CAMBRIDGE

Cambridge University Press & Assessment
978-1-316-51967-7 — Linear State/Signal Systems
Damir Z. Arov , Olof J. Staffans
Table of Contents
More Information

Contents

Preface *page* xxi
List of Notations xxiii

1 Introduction and Overview **1**
1.1 Linear Time-Invariant Dynamical Systems 1
1.1.1 State Systems 2
1.1.2 Systems That Interact with the Outside World 3
1.1.3 Input/State/Output Systems 3
1.1.4 Input/Output Systems 4
1.1.5 Classical (Sub)networks 5
1.1.6 Port-Hamiltonian Systems 5
1.1.7 Behavioral Systems 6
1.1.8 State/Signal Systems 7
1.1.9 State/Signal versus I/S/O Systems 8
1.1.10 Frequency Domain Systems 10
1.1.11 Boundary Triplets 11
1.1.12 State/Signal versus Behavioral Systems 11
1.1.13 How to Read This Book 12
1.1.14 H-Spaces 12
1.1.15 Where to Go from Here? 13
1.2 An Overview of State/Signal and Input/State/Output Systems 14
1.2.1 Input/State/Output Systems 14
1.2.2 Well-Posed I/S/O Systems 17
1.2.3 State/Signal Systems 18
1.2.4 I/S/O Representations 19
1.2.5 Similarity of I/S/O and State/Signal Systems 20
1.2.6 Input/Output Invariant Properties of I/S/O Systems 23
1.2.7 Properties of I/S/O Systems in the State/Signal Sense 25

© in this web service Cambridge University Press & Assessment www.cambridge.org

CAMBRIDGE
Cambridge University Press & Assessment
978-1-316-51967-7 — Linear State/Signal Systems
Damir Z. Arov , Olof J. Staffans
Table of Contents
More Information

1.2.8 Static Transformations of I/S/O and State/Signal Systems 26
1.2.9 Invariant Subspaces of I/S/O and State/Signal Systems 28
1.2.10 Interconnections of I/S/O and State/Signal Systems 29
1.2.11 External Characteristics of I/S/O and State/Signal Systems 31
1.2.12 Restrictions, Projections, and Compressions 31
1.2.13 I/S/O and State/Signal Systems in Discrete Time 33
1.2.14 The Resolvent Matrix of an I/S/O System 35
1.2.15 The Resolvent Set and the Characteristic Bundles of State/Signal Systems 37
1.2.16 Well-Posed I/S/O and State/Signal Systems in the Frequency Domain 39
1.2.17 General Resolvable Frequency Domain I/S/O and State/Signal Systems 40
1.2.18 Dual and Adjoint I/S/O and State/Signal Systems 42
1.2.19 Passive I/S/O and State/Signal Systems 44
1.2.20 Passive Finite-Dimensional Electrical *n*-Ports 45
1.2.21 Some Finite-Dimensional Passive 2-Ports 51
1.2.22 Some Distributed Parameter Passive Systems 59
1.3 Notes and Comments 68

2 State/Signal Systems: Trajectories, Transformations, and Interconnections 71
2.1 State/Signal Nodes and State/Signal Systems 71
2.1.1 The State/Signal System and Its Trajectories 71
2.1.2 Regular and Semiregular State/Signal Nodes 75
2.1.3 Kernel and Image Representations of Closed State/Signal Nodes 80
2.1.4 Bounded State/Signal Nodes and Systems 84
2.2 Some Basic Transformations of State/Signal Nodes 87
2.2.1 Similarity of Two State/Signal Nodes 87
2.2.2 Time Reflection of a State/Signal Node 89
2.2.3 Time Rescaling of a State/Signal Node 91
2.2.4 Exponentially Weighted State/Signal Nodes 92
2.3 Properties of Trajectories of State/Signal Systems 93
2.3.1 Classical, Generalized, and Mild Trajectories 93
2.3.2 Existence and Uniqueness of Trajectories 95

© in this web service Cambridge University Press & Assessment www.cambridge.org

CAMBRIDGE
Cambridge University Press & Assessment
978-1-316-51967-7 — Linear State/Signal Systems
Damir Z. Arov , Olof J. Staffans
Table of Contents
More Information

