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978-1-316-51895-3 — Algebraic Number Theory for Beginners Algebraic Number Theory for Beginners
John Stillwell
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