and R is the total resistance of the galvanometer circuit in ohms. This quantity of electricity can be measured directly in terms of the deflection and known constants of the galvanometer. If, however, the solenoid were embedded in a medium of permeability μ, the quantity of electricity discharged through the galvanometer would be μ times the value observed when the solenoid was *in vacuo*, although the galvanometer deflection would be but imperceptibly changed unless the medium were ferromagnetic. Now, μ is here assumed to be the same quantity as that occurring in the expression for the force between two poles. We have, however, no means of proving this identity by experiment. Indeed, Abraham* has pointed out that μ may be defined in at least six different ways, and that it is impossible to prove experimentally that any two of them are the same.

A mathematical formula for μ may be obtained as follows. We consider such a long solenoid that when it is filled with a cylinder of magnetic material any free poles generated at the ends of the cylinder will produce no measurable change in the field acting at the middle portions of the cylinder. Let $\mathscr{H}$ be the field acting in the absence of magnetic material. The field inside a long narrow tunnel parallel to the axis of the cylinder will still be $\mathscr{H}$. If we cut a crevasse in the cylinder, or, rather, if we cut through the cylinder perpendicular to its axis and separate the two portions slightly, the field inside this gap will be augmented by the field arising from the magnetic poles, I per unit area, on either side of the gap. It is clear that the lines of force joining these poles must traverse the gap normally, and, hence, $4\pi I$ lines of force must pass through each sq. cm. of the gap cross-section. The total number of lines per unit area is therefore $\mathscr{H}+4\pi I$, a quantity which we call the *magnetic induction* and denote by B. Hence we see that the ratio of

* H. Abraham, *Bull. Nat. Res. Coun., Washington*, No. 93.

the quantity of electricity discharged through the galvanometer is μ to 1 or B to $\mathscr{H}$. Hence

$$\mu = 1 + 4\pi I/\mathscr{H} = 1 + 4\pi k. \qquad \text{......(1)}$$

We shall assume this expression to be correct for all isotropic substances. It is incorrect, in general, for crystalline substances, since in the latter cases I and $\mathscr{H}$ do not coincide in direction; we shall return to this matter in Chapter IV.

We may further imagine that *lines of induction* analogous to lines of force pass through a magnetised substance. If we consider lines of induction passing through an area perpendicular to the lines, then we define the product of the induction and the area as the *normal flux*, usually abbreviated to flux, so that the induction may be termed the *flux density* and measured in lines per sq. cm. In air or in vacuum the flux density is equal to the field strength (or intensity).

The ratio $I/\mathscr{H}$ or k is termed the *magnetic susceptibility*, and according to their susceptibilities substances may be roughly divided into three classes—*diamagnetic*, *paramagnetic* and *ferromagnetic*. In general, the susceptibilities of substances in the first two classes are small, diamagnetic susceptibilities being negative and paramagnetic susceptibilities positive. The susceptibility of a ferromagnetic is usually very large, unless the material is *saturated*—i.e. magnetised as strongly as possible in a powerful field—and depends upon the applied field and the previous magnetic history of the material. The magnetisation of diamagnetics and paramagnetics disappears when the exciting field is removed, but ferromagnetics then exhibit residual or permanent magnetisation.

In the English and foreign literatures certain symbols are commonly used in the description of susceptibility values. First, *the susceptibility per unit volume*, or the intensity of magnetisation per unit field, which is the quantity most commonly measured directly by experiment, is usually denoted by k or κ or x. Secondly, the *susceptibility per unit mass*, a quantity frequently required in theoretical discussions, is denoted by χ, and is equal to k/ρ, where ρ is the density of the substance. Thirdly, the *atomic or molar susceptibility* χ_A or χ_M

is respectively equal to the product of χ and the atomic or molecular weight, or to the product of k and the atomic or molecular volume. Thus, in the case of mercury, we have $k = -2{\cdot}285 \times 10^{-6}$ e.m.u. per c.c., $\rho = 13{\cdot}6$ gm. per c.c., and the atomic weight $= 200{\cdot}6$, so that, numerically,

$$\chi = 2{\cdot}285 \times 10^{-6} \div 13{\cdot}6 = 0{\cdot}168 \times 10^{-6} \text{ e.m.u. per gm.}$$

and

$$\chi_A = 0{\cdot}168 \times 10^{-6} \times 200{\cdot}6 = 33{\cdot}7 \times 10^{-6} \text{ e.m.u. per gm. atom.}$$

Energy of Magnetisation. There are several ways in which we may derive an expression for the energy stored in a magnetised medium, and one of the most satisfactory is the following. A thin ribbon of metal is wound as a uniform solenoid upon a very long cylinder of the magnetic material. Let n be the number of turns per cm. and l the total length of the cylinder, whose area of cross-section is α. Let R be the resistance of the winding. Then, the self-inductance L of the winding, i.e. the total number of lines of force linked with it when unit current flows, is $4\pi n(n\alpha l)\,\mu$. Hence, on applying an electromotive force E to the ends of the solenoid, we have, if i is the current at any instant,

$$E = Ri + L\,.\,di/dt,$$

so that

$$\int Ei\,dt = \int Ri^2 dt + \int Li\,di.$$

The second term on the right-hand side of the last equation is the work done in establishing the magnetic energy associated with the current i in the solenoid; on integration it gives

$$\tfrac{1}{2}Li^2 = \frac{1}{2}\frac{(4\pi n i)^2}{4\pi}\alpha l \mu = \tfrac{1}{2}\mu\frac{\mathscr{H}^2}{4\pi}\alpha l.$$

But αl is the total volume of the material upon which the solenoid is wound. Therefore the work done or energy stored per unit volume is $\mu\mathscr{H}^2/8\pi$. In the case of diamagnetics and paramagnetics this expression has a definite meaning, but in the case of ferromagnetics it is practically indeterminate as μ is almost an unknown function of $\mathscr{H}$.