2.3.3 Connections between Classical, Generalized, and Mild Trajectories 100
2.4 Some Additional Transformations of State/Signal Nodes 104
2.4.1 The (P, Q)-Image of a State/Signal Node 104
2.4.2 Parts and Static Projections of a State/Signal Node 107
2.4.3 Adding Inputs and Output to a State/Signal Node 111
2.5 Interconnections of State/Signal Nodes 118
2.5.1 The Cross Product of Two State/Signal Nodes 118
2.5.2 (P, Q)-Interconnections of State/Signal Nodes 119
2.5.3 A Short Circuit Connection of State/Signal Nodes 120
2.5.4 Examples of Interconnections of State/Signal Nodes 121
2.6 Examples of Infinite-Dimensional State/Signal Systems 123

3 State/Signal Systems: Dynamic and Frequency Domain Properties 132
3.1 Signal Behaviors and Their State/Signal Realizations 132
3.1.1 Future Signal Behaviors 132
3.1.2 External Equivalence of State/Signal Systems 133
3.2 Dynamic Properties of State/Signal Systems 133
3.2.1 Controllability and Observability of State/Signal Systems 134
3.2.2 Intertwinements of State/Signal Systems 140
3.2.3 Compressions, Restrictions, and Projections of State/Signal Systems 142
3.2.4 Examples of Minimal Compressions 148
3.2.5 State/Signal Systems with the Continuation Property 153
3.3 State Systems 156
3.3.1 A State System and Its Trajectories 157
3.3.2 The Homogeneous Cauchy Problem 158
3.3.3 Bounded State Systems and Uniformly Continuous Groups 159
3.3.4 Well-Posed State Systems and Strongly Continuous Semigroups 163
3.3.5 Transformations and Interconnections of State Nodes 169
3.3.6 Invariant Subspaces of State Nodes 171
3.3.7 Intertwinement of State Nodes 173
3.4 Frequency Domain Characteristics of State/Signal Nodes 174
3.4.1 The Characteristic Node Bundle 174
3.4.2 The Characteristic Control Bundle 178
3.4.3 The Characteristic Observation Bundle 179
3.4.4 The Characteristic Signal Bundle 180

© in this web service Cambridge University Press & Assessment www.cambridge.org

CAMBRIDGE
Cambridge University Press & Assessment
978-1-316-51967-7 — Linear State/Signal Systems
Damir Z. Arov , Olof J. Staffans
Table of Contents
More Information

3.4.5 The Characteristic Bundles of Transformed State/Signal Systems 182
3.4.6 The Resolvent of a Regular State Node 189
3.4.7 The Resolvent Set of a State/Signal Node 191
3.5 Invariance with Respect to Similarities 192
3.6 Dual and Adjoint State/Signal Nodes and Systems 194
3.6.1 The Dual of a State/Signal System 194
3.6.2 The Duals of Some Transformed State/Signal Nodes 202
3.6.3 The Characteristic Bundles of Dual State/Signal Nodes 206
3.6.4 The Adjoint State/Signal System 207
3.7 Notes to Chapters 2 and 3 213

4 Input/State/Output Representations 217
4.1 Input/State/Output Nodes and Systems 217
4.1.1 Regular I/S/O Nodes 217
4.1.2 General I/S/O Nodes and Systems 220
4.1.3 Kernal and Image Representations of Closed I/S/O Nodes 223
4.1.4 State, Input/State and State/Output Nodes and Systems 226
4.1.5 Input/State and State/Output Representations of State/Signal Systems 228
4.1.6 Free Inputs and Continuously Determined Outputs 229
4.1.7 Existence and Uniqueness of Trajectories 230
4.1.8 Bounded I/S/O Nodes and Systems 232
4.2 Input/State/Output Representations of State/Signal Nodes and Systems 234
4.2.1 The State/Signal Node Induced by an I/S/O Node 234
4.2.2 I/O Representations of the Signal Space 236
4.2.3 General I/S/O Representations of a State/Signal Node 239
4.2.4 Semiregular I/S/O Representations of a Semiregular State/Signal Node 241
4.2.5 Regular I/S/O Representations of a Regular State/Signal Node 242
4.2.6 Parametrization of I/S/O Representations 246
4.2.7 Bounded I/S/O Representations of Bounded State/Signal Nodes 248
4.2.8 Parametrization of Bounded I/S/O Representations 251
4.3 State Feedback and Output Injection Representations 256
4.3.1 State Feedback Representations 256
4.3.2 Output Injection Representations 259

