Point Processes and Jump Diffusions

The theory of marked point processes on the real line is of great and increasing importance in areas such as insurance mathematics, queuing theory and financial economics. However, the theory is often viewed as technically and conceptually difficult and has proved to be a block for PhD students looking to enter the area.

This book gives an intuitive picture of the central concepts as well as the deeper results, while presenting the mathematical theory in a rigorous fashion and discussing applications in filtering theory and financial economics. Consequently, readers will get a deep understanding of the theory and how to use it. A number of exercises of differing levels of difficulty are included, providing opportunities to put new ideas into practice. Graduate students in mathematics, finance and economics will gain a good working knowledge of point-process theory, allowing them to progress to independent research.

Tomas Björk was Professor Emeritus of Mathematical Finance at the Stockholm School of Economics and previously worked at the Mathematics Department of the Royal Institute of Technology, Stockholm. Björk served as co-editor of *Mathematical Finance*, on the editorial board for *Finance and Stochastics* and several other journals, and was President of the Bachelier Finance Society. He was particularly known for his research on point-process-driven forward-rate models, finite-dimensional realizations of infinite-dimensional SDEs, and time-inconsistent control theory. He was the author of the well-known textbook *Arbitrage Theory in Continuous Time* (1998), now in its fourth edition.

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An Introduction with Finance Applications

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Preface

This text is intended as an introductory overview of stochastic calculus for marked point processes and jump diffusions with applications to filtering, stochastic control and finance.

The pedagogical approach is that I have tried to provide as much intuition as possible while being reasonably precise. In more concrete terms this means that most major results are preceded by a heuristic argument, often including "infinitesimal" arguments, which leads up to the formulation of a conjecture. As an example, when discussing measure transformations, some rather simple arguments will lead us to conjecture the formulation of the Girsanov theorem. This conjecture is then given a formal proof. Most results are given full formal proofs, apart from the fact that I go lightly on technical details like integrability and regularity assumptions. This is done in order to increase readability and to highlight the main ideas in the proofs. For more technical points the reader is referred to the specialist literature. A few results, where the proofs are really hard, are presented without proofs, but often with a heuristic argument to make them believable. The reader is again referred to the literature.

The dependence structure of the text is that Parts II–V on control, filtering and finance build on Part I. Parts II–V can, however, be read independently of each other.

The reader is assumed to be familiar with basic measure theory and stochastic calculus for Wiener processes. No previous knowledge of finance is assumed.

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