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René L. Schilling , Franziska Kühn
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This should be used in conjunction with the references, the table of contents, the lists of symbols and the list of topics. Generally, index entries are theorems, concepts and methods, but not concrete counterexamples. These can be found using the table of contents or the list of topics. Numbers following entries are either chapter numbers (§n) or page numbers (n), bold numbers refer to definitions and important statements. The symbols 👁 and ▦ refer to pictures and tables. Unless otherwise stated, 'integral', 'integrability', etc. always mean the (abstract) Lebesgue integral. Within the index we use 'L…' and 'R…' as a shorthand for '(abstract) Lebesgue…' and 'Riemann…'.

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