© in this web service Cambridge University Press & Assessment www.cambridge.org

CAMBRIDGE
Cambridge University Press & Assessment
978-1-316-51967-7 — Linear State/Signal Systems
Damir Z. Arov , Olof J. Staffans
Table of Contents
More Information

4.4 Basic Transformations of Input/State/Output Nodes 261
4.4.1 Similarity of I/S/O Nodes 261
4.4.2 Time Reflection of an I/S/O Node 263
4.4.3 Time Rescaling of an I/S/O Node 264
4.4.4 Exponentially Weighted I/S/O Nodes 265
4.5 Properties of Trajectories of Input/State/Output Systems 267
4.5.1 Basic Properties of the Sets of Classical and Generalized Trajectories 267
4.5.2 Solvability and the Uniqueness Property 268
4.5.3 Connections between Classical, Generalized, and Mild Trajectories 270
4.6 Some Simple Input/State/Output Examples 273

5 Input/State/Output Systems: Dynamic and Frequency Domain Properties 276
5.1 Additional Transformations of Input/State/Output Nodes 276
5.1.1 Adding a Feedthrough Term to an I/S/O Node 276
5.1.2 Modifying Inputs and Outputs of an I/S/O Node 277
5.1.3 The (P, R, Q)-Image of an I/S/O Node 278
5.1.4 Parts and Static Projections of an I/S/O Node 281
5.1.5 Static Output Feedback 285
5.1.6 Adding Inputs and Outputs to an I/S/O Node 287
5.1.7 A Second Look at State Feedbacks and Output Injections 297
5.2 Interconnections of Input/State/Output Nodes 303
5.2.1 The Cross Product of Two I/S/O Nodes 303
5.2.2 (P, R, Q)-Interconnections of I/S/O Nodes 305
5.2.3 A Short Circuit Connection of I/S/O Nodes 306
5.2.4 T-Junctions, Sum Junctions, and Difference Junctions 307
5.2.5 Parallel and Difference Connections 311
5.2.6 Cascade Connections 313
5.2.7 Dynamic Feedback 316
5.2.8 Examples of I/S/O Interconnections 317
5.3 Realizations of Input/Output Behaviors 318
5.3.1 Future I/O Behaviors 318
5.3.2 External Equivalence of I/S/O Systems 319
5.4 Dynamic Properties of Input/State/Output Systems 320
5.4.1 Controllability and Observability of I/S/O Systems 320
5.4.2 Intertwinements of I/S/O Systems 325

© in this web service Cambridge University Press & Assessment www.cambridge.org

CAMBRIDGE
Cambridge University Press & Assessment
978-1-316-51967-7 — Linear State/Signal Systems
Damir Z. Arov , Olof J. Staffans
Table of Contents
More Information

5.4.3 Compressions, Restrictions, and Projections of I/S/O Systems 326
5.4.4 I/S/O Systems with the Continuation Property 330
5.5 Frequency Domain Characteristics of Input/State/Output Nodes 331
5.5.1 The Characteristic Node Bundle of an I/S/O Node 331
5.5.2 The I/S/O Resolvent Matrix of an I/S/O Node 334
5.5.3 Resolvability of Transformed I/S/O Nodes 338
5.5.4 Frequency Domain I/S/O-Admissible I/O Representations 341
5.6 The Correspondence between State/Signal and Input/State/Output Notions 343
5.6.1 I/O Invariant Notions 343
5.6.2 Properties of I/S/O Systems in the State/Signal Sense 349
5.7 Adjoint and Dual Input/State/Output Nodes and Systems 351
5.7.1 The Adjoint and the Dual of an I/S/O Node 351
5.7.2 Adjoint and Dual I/S/O Representations 354
5.7.3 I/S/O Lagrange Identities 355
5.7.4 Properties of Adjoint and Dual I/S/O Nodes and Systems 359
5.7.5 The Adjoints and Duals of Some Transformed I/S/O Nodes 360
5.7.6 The Adjoints and Duals of Some Interconnected I/S/O Nodes 364
5.8 Notes to Chapters 4 and 5 366