It should be realised that the quantity $\mu\mathscr{H}^2/8\pi$ also represents the magnetic potential energy of unit volume of the material with respect to the applied magnetic field $\mathscr{H}$. It should therefore *in this connection* be preceded by a minus sign to indicate that it is negative, for we should have either to do work to turn the material, without change in magnetisation, through 180 degrees with respect to the lines of force of $\mathscr{H}$, or to supply thermal energy in order sufficiently to agitate the magnetic particles and so demagnetise the material. We shall see in Chapter III how this enables us to determine the direction in which diamagnetic and paramagnetic bodies tend to move in a non-uniform field.

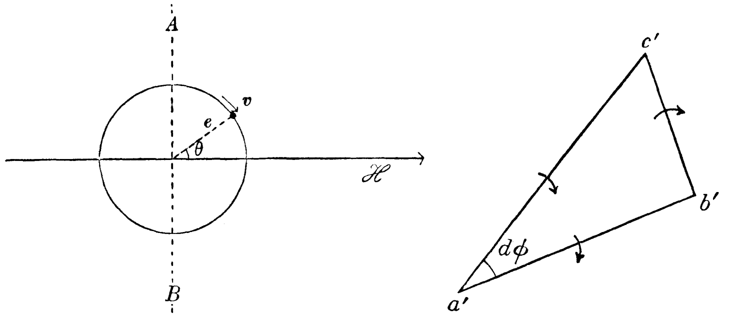
Classical Theory of Diamagnetism. In 1854 Weber showed that all matter should exhibit diamagnetism, on the assumption that Ampèrian currents circulated within its molecules, but it was not until 1905 that Langevin showed that the electron theory provided a more satisfactory picture. Langevin examined the behaviour of an electron moving in a circular orbit of radius r when a magnetic field $\mathscr{H}$ was slowly established perpendicular to the plane of the orbit. He found that, unless the electron moved under an inverse cube-law force, the radius of the orbit remained unchanged but the velocity of the electron increased or diminished in accordance with Lenz's law, giving a change in magnetic moment of the system equal to $\Delta M = -\dfrac{e^2}{4m}\mathscr{H}r^2$. If, therefore, an atom contains a large number of electrons with their orbits orientated in all possible directions the total contribution made to the induced magnetic moment is

$$M_a = -\frac{e^2\mathscr{H}}{4m}\Sigma\bar{r}^2, \qquad \ldots\ldots(2)$$

where $\bar{r}^2$ represents the mean of the squares of the radii of the projections of the orbits on a plane perpendicular to the field. If we write $\bar{r}_0^2 = \bar{x}^2+\bar{y}^2+\bar{z}^2$, then $\bar{r}^2 = \bar{x}^2+\bar{y}^2 = \frac{2}{3}\bar{r}_0^2$, so that the molar susceptibility is given by

$$\chi_M = N\frac{M_a}{\mathscr{H}} = -\frac{Ne^2}{6m}\Sigma\bar{r}_0^2. \qquad \ldots\ldots(3)$$

The last expression may be obtained in a more instructive manner; for, let us consider the special case of a circular orbit in the plane of the paper with a field $\mathscr{H}$ applied in the direction shown in Fig. 3a. Then it follows that when the electron is to the right of the line AB it experiences a force pushing it into the plane of the paper, and when to the left of AB a force pushing it out. Hence we may suppose that a



(a) (b)

Fig. 3

torque acts on the system about the axis AB. The orbit accordingly precesses about $\mathscr{H}$. The angular momentum associated with the circular orbit is $U = mvr = 2m\alpha$, where α is the areal velocity, and the magnetic moment M, equal to the product of the equivalent current and the area A of the orbit, is $A\dfrac{e}{t} = \dfrac{A}{t}e = \alpha e$, where t is the time of description of the orbit. Now the torque Γ acting on the orbit is

$$M\mathscr{H} = -e\alpha\mathscr{H},$$

and if in Fig. 3b, $a'b'$ represents U at a given instant and $a'c'$ represents U a short time interval dt later, when the orbit has precessed through an angle $d\phi$, then $b'c' = U\,d\phi = \Gamma dt$. Hence $\Gamma = U\,d\phi/dt = U\sigma$, where σ is the angular velocity of precession.

Thus $\quad \Gamma = -e\alpha\mathscr{H} = 2m\alpha\sigma \quad$ or $\quad \sigma = -\dfrac{e}{2m}\mathscr{H}. \quad$ ......(4)

The change in magnetic moment brought about by this precession is now equal to

$$\Delta M = \frac{er^2}{2}\sigma = -\frac{e^2\mathscr{H}}{4m}r^2. \qquad ......(5)$$