

Index

- 3J symbols, *see* vector-coupling coefficients
- 6J symbols, 536–538, 540
- 9J symbols, 538–540
- abelian invariant subgroup, 25, 26, 120, 152  
215, 218
- absolute derivative, 455
- accidental degeneracy, 19, 174, 521
- adiabatic continuity, *see* Fermi liquid
- adjoint representation
  - construction of, 52
  - for isospin, 58
  - generated by structure constants, 44
  - SU(2), 52
- affine connection, *see* connection coefficients
- Aharonov–Bohm effect
  - and 4-vector potential, 464
  - and gauge invariance, 466, 467
  - experimental setup, 464
  - magnetic fields, 465
  - phase of electron wavefunction, 465
  - topological origin, 468
- alpha-string, *see* Lie algebras
- ammonia molecule, 84, 87, 88
- Anderson localization, 486, 488
- angular momentum
  - adjoint representation, 45
  - and SU(2), 19, 45
  - fundamental representation, 45
  - Lie algebra, 43
  - operators as spherical tensors, 114, 116
  - orbital angular momentum operator, 176, 177
  - recoupling of four angular momenta, 538–540
  - recoupling of three angular momenta, 535–537
  - vector-coupling coefficients, 109–112
- anholonomy, *see* holonomy
- anomalies in quantum field theory, 301, 585, 586
- antisymmetric rank-3 tensor, 46
- anyons
  - abelian, 528
  - and fractional statistics, 500, 525
  - in fractional quantum Hall states, 500
  - irreps of the braid group, 528
  - non-abelian, 528
- asymptotic freedom, *see* QCD
- atlas, *see* manifolds
- automorphism
  - generators and complex conjugates, 149
  - identifying, 149
  - inner, 15, 22
- axis-angle parameterization, 119
- Baker–Campbell–Hausdorff (BCH) formula, 63
- balls (open or closed), *see* topology
- band insulators, 81, 82, 560
- band theory of solids, 81, 82
- baryon, definition, 326
- BCS, *see* superconductivity
- Berry phase
  - and adiabatic curvature, 490
  - and gauge invariance, 471, 472
  - and topology of Brillouin zone, 495
  - Berry connection, 470, 471, 507, 508
  - Berry curvature, 472, 474
  - Berry flux, 474
  - Berry potential, 471
  - fast and slow degrees of freedom, 468, 470
  - geometrical origin, 473
  - in integer quantum Hall effect, 490
  - physical reality, 474
  - spin-1/2 in a magnetic field, 474, 475
- beta-string, *see* Lie algebras
- Bloch’s theorem, 79, 80, 92, 486
- Bragg planes, 77, 78
- braid groups
  - definition, 526
  - multiplication rule, 526, 528
- Brillouin zone
  - and Bloch’s theorem, 80
  - and Chern numbers, 496
  - and lattice wavefunctions, 78, 496
  - definition, 77, 78
  - first Brillouin zone, 78
  - magnetic, 491
  - topology of, 495, 496
- bulk–boundary correspondence, 498, 512, 522
- bundle, *see* fiber bundle
- Cartan matrix, 138, 139
- Cartan subalgebra, 47, 126
- Cartan–Dynkin method, 126
  - and SU(2), 48, 49
  - and SU(3), 49, 50

- Cartan–Dynkin method (Cont.)