6 Bounded Input/State/Output Systems in Continuous and Discrete Time 370
6.1 Bounded State Operators and Nodes 370
6.1.1 The Spectral Radius of a Bounded State Operator 370
6.1.2 Invariant Subspaces of Bounded State Operators and Uniformly Continuous Groups 372
6.1.3 Parts and Projections of Bounded State Operators 373
6.1.4 Parts and Projections of Uniformly Continuous Groups 375
6.1.5 Intertwinements of Bounded State Operators and Uniformly Continuous Groups 377
6.1.6 Compressions of Bounded State Operators and Uniformly Continuous Groups 379
6.1.7 The General Structure of a Compression of a Bounded State Operator 385

© in this web service Cambridge University Press & Assessment www.cambridge.org

CAMBRIDGE
Cambridge University Press & Assessment
978-1-316-51967-7 — Linear State/Signal Systems
Damir Z. Arov , Olof J. Staffans
Table of Contents
More Information

6.1.8 The Adjoints of Bounded State Operators and Uniformly Continuous Groups 390
6.2 Static Properties of Bounded Input/State/Output Nodes 393
6.2.1 Transformations of Bounded I/S/O Nodes 393
6.2.2 Interconnections of Bounded I/S/O Nodes 403
6.2.3 The I/S/O Resolvent Matrix of a Bounded I/S/O Node 406
6.3 Dynamic Properties of Bounded Input/State/Output Systems 407
6.3.1 Strongly Invariant and Unobservably Invariant Subspaces 407
6.3.2 External Equivalence of Bounded I/S/O Systems 415
6.3.3 Intertwinements of Bounded I/S/O Systems 417
6.3.4 Restrictions and Projections of Bounded I/S/O Systems 421
6.3.5 Compressions of Bounded I/S/O Systems 423
6.3.6 The General Structure of a Bounded I/S/O Compression 428
6.3.7 Compressions into Minimal Bounded I/S/O Systems 435
6.4 The Adjoint and the Dual of a Bounded Input/State/Output Node 439
6.5 Discrete Time Input/State/Output Systems 444
6.5.1 Introduction to Discrete Time I/S/O Systems 444
6.5.2 Properties of Discrete Time I/S/O Systems 445
6.5.3 Time Reflection of Discrete Time I/S/O Systems 448
6.5.4 Power Weightings of Discrete Time I/S/O Systems 449
6.5.5 Frequency Domain Shifts of Discrete Time I/S/O Systems 450
6.5.6 Stable Discrete Time I/S/O Systems 451
6.5.7 Connections between Continuous and Discrete Time I/S/O Properties 453
6.5.8 Dynamic Notions for Bounded I/S/O Nodes 454
6.6 Bounded Input/State/Output Realizations 456
6.6.1 Analyticity at Infinity of the I/S/O Resolvent Matrix 456
6.6.2 Existence of a Bounded I/S/O Realization 457

7 Bounded State/Signal Systems in Continuous and Discrete Time 460
7.1 Static Properties of Bounded State/Signal Nodes 460
7.1.1 The I/S/O-Bounded Resolvent Set of a Bounded State/Signal Node 460
7.1.2 Transformations of Bounded State/Signal Nodes 462
7.1.3 Resolvability of Transformations of State/Signal Nodes 476
7.2 Dynamic Properties of Bounded State/Signal Systems 481

© in this web service Cambridge University Press & Assessment www.cambridge.org

CAMBRIDGE
Cambridge University Press & Assessment
978-1-316-51967-7 — Linear State/Signal Systems
Damir Z. Arov , Olof J. Staffans
Table of Contents
More Information

7.2.1 Strongly Invariant and Unobservably Invariant Subspaces 481
7.2.2 External Equivalence of Bounded State/Signal Systems 489
7.2.3 Intertwinements of Bounded State/Signal Systems 490
7.2.4 Restrictions and Projections of Bounded State/Signal Systems 494
7.2.5 Compressions of Bounded State/Signal Systems 496
7.2.6 The General Structure of a Bounded State/Signal Compression 498
7.2.7 Compressions into Minimal Bounded State/Signal Systems 504
7.2.8 Bounded State/Signal Realizations 505
7.3 The Dual and the Adjoint of a Bounded State/Signal Node 506
7.4 Discrete Time State/Signal Systems 510
7.4.1 Introduction to Discrete Time State/Signal Systems 510
7.4.2 Properties of Discrete Time State/Signal Systems 511
7.4.3 Time Reflection of Discrete Time State/Signal Systems 513
7.4.4 Power Weightings of Discrete Time State/Signal Systems 513
7.4.5 Frequency Domain Shifts of Discrete Time State/Signal Systems 514
7.4.6 Stable Discrete Time State/Signal Systems 515
7.4.7 Connections between Continuous and Discrete Time State/Signal Properties 515
7.4.8 Dynamic Notions for Bounded State/Signal Nodes 516
7.5 Notes to Chapters 6 and 7 518

