

Contents

<i>Preface</i>	<i>page</i> ix
<i>Acknowledgments</i>	x
1 Introduction	1
1.1 Function-Theoretic Operator Theory on Vectorial Hardy Spaces, Reproducing Kernel Hilbert Spaces, and Discrete-Time Linear Systems: Background	1
1.2 The Synthesis of the Systems-Theory and Reproducing Kernel Approaches	3
1.3 Standard Weighted Bergman Spaces	11
1.4 The Hardy–Fock Space Setting	14
1.5 Weighted Bergman–Fock Spaces	16
1.6 Overview	19
1.7 Notes	25
2 Formal Reproducing Kernel Hilbert Spaces	26
2.1 Basic Definitions	26
2.2 Weighted Hardy–Fock Spaces	35
2.3 Notes	41
3 Contractive Multipliers	42
3.1 Contractive Multipliers in General	43
3.2 Contractive Multipliers between Hardy–Fock Spaces	49
3.3 A Noncommutative Leech’s Theorem	72
3.4 Contractive Multipliers from $H_{\mathcal{U}}^2(\mathbb{F}_d^+)$ to $H_{\omega, \mathcal{Y}}^2(\mathbb{F}_d^+)$ for Admissible ω	78
3.5 $H_{\omega, \mathcal{Y}}^2(\mathbb{F}_d^+)$ -Bergman-Inner Formal Power Series	95
3.6 Notes	100

4	Stein Relations and Observability Range Spaces	103
4.1	Preliminaries on Functional Calculus for the Operator B_A	104
4.2	Observability, Defect and Shifted Defect Operators	113
4.3	Shifted Observability Operators and Observability Gramians	136
4.4	The Model Shift-Operator Tuple on $H_{\omega, \mathcal{Y}}^2(\mathbb{F}_d^+)$	139
4.5	A Wold Decomposition for ω -Isometric-like Operator Tuples	146
4.6	Observability-Operator Range Spaces	157
4.7	Notes	168
5	Beurling–Lax Theorems Based on Contractive Multipliers	180
5.1	Beurling–Lax Representations with Model Space $H_{\mathcal{U}}^2(\mathbb{F}_d^+)$	180
5.2	Beurling–Lax Representations Based on Contractive Multipliers from $H_{\omega, \mathcal{U}}^2(\mathbb{F}_d^+)$ to $H_{\omega, \mathcal{Y}}^2(\mathbb{F}_d^+)$	202
5.3	Representations with Model Space of the Form $\bigoplus_{j=1}^n \mathcal{A}_{j, \mathcal{U}_j}(\mathbb{F}_d^+)$	205
5.4	Notes	212
6	Non-orthogonal Beurling–Lax Representations Based on Wandering Subspaces	215
6.1	Beurling–Lax Quasi-Wandering Subspace Representations	216
6.2	Non-orthogonal Beurling–Lax Representations Based on Wandering Subspaces	221
6.3	Notes	229
7	Orthogonal Beurling–Lax Representations Based on Wandering Subspaces	230
7.1	Transfer Functions $\Theta_{\omega, \mathcal{U}_\beta}$ and Metric Constraints	230
7.2	Beurling–Lax Representations Based on Bergman-Inner Families	242
7.3	Expansive Multiplier Property	265
7.4	Bergman-Inner Multipliers as Extremal Solutions of Interpolation Problems	278
7.5	Notes	284
8	Models for ω-Hypercontractive Operator Tuples	285
8.1	Model Theory Based on Observability Operators	286
8.2	The Characteristic Function Approach	291
8.3	Model Theory for n -Hypercontractions	309
8.4	Notes	313
9	Weighted Hardy–Fock Spaces Built from a Regular Formal Power Series	315
9.1	Preliminaries	315
9.2	The Spaces $H_{\omega_{p,n}, \mathcal{Y}}^2(\mathbb{F}_d^+)$ and Their Contractive Multipliers	322

Contents

vii

9.3	Output Stability, Stein Equations, and Inequalities	354
9.4	The $\omega_{p,n}$ -Shift Model Operator Tuple $\mathbf{S}_{\omega_{p,n},R}$	367
9.5	Observability Operator Range Spaces in $H^2_{\omega_{p,n},\mathcal{Y}}(\mathbb{R}_d^+)$	370
9.6	Beurling–Lax Theorems Based on Contractive Multipliers	372
9.7	Beurling–Lax Representations via Quasi-Wandering Subspaces	383
9.8	Beurling–Lax Representations Based on Bergman-Inner Families	385
9.9	Operator Model Theory for c.n.c. $*$ - (p,n) -Hypercontractive Tuples	399
9.10	Notes	411
	<i>References</i>	415
	<i>Notation Index</i>	425
	<i>Subject Index</i>	427