  - Cartan subalgebra, 47
  - Casimir operators, 47
  - classification of algebras, 126
  - multiplicity of weights, 47
  - roots, 47, 48
  - weight diagram, 48
  - weight space, 49
  - weight vectors, 48
  - weights, 47
- Cartan–Killing form, *see* Lie algebras
- Casimir operator
  - SU(2), 47
  - $E_2$ , 214
  - Lie groups, 130
  - Poincaré group, 266
  - SU(3), 145
- Cayley’s theorem, 36, 65
- center of group, *see* abelian invariant subgroup
- character tables, 34, 35
- charge conjugation (C)
  - definition, 260
  - for spinors, 257, 260
- charge independence hypothesis, *see* isospin
- chart, *see* manifolds
- Chern insulators, 513
- Chern numbers
  - and Berry curvature, 475, 476
  - and level crossings, 495
  - and time-reversal symmetry, 508, 509
  - and topology of Brillouin zone, 495, 496, 509, 511
  - Chern theorem, 493, 494
  - generalization of Gauss–Bonnet theorem, 491
  - in the integer quantum Hall effect, 491–493, 495–498
  - TKNN invariant, 489
- Chern theorem, 464, 476, 493, 494
- chiral invariance
  - and current algebra, 578
  - and helicity, 250, 252, 253
  - and interactions, 254
  - and weak interactions, 259
  - chiral edge states, 488, 500, 501, 513
  - explicit breaking of, 582
  - projection operators for chiral fermions, 252
- classes
  - conjugate, 22, 23
  - rotation group, 119
- Clebsch–Gordan coefficients, *see* vector-coupling coefficients
- Clebsch–Gordan series
  - and outer product of permutation group, 156, 157
  - and representation characters, 71
  - and Young diagrams for SU( $N$ ), 156, 157
  - for SO(3), 109
  - for SU(2), 50, 51
  - for SU(3), 156
  - product of SU(3) adjoint representations, 156
- Clifford algebra, 248, 249
- coherent states
  - and dynamical symmetry, 369, 372, 373
  - atoms interacting with radiation, 373–375
  - coset space, 371
  - definition, 368, 369
  - electromagnetic, 369
  - extremal state, 370
  - fermionic, 375, 376
  - generalized algorithm, 371
  - Glauber, 368, 369
  - stability subgroup, 370
- color degree of freedom, *see* QCD
- compactness
  - and invariant integration, 103
  - and semisimplicity for Lie algebras, 130
  - and the Heine–Borel theorem, 425
  - and the metric tensor for Lie algebras, 130
  - as topological invariant, 427
  - topological definition, 424, 425
- composition, 7, 99
- Condon–Shortley phase convention, 106
- conduction band, 81, 82
- confinement, *see* QCD
- conjugate classes, 22, 23
- conjugate representation, 149
- connectedness
  - as topological invariant, 427
  - for SU(2) and SO(3), 119, 120
  - intuitive definition, 101
  - simply connected spaces, 432
  - topological definition, 426
- connection coefficients, 453, 454
- conservation laws, 5, 6
- conserved vector current (CVC), *see* current algebra
- Cooper instability, 414
  - in conventional superconductors, 557, 559
  - of a Fermi liquid, 557, 559
  - of doped AF Mott insulator, 573, 574
- Cooper pairs
  - as instability of Fermi liquid, 557, 559
  - charge carriers for superconductivity, 318
  - in doped Mott insulators, 573
  - topologically non-trivial, 524
- coordinate systems
  - anholonomic, 451
  - holonomic, 451
- cosets
  - and factor groups, 26–29
  - and generalized coherent states, 29
  - and partitioning of group elements, 22
  - left, 27–29
  - multiplication law, 28
  - right, 27–29

- cotangent bundle, *see* fiber bundle
- covariant derivative
  - and minimal substitution, 288