8 Semi-bounded Input/State/Output Systems 521
8.1 C_0 Semigroups and Well-Posed State Systems 521
8.1.1 On the Resolvents of Generators of C_0 Semigroups 521
8.1.2 The Inhomogeneous Cauchy Problem 524
8.1.3 Invariant Subspaces of C_0 Semigroups 530
8.1.4 Parts, Projections, and Restrictions of Single-Valued Resolvable Main Operators 530
8.1.5 Parts and Projections of C_0 Semigroups 533
8.1.6 Intertwinements of C_0 Semigroups 535
8.1.7 Compressions of C_0 Semigroups 536
8.1.8 The General Structure of a Compression of a C_0 Semigroup 539

© in this web service Cambridge University Press & Assessment www.cambridge.org

CAMBRIDGE
Cambridge University Press & Assessment
978-1-316-51967-7 — Linear State/Signal Systems
Damir Z. Arov , Olof J. Staffans
Table of Contents
More Information

8.1.9 The Adjoint of a C_0 Semigroup 542
8.2 Semi-bounded Input/State/Output Systems 544
8.2.1 Introduction to Semi-bounded I/S/O Systems 544
8.2.2 Transformations of Semi-bounded I/S/O Nodes 547
8.2.3 Interconnections of Semi-bounded I/S/O Nodes 551
8.2.4 The I/S/O Resolvent Matrix of a Semi-bounded I/S/O Node 552
8.3 Dynamic Properties of Semi-bounded Input/State/Output Systems 553
8.3.1 Strongly Invariant and Unobservably Invariant Subspaces 553
8.3.2 External Equivalence of Semi-bounded I/S/O Systems 559
8.3.3 Intertwinements of Semi-bounded I/S/O Systems 559
8.3.4 Restrictions and Projections of Semi-bounded I/S/O Systems 562
8.3.5 Compressions of Semi-bounded I/S/O Systems 563
8.3.6 The General Structure of a Semi-bounded I/S/O Compression 565
8.3.7 Compressions into Minimal Semi-bounded I/S/O Systems 570
8.4 The Adjoint of a Semi-bounded Input/State/Output Node 572

9 Semi-bounded State/Signal Systems 576
9.1 Static Properties of Semi-bounded State/Signal Nodes 576
9.1.1 Introduction to Semi-bounded State/Signal Nodes and Systems 576
9.1.2 The I/S/O Semi-bounded Resolvent Set of a Semi-bounded State/Signal Node 580
9.1.3 Transformations and Interconnections of Semi-bounded State/Signal Nodes 581
9.2 Dynamic Properties of Semi-bounded State/Signal Systems 581
9.2.1 Strongly Invariant and Unobservably Invariant Subspaces 581
9.2.2 External Equivalence of Semi-bounded State/Signal Systems 584
9.2.3 Intertwinements of Semi-bounded State/Signal Systems 585
9.2.4 Restrictions and Projections of Semi-bounded State/Signal Systems 587
9.2.5 Compressions of Semi-bounded State/Signal Systems 588

© in this web service Cambridge University Press & Assessment www.cambridge.org

CAMBRIDGE
Cambridge University Press & Assessment
978-1-316-51967-7 — Linear State/Signal Systems
Damir Z. Arov , Olof J. Staffans
Table of Contents
More Information

9.2.6 The General Structure of a Semi-bounded State/Signal Compression 589
9.2.7 Compressions into Minimal Semi-bounded State/Signal Systems 593
9.3 The Adjoint of a Semi-bounded State/Signal Node 594
9.4 Notes to Chapters 8 and 9 596

