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Cambridge International
AS and A Level Mathematics:
Pure Mathematics 2 & 3

Coursebook

Revised Edition

CAMBRIDGE UNIVERSITY PRESS

University Printing House, Cambridge CB2 8BS, United Kingdom

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning and research at the highest international levels of excellence.

Information on this title: education.cambridge.org

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First published 2002

Second edition 2016

Printed in the United Kingdom by Latimer Trend

A catalogue record for this publication is available from the British Library

ISBN 978-1-316-60022-1 Paperback

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Introduction

Cambridge International AS and A Level Mathematics has been written especially for the Cambridge International Examinations syllabus 9709. There is one book corresponding to each syllabus unit, except for this book which covers two units, the second and third Pure Mathematics units, P2 and P3.

The syllabus content is arranged by chapters which are ordered so as to provide a viable teaching course. The first eleven chapters are required for unit P2; all the chapters are required for unit P3. This is indicated by the vertical grey bars on the contents page.

A few sections include important results that are difficult to prove or outside the syllabus. These sections are marked with a vertical orange bar and there is usually a sentence early on explaining precisely what it is that the student needs to know.

Some paragraphs within the text appear in *this type style*. These paragraphs are usually outside the main stream of the mathematical argument, but may help to give insight, or suggest extra work or different approaches.

Graphic calculators are not permitted in the examination, but they are useful aids in learning mathematics. In the book the authors have noted where access to a graphic calculator would be especially helpful but have not assumed that they are available to all students.

Numerical work is presented in a form intended to discourage premature approximation. In ongoing calculations inexact numbers appear in decimal form like 3.456..., signifying that the number is held in a calculator to more places than are given. Numbers are not rounded at this stage; the full display could be, for example, 3.456 123 or 3.456 789. Final answers are then stated with some indication that they are approximate, for example '1.23 correct to 3 significant figures'.

There are plenty of exercises, and each chapter ends with a Miscellaneous exercise which includes some questions of examination standard. There are two Revision exercises for the material common to units P2 and P3, and a further Revision exercise for unit P3. There are also two Practice exam-style papers for unit P2 at the end of P2&3, and two Practice exam-style papers for unit P3 at the end of P3.

Some exercises include questions that go beyond the likely requirements of the examinations, either in difficulty or in length or both. In the P2&3 chapters some questions may be more appropriate for P3 than for P2 students. Questions marked with a coloured bar require knowledge of results or techniques outside the syllabus.

Cambridge University Press would like to thank Cambridge International Examinations, for permission to use past examination questions set internationally.

The authors thank Cambridge International Examinations and Cambridge University Press, in particular Diana Gillooly, for their help in producing this book. However, the responsibility for the text, and for any errors, remains with the authors.

Hugh Neill and Douglas Quadling, 2002

Introduction

Introduction to the revised edition

This revised edition has been prepared to bring this textbook in line with the current version of the Cambridge International Examinations specification. As much as possible of the original edition has been left unchanged to assist teachers familiar with the original edition; this includes section numbers, question numbers and so on. A number of new examples have been added; these appear in Sections 1.4 (polynomial division by long division), 13.2 (intersection of a line and a plane), 13.3 (intersection of two planes) and 18.3 (integration of $\tan x$). A small number of additional questions have been added to some of the exercises where there appeared to be gaps in the coverage of the specification. The argument at the start of Chapter 16 proving that the complex numbers cannot form an ordered field has been corrected and moved to an appendix to the chapter. All questions and answers taken from Cambridge International Examinations past papers have been clearly referenced. All other questions and answers have been written by the authors of this book.

The other major change in this edition is the replacement of all of the older OCR examination questions in the Miscellaneous exercises with more recent Cambridge questions. This will be of benefit to students preparing for the current style of examination questions. In order to maintain the numbering of the other questions, the newer questions have been slotted in to the exercises. While this has inevitably meant some loss of order within the miscellaneous exercises, this was felt to be more than compensated for by the preservation of the original numbering.

The editor of this edition thanks Cambridge International Examinations and Cambridge University Press, in particular Cathryn Freear and Andrew Briggs, for their great help in preparing this revised edition.

Julian Gilbey
London, 2016