  - for electromagnetic coupling, 288
  - in general relativity, 454
- covering group, *see* universal covering group
- covering operations, *see* symmetry
- Coxeter matrix, *see* Cartan matrix
- CP symmetry
  - $\theta$ -vacuum and the strong CP problem, 448
  - and Weyl spinors, 259
  - conserved by  $V - A$  interaction, 260
  - definition, 260
  - in weak interactions, 260
- CPT symmetry, 260
- critical dynamical symmetry, *see* dynamical symmetry
- critical exponent, *see* phase transitions
- crystal momentum, 96
- current algebra
  - $SU(2)_L \times SU(2)_R$ , 578
  - $SU(3)_L \times SU(3)_R$ , 578
  - and chiral symmetry, 578
  - and parity, 579
  - conserved vector current (CVC), 578
  - explicit breaking of chiral symmetry, 582
  - partially conserved axial current (PCAC), 578, 579
- curvature
  - and parallel transport, 450, 454, 490
  - Berry, 472, 474
  - extrinsic, 454
  - intrinsic, 454
- cyclotron frequency, 481
- $D$ -matrices, 106–108, 605
- d'Alembertian operator, 242, 276, 282
- Debye frequency, 557, 559
- Dirac equation
  - and chirality, 252, 253
  - and Dirac 4-spinors, 237, 244
  - and helicity, 252
  - and positrons, 252
  - and the Lorentz group, 243
  - as wave equation, 276
  - bilinear covariants, 245–249
  - chiral invariance, 250
  - Clifford algebra, 248, 249
  - covariance properties, 246–249
  - Dirac matrices, 245
  - in condensed matter, 347, 523
  - in graphene, 345, 347, 348
  - Lorentz-boosted spinors, 244
  - negative-energy states, 252
  - Pauli–Dirac representation, 245, 251, 252
  - quaternions, 248, 249
  - single-particle interpretation, 252
  - Weyl equation for massless fermions, 248
  - Weyl representation, 245
- Dirac mass, *see* neutrinos
- Dirac notation, *see* second quantization
- direct product
  - and  $SU(N)$  Young diagrams, 156
  - of representations, 31
  - of  $SO(3)$  representations, 109
  - of  $SU(N)$  representations, 156
  - of  $SU(2)$  representations, 51
  - representations of direct product group, 31, 152
- direct product group, 29, 30
- direct sum
  - multiplicity of irreps, 51
  - of irreps, 19, 109
- directional derivative, 451
- divergence theorem, 297
- Drude model, 502
- dynamical Pauli effect, *see* fermion dynamical symmetry model (FDSM)
- dynamical symmetry
  - and dynamics, 59, 60, 277, 343, 560
  - and local gauge invariance, 560
  - as a microscopic method, 340–343
  - as Hilbert space truncation, 340–343, 365
  - coset space, 373
  - critical dynamical symmetry, 355, 359–361, 549, 567, 568, 575
  - dimensionality constraint, 362–364
  - effective interaction, 342, 343
  - FDSM, 543, 546, 547
  - generalized coherent states, 370–373
  - Ginocchio model, 543, 547
  - graphene in strong magnetic field, 343–361
  - maximum stability subgroup, 373
  - solution algorithm, 341, 342
  - topological and algebraic constraints, 362–364
  - universality of emergent states, 361
  - validity, 342
- Dynkin diagrams, 608
  - and Cartan matrix, 138, 139
  - and Lie algebras, 141
  - constructing all roots from, 139
  - constructing Lie algebras from, 139–141
  - construction of, 138
  - definition, 137, 138
- eigenoperators, 128
- Einstein summation convention, *see* summation convention (repeated index)
- Elliott model
  - and nuclear collective motion, 181, 187
  - and nuclear shell model, 183
  - angular momentum content, 186
  - band terminations, 206
  - classification of nuclear states, 179, 181–185

- Elliott model (cont.)