10 Resolvable Input/State/Output and State/Signal Nodes 599
10.1 Resolvable State Nodes 599
10.1.1 Linear Operator-Valued Pencils 599
10.1.2 The Resolvent of a State Node 601
10.1.3 The Interpolation Space of a Semiregular State Node 606
10.1.4 The Extrapolation Space of a Regular Resolvable State Node 607
10.1.5 The Duals of the Interpolation and Extrapolation Spaces 610
10.1.6 The Interpolation and Extrapolation Spaces of a Semigroup Generator 613
10.2 Resolvable Input/State/Output Nodes 614
10.2.1 Resolvability of an I/S/O Node 615
10.2.2 Kernel and Image Representations of the I/S/O Resolvent Matrix 618
10.2.3 The I/S/O Resolvent Identity 620
10.2.4 Representations of the System Operator 624
10.2.5 Semiregular and Regular Resolvable I/S/O Nodes 628
10.2.6 The Observation and Control Operators of a Regular Resolvable I/S/O Node 631
10.2.7 Some Examples of Regular Resolvable I/S/O Nodes 637
10.2.8 Resolvability of Transformed I/S/O Nodes 640
10.2.9 Resolvability of Interconnected I/S/O Nodes 647
10.2.10 The Resolvent Family of Bounded I/S/O Nodes 652
10.2.11 A Finite-Dimensional Nonregular Resolvable I/S/O Node 653
10.2.12 The Adjoint and the Dual of a Resolvable I/S/O Node 655
10.3 Resolvable State/Signal Nodes 658
10.3.1 On the Resolvent Set of a Closed State/Signal Node 658
10.3.2 Frequency Domain I/S/O-Admissible I/O Representations 662
10.3.3 Resolvability of Transformed State/Signal Nodes 669
10.3.4 The Resolvent Family of Bounded State/Signal Nodes 673

© in this web service Cambridge University Press & Assessment www.cambridge.org

CAMBRIDGE
Cambridge University Press & Assessment
978-1-316-51967-7 — Linear State/Signal Systems
Damir Z. Arov , Olof J. Staffans
Table of Contents
More Information

10.3.5 The Dual and the Adjoint of a Resolvable State/Signal System 674
10.4 Notes and Comments 676

11 Frequency Domain Input/State/Output Systems 679
11.1 Frequency Domain Input/State/Output Systems 679
11.1.1 Introduction to Frequency Domain I/S/O Systems 679
11.1.2 Frequency Domain Controllability and Observability 681
11.1.3 Frequency Domain Invariance 682
11.1.4 The Frequency Domain Behavior and External Equivalence 689
11.1.5 Frequency Domain Intertwinements 690
11.1.6 Frequency Domain Compressions, Restrictions, and Projections 696
11.1.7 Resolvable Frequency Domain Compressions, Restrictions, and Projections 698
11.1.8 The General Structure of a Resolvable Frequency Domain Compression 704
11.1.9 Compressions into Ω-Minimal I/S/O Systems 712
11.1.10 Results for Connected Frequency Domains 715
11.2 The Adjoint and the Dual of a Frequency Domain Input/State/Output System 725
11.2.1 Frequency Domain Lagrange Identities 726
11.2.2 Properties of Adjoint and Dual Frequency Domain I/S/O Systems 728
11.3 Frequency Domain Notions for Ω-Resolvable Input/State/Output Nodes 730
11.3.1 Dynamic Properties of the Resolvent Family of Bounded I/S/O Nodes 730
11.4 Resolvable Frequency Domain State Systems 733
11.4.1 Frequency Domain Invariance 734
11.4.2 Frequency Domain Intertwinements and Compressions 734
11.4.3 Results for Connected Frequency Domains 738
11.4.4 Frequency Domain Duality 740
11.5 Notes and Comments 741

12 Frequency Domain State/Signal Systems 743
12.1 Frequency Domain State/Signal Systems 743
12.1.1 Introduction to Frequency Domain State/Signal Systems 743

© in this web service Cambridge University Press & Assessment www.cambridge.org

CAMBRIDGE
Cambridge University Press & Assessment
978-1-316-51967-7 — Linear State/Signal Systems
Damir Z. Arov , Olof J. Staffans
Table of Contents
More Information

