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978-1-316-51993-6 — Transcendence and Linear Relations of 1-Periods
Annette Huber , Gisbert Wüstholz
Table of Contents
More Information

# Contents


| | | | |
|---|---|---|---|
| | *Prologue* | | *page* xi |
| | *Acknowledgements* | | xx |
| **1** | **Introduction** | | 1 |
| | 1.1 | Transcendence | 1 |
| | 1.2 | Relations Between Periods | 2 |
| | 1.3 | Dimensions of Period Spaces | 4 |
| | 1.4 | Method of Proof | 7 |
| | 1.5 | Why 1-Motives? | 8 |
| | 1.6 | The Case of Elliptic Curves | 8 |
| | 1.7 | Values of Hypergeometric Functions | 9 |
| | 1.8 | Structure of the Monograph | 10 |
| | **PART ONE FOUNDATIONS** | | 13 |
| **2** | **Basics on Categories** | | 15 |
| | 2.1 | Additive and Abelian Categories | 15 |
| | 2.2 | Subcategories | 17 |
| | 2.3 | Functors | 18 |
| **3** | **Homology and Cohomology** | | 21 |
| | 3.1 | Singular Homology | 21 |
| | 3.2 | Algebraic de Rham Cohomology | 24 |
| | 3.3 | The Period Pairing | 29 |
| **4** | **Commutative Algebraic Groups** | | 31 |
| | 4.1 | The Building Blocks | 31 |
| | 4.2 | Group Extensions | 33 |




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CAMBRIDGE
Cambridge University Press & Assessment
978-1-316-51993-6 — Transcendence and Linear Relations of 1-Periods
Annette Huber , Gisbert Wüstholz
Table of Contents
More Information



4.3 Semi-abelian Varieties 37
4.4 Universal Vector Extensions 39
4.5 Generalised Jacobians 40

**5 Lie Groups** 43
5.1 The Lie Algebra 43
5.2 The Exponential Map 44
5.3 Integration over Paths 45

**6 The Analytic Subgroup Theorem** 48
6.1 The Statement 48
6.2 Analytic vs Algebraic Homomorphisms 50

**7 The Formalism of the Period Conjecture** 54
7.1 Periods 54
7.2 The Period Conjecture 59

**PART TWO PERIODS OF DELIGNE 1-MOTIVES** 67

**8 Deligne's 1-Motives** 69
8.1 The Category and the Realisation Functors 69
8.2 The Functor to Mixed Hodge Structures 75
8.3 The Key Comparison 79

**9 Periods of 1-Motives** 82
9.1 Definition and First Properties 82
9.2 Relations Between Periods 84
9.3 Transcendence of Periods of 1-Motives 87
9.4 Fullness 89

**10 First Examples** 93
10.1 Squaring the Circle 93
10.2 Transcendence of Logarithms 95
10.3 Hilbert's Seventh Problem 97
10.4 Abelian Periods for Closed Paths 99

**11 On Non-closed Elliptic Periods** 101
11.1 The Setting 101
11.2 Without CM 102
11.3 The CM-Case 104
11.4 Transcendence 106

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CAMBRIDGE
Cambridge University Press & Assessment
978-1-316-51993-6 — Transcendence and Linear Relations of 1-Periods
Annette Huber , Gisbert Wüstholz
Table of Contents
More Information



**PART THREE PERIODS OF ALGEBRAIC VARIETIES** 109

**12 Periods of Algebraic Varieties** 111
12.1 Spaces of Cohomological 1-Periods 111
12.2 Periods of Curve Type 112
12.3 Comparison with Periods of 1-Motives 115
12.4 The Motivic Point of View 117

**13 Relations Between Periods** 120
13.1 Kontsevich's Period Conjecture 120
13.2 The Case of Curves 123

**14 Vanishing of Periods of Curves** 128
14.1 Classical Periods 128
14.2 The Setting 130
14.3 Forms of the First Kind 134
14.4 Forms of the Second Kind 136
14.5 Forms of the Third Kind 138
14.6 Arbitrary Differential Forms 140
14.7 Vanishing of Simple Periods 141

**PART FOUR DIMENSIONS OF PERIOD SPACES** 145

**15 Dimension Computations: An Estimate** 147
15.1 Set-up and Terminology 147
15.2 The Saturated Case 150
15.3 Special Cases 155
15.4 Proof of the Dimension Estimate 159

**16 Structure of the Period Space** 163

**17 Incomplete Periods of the Third Kind** 167
17.1 Relation Spaces 167
17.2 Alternative Description of $\delta_{\mathrm{inc}3}(M)$ 174

**18 Elliptic Curves** 178
18.1 Classical Theory of Periods 178
18.2 Elliptic Periods 181
18.3 A Calculation 183
18.4 Transcendence of Incomplete Periods 184
18.5 Elliptic Period Space 186

**19 Values of Hypergeometric Functions** 191
19.1 Elliptic Integrals 191
19.2 Abelian Integrals 197

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CAMBRIDGE
Cambridge University Press & Assessment
978-1-316-51993-6 — Transcendence and Linear Relations of 1-Periods
Annette Huber , Gisbert Wüstholz
Table of Contents
More Information



**PART FIVE APPENDICES** 207

***Appendix A*** **Nori Motives** 209

***Appendix B*** **Voevodsky Motives** 217

***Appendix C*** **Comparison of Realisations** 221

*List of Notation* 229

*References* 235

*Index* 241

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