  - electromagnetic transitions, 204
  - group-theoretical solution, 188, 189
  - Hamiltonian, 187
  - SO(3) subgroups, 184
  - spectrum, 189
  - SU(2) groups, 185
  - weight space operators, 185, 186
- emergent states
  - and adiabatic continuity, 341, 505
  - and dynamical symmetry, 340, 341
  - and quantum phases, 341, 355
  - and spontaneous symmetry breaking, 341
  - and universality, 141, 361, 362
  - definition, 341
  - dynamical symmetry algorithm, 341
  - fermion dynamical symmetry model, 543
  - graphene in a magnetic field, 349
  - Lie group universality, 414
  - topological and algebraic constraints, 362
- endomorphism, 15
- entanglement, 342, 530
- equivalence classes
  - and topological invariants, 426
  - conjugate classes, 22
  - definition and examples, 23
  - for homotopies, 429
  - for topological spaces, 426
- essential degeneracy, 521
- euclidean groups
  - as group of motion, 213
  - definition, 212
  - $E_2$ , 212, 214
  - $E_3$ , 212
  - invariant subgroup of translations, 215
  - semidirect product, 213–215
- Euler angles, 105, 106, 381, 383
- Euler characteristic, 492
- Euler–Lagrange equations, 283
- factor groups, 26–29
- faithful representations, 16, 45, 59, 92, 101, 126
- FDSM, *see* fermion dynamical symmetry model
- Fermi liquid
  - adiabatic continuity, 504, 505, 558
  - and quasiparticles, 558
  - Cooper instability, 559
  - definition, 557
  - departures from, 501, 558
  - precursor to conventional superconductors, 557
- Fermi surface
  - and Pauli principle, 558
  - definition, 81
  - role in Cooper mechanism, 559
  - scattering at, 558
- Fermi velocity, 345, 521, 523
- fermion dynamical symmetry model (FDSM)
  - and nuclear structure physics, 546, 547
  - and the Ginocchio model, 543
  - as shell model truncation, 543
  - corroboration by projected shell model, 552
  - dynamical Pauli effect, 551, 552
  - dynamical symmetries, 548, 549
  - general assumptions, 181, 546, 547
  - generators, 547
  - irreducible representations, 549–551
  - Lie algebra, 548
  - matrix elements, 550, 551
  - quantitative calculations, 551
  - SO(5) dynamical symmetry, 549
  - SO(6) dynamical symmetry, 548
  - SO(7) critical dynamical symmetry, 549
  - SU(2) dynamical symmetry, 548
  - SU(3) dynamical symmetry, 548
- Feynman diagrams
  - and Feynman rules, 282
  - definition, 257
  - non-abelian gauge bosons, 294
- Feynman slash notation, *see* quantum field theory
- fiber bundle
  - base space, 457
  - cotangent bundle, 456
  - fiber space, 457
  - for  $S^1$ , 457, 458
  - for a Möbius strip, 457, 458
  - in general relativity, 456
  - locally a product of two spaces, 457
  - tangent bundle, 456, 457
  - trivial and non-trivial, 457
- flavor oscillations, *see* neutrinos
- fractional quantum Hall effect, *see* quantum Hall effect
- fractional statistics (anyons), 500, 525
- fractionalization of quantum numbers
  - in Luttinger liquids, 501, 522
  - in Majorana quasiparticles, 524
- function, *see* maps
- fundamental group, *see* homotopies
- gauge fields
  - QCD, 332
  - running coupling, 332, 335–337
  - Standard Electroweak Model, 324
- gauge hierarchy problem, 588
- gauge invariance
  - Aharonov–Bohm effect, 466, 467
  - and conservation of charge, 505
  - and conserved particle number, 390, 391
  - and longitudinal polarization, 320, 321
  - and photon mass, 289, 290
  - and quantum field theory, 280

- covariant derivatives, 292
- dynamical content for local invariance, 290
- gauge bosons, 290
- gauge charges, 255
- gauging the symmetry, 290
- global, 290, 390, 391
- in lattice QCD, 459
- in quantum mechanics, 287–289
- local, 59, 60, 290
- minimal substitution, 287–289, 465, 471
- non-abelian (Yang–Mills fields), 291–295
- path-dependent representations, 458, 459
- spontaneous symmetry breaking, 295
- gauge transformations
  - Coulomb gauge, 242
  - covariant notation, 242
  - in Maxwell equations, 241, 242
  - Landau gauge, 481
  - Lorenz gauge, 241