12.1.2 Separately and Jointly I/S/O Admissible Frequency Domains 745
12.1.3 Frequency Domain Controllability and Observability 747
12.1.4 Frequency Domain Invariance 748
12.1.5 The Frequency Domain Behavior and External Equivalence 752
12.1.6 Frequency Domain Intertwinements 755
12.1.7 Frequency Domain Compressions, Restrictions, and Projections 761
12.1.8 Resolvable Frequency Domain Compressions, Restrictions, and Projections 763
12.1.9 The General Structure of a Resolvable Frequency Domain Compression 769
12.1.10 Compressions into Ω-Minimal State/Signal Systems 773
12.2 Local Frequency Domain Notions 775
12.2.1 Local Frequency Domain Notions for Ω-Resolvable State/Signal Systems 776
12.2.2 Connected Frequency Domains 783
12.3 The Dual and the Adjoint of a Frequency Domain State/Signal System 793
12.3.1 Frequency Domain Lagrange Identities 794
12.3.2 Properties of Dual and Adjoint Frequency Domain State/Signal Systems 795
12.4 Frequency Domain Notions for Ω-Resolvable State/Signal Nodes 798
12.4.1 Dynamic Properties of the Resolvent Family of Bounded State/Signal Nodes 798
12.5 Notes and Comments 801

13 Internally Well-Posed Systems 802
13.1 Internally Well-Posed Input/State/Output Systems 802
13.1.1 Basic Definitions and Properties 802
13.1.2 Transformations and Interconnections 804
13.2 Frequency-Domain Internally Well-Posed Input/State/Output Systems 805
13.2.1 Frequency Domain Invariance 806
13.2.2 Frequency Domain Intertwinements 806
13.2.3 Frequency-Domain Restrictions, Projections, and Compressions 807
13.2.4 The General Structure of $\rho_{+\infty}(\Sigma)$-Compressions 809

© in this web service Cambridge University Press & Assessment www.cambridge.org

CAMBRIDGE
Cambridge University Press & Assessment
978-1-316-51967-7 — Linear State/Signal Systems
Damir Z. Arov , Olof J. Staffans
Table of Contents
More Information

13.3 Internally Well-Posed State/Signal Systems 812
13.3.1 Basic Definitions and Properties 812
13.3.2 Frequency-Domain Compressions of Internally Well-Posed State/Signal Systems 813
13.4 Notes and Comments 814

14 Well-Posed Input/State/Output Systems 816
14.1 Basic Properties of Well-Posed Input/State/Output Systems 816
14.1.1 The Definition of a Well-Posed I/S/O System 816
14.1.2 Alternative Conditions for Well-Posedness 819
14.1.3 The Fundamental I/S/O Solution of a Well-Posed I/S/O System 825
14.2 The Growth Bound of a Well-Posed Input/State/Output System 833
14.2.1 The Growth Bound of a Well-Posed I/S/O System 833
14.2.2 Stable I/S/O Systems 838
14.3 Resolvability of Well-Posed Input/State/Output Systems 842
14.3.1 Well-Posed I/S/O Systems Are Resolvable 842
14.3.2 Growth Estimates for the I/S/O Resolvent Matrix 847
14.4 Realizations of Shift-Invariant Causal Linear Operators 850
14.4.1 Shift Invariant Causal Linear Operators 850
14.4.2 Realizations of Shift Invariant Causal Linear Operators 852
14.4.3 Toeplitz and Hankel Operators 853
14.5 Transformations and Interconnections of Well-Posed Input/State/Output Systems 857
14.5.1 Well-Posedness and Stability of Transformed I/S/O Systems 857
14.5.2 Well-Posedness and Stability of Interconnected I/S/O Systems 864
14.5.3 Stabilizable and Detectable I/S/O Systems 867
14.6 Dynamic Properties of Well-Posed Input/State/Output Systems 869
14.6.1 Strongly Invariant and Unobservably Invariant Subspaces 869
14.6.2 Intertwinements of Well-Posed I/S/O Systems 870
14.6.3 Restrictions, Projections, and Compressions 874
14.6.4 The General Structure of a Well-Posed I/S/O Compression 879
14.6.5 Compressions Into Minimal Well-Posed I/S/O Systems 886
14.7 Well-Posed Input/State/Output Systems in the Frequency Domain 887
14.7.1 Time and Frequency Domain External Equivalence 888
14.7.2 Time and Frequency Domain Invariance 888

© in this web service Cambridge University Press & Assessment www.cambridge.org

CAMBRIDGE
Cambridge University Press & Assessment
978-1-316-51967-7 — Linear State/Signal Systems
Damir Z. Arov , Olof J. Staffans
Table of Contents
More Information

14.7.3 Time and Frequency Domain Compressions and Intertwinements 889
14.7.4 Frequency Domain Stability 891
14.8 The Adjoint of a Well-Posed Input/State/Output System 893
14.9 Scattering Passive Input/State/Output Systems 897
14.9.1 Hilbert Space I/S/O Nodes and Systems 897
14.9.2 Scattering Passive I/S/O Systems 898
14.9.3 The Internal I/S/O Cayley Transform 901
14.9.4 The Adjoint of a Passive Scattering System 904
14.10 Notes and Comments 904

15 Well-Posed State/Signal Systems 907
15.1 Basic Properties of Well-Posed State/Signal Systems 907
15.1.1 Basic Definitions 907
15.1.2 Well-Posedness and Stability of Transformed I/S/O Systems 913
15.1.3 The Behaviors Induced by a Well-Posed State/Signal System 914
15.1.4 The Past/Present and Present/Future Maps of a Well-Posed State/Signal System 916
15.2 Stable State/Signal Systems 920
15.2.1 Stable State/Signal Trajectories 920
15.2.2 Stable State/Signal Behaviors 921
15.2.3 Stabilizable and Detectable State/Signal Systems 922
15.3 Realizations of Well-Posed Behaviors 925
15.3.1 Well-Posed Future, Past, and Two-Sided Behaviors 925
15.3.2 State/Signal Realizations of Well-Posed Behaviors 929
15.3.3 The Past/Future Map of a Well-Posed Behavior 929
15.4 Dynamic Properties of Well-Posed State/Signal Systems 930
15.4.1 Strongly Invariant and Unobservably Invariant Subspaces 930
15.4.2 Intertwinements of Well-Posed State/Signal Systems 932
15.4.3 Restrictions, Projections, and Compressions of Well-Posed State/Signal Systems 933
15.4.4 The General Structure of a Compression 936
15.4.5 Compressions into Minimal Well-Posed State/Signal Systems 939
15.5 Well-Posed State/Signal Systems in the Frequency Domain 940
15.6 The Adjoint of a Well-Posed State/Signal Node 942
15.7 Passive State/Signal Systems 944

© in this web service Cambridge University Press & Assessment www.cambridge.org

CAMBRIDGE
Cambridge University Press & Assessment
978-1-316-51967-7 — Linear State/Signal Systems
Damir Z. Arov , Olof J. Staffans
Table of Contents
More Information

15.7.1 Kreĭn Spaces 944
15.7.2 The Kreĭn Node Space of a Scattering Passive I/S/O System 945
15.7.3 Passive State/Signal Systems 946
15.8 Notes and Comments 949

Appendix A Operators and Analytic Vector Bundles in *H*-Spaces 950
A.1 *H*-Spaces 950
A.1.1 Using More than One Norm in a Vector Space 950
A.1.2 Introduction to *H*-Spaces 952
A.1.3 Linear Operators in *H*-Spaces 953
A.1.4 Closed Linear Operators in *H*-Spaces 955
A.1.5 Complementary Projections and Coordinate Respresentations of *H*-Spaces 956
A.1.6 Isomorphisms in *H*-Spaces 960
A.1.7 Partial Inverses of Bounded Linear Operators 961
A.1.8 Inversion of Block Matrix Operators 964
A.1.9 The Graph Norm and Graph Topology 965
A.1.10 Linear Multivalued Operators in *H*-Spaces 966
A.1.11 The Single-Valued and Injective Parts of a Multivalued Operator 969
A.1.12 On the Resolvent of a Bounded Operator 970
A.2 Duality in *H*-Spaces 971
A.2.1 The Dual of an *H*-Space 971
A.2.2 The Adjoint of a Bounded Linear Operator 973
A.2.3 Duals of Product Spaces 976
A.2.4 The Duals of the Components of a Direct Sum Decomposition 979
A.2.5 The Adjoint of a Linear Operator with Dense Domain 982
A.2.6 The Dual of a Continuous Dense Embedding 983
A.2.7 The Adjoint of a Multivalued Operator 984
A.3 Analytic Vector Bundles and Analytic Operator-Valued Functions 988
A.3.1 The Dual Vector Bundle 992

References 994
Index 1005

© in this web service Cambridge University Press & Assessment www.cambridge.org