  - radiation gauge, 242
  - symmetric gauge, 481
  - to unitary gauge, 330
- Gauss’ law, 520
- Gauss–Bonnet theorem
  - definition, 491, 492
  - generalization to Chern numbers, 491
- Gauss–Bonnet–Chern theorem, *see* Chern theorem
- Gaussian curvature
  - formula, 462
  - is intrinsic, 454, 462
  - special case of Riemann curvature, 454
- Gell-Mann, Okubo mass formula
  - masses in the baryon decuplet, 201, 202
  - masses in the nucleon octet, 201
  - physical motivation, 200
  - prediction of the  $\Omega^-$  particle, 202
- generalized coherent states, *see* coherent states
- generator coordinate method
  - and the variational principle, 379
  - definition, 378, 379
  - generating functions, 378, 379
  - generator coordinates, 378, 379
  - Hill–Wheeler equation, 379, 381
  - weight function, 379
- generators
  - of continuous groups, 22
  - of groups, 21
  - of permutation group  $S_3$ , 21
  - of  $SU(3)$ , 143
  - of translations, 22
- genus, *see* topology
- geometrical phase
  - Aharonov–Bohm effect, 464–468
  - Berry connection, 470, 471, 473
  - Berry curvature, 472, 474
  - Berry flux, 474
  - Berry phase, 468–471, 473
- gerade, 91
- Ginocchio model
  - $k$ – $i$  coupling scheme, 543
  - and symmetry dictated truncation, 545
  - basis for FDSM coupling scheme, 547
  - dynamical symmetry of, 543
- Ginzburg–Landau theory
  - and broken symmetry phases, 506
  - order parameters, 506
- gluons, *see* QCD
- Goldstone bosons, *see* spontaneous symmetry breaking
- Goldstone theorem
  - and breaking of continuous global symmetry, 307
  - and Goldstone modes, 308
  - and massless scalars, 309
  - and spontaneous symmetry breaking, 308
  - Higgs loophole, 311, 312
  - limitations of, 311, 312
- grand unified theories (GUTs)
  - and evolution of coupling constants, 584
  - and the Standard Model, 338
  - anomaly-free representations, 585, 586
  - extensions of  $SU(5)$ , 588
  - gauge hierarchy problem, 588
  - leptoquark bosons  $X$  and  $Y$ , 586–588
  - minimal criteria, 585
  - proton decay, 586
  - quantization of electrical charge, 586, 587
  - $SO(10)$ , 588
  - $SU(5)$ , 295, 586–588
  - supersymmetric, 588
  - symmetry breaking hierarchy, 587
  - violation of baryon number, 586
  - violation of lepton number, 586
  - Weinberg angle, 332
- graphene in magnetic field
  - coherent states, 355–359
  - Dirac points, 345
  - dispersion, 344
  - dynamical symmetry of, 343
  - energy surfaces, 357–359
  - Landau levels, 345, 347, 348
  - massless Dirac equation, 345, 347, 348
  - order parameters, 352, 353
  - pair states, 351, 352
  - quantum Hall effects, 346
  - quantum phase transitions, 359–361
  - real space lattice, 344
  - $SO(8)$  collective subspace, 353
  - $SO(8)$  dynamical symmetries, 353–355
  - sublattice pseudospin, 344
  - valley isospin, 345, 349, 351
- Grassmann variables, 443

- group integration, *see* invariant group integration
- groups
  - 4-group ( $D_2$ ), 36
  - abelian, 10
  - abstract nature of, 9
  - additive group of integers, 8
  - and tensors, 112–114
  - braid groups, 526
  - $C_{3v}$ , 88–92
  - $C_2$ , 9, 29
  - $C_3$ , 29
  - $C_6$ , 29
  - $C_2$ , 167
  - $C_3$ , 167
  - $C_6$ , 167
  - compact semisimple, 144
  - continuous, 10
  - cosets, 26–29
  - cyclic, 9, 25, 28–30, 80, 83, 167
  - definition, 7
  - dihedral, 36
  - direct product, 29, 30
  - euclidean, 212, 213
  - examples of matrix, 20
  - factor groups, 26–29
  - generators, 21, 22, 40, 42
  - inner automorphism, 22
  - invariant integration, 103, 104
  - invariant subgroups, 24, 25, 28, 29
  - irreducible representations, 68
  - irrep labels for point groups, 91
  - isometry, 267
  - Lagrange’s theorem, 27
  - Lie, 10, 40
  - linear, 20
  - Lorentz, 220
  - non-abelian, 10
  - non-compact, 209
  - number of parameters, 10
  - order of, 10
  - orthogonal, 21, 97
  - orthogonality and completeness, 104
  - permutations, 11, 12
  - point groups, 82–84, 87, 88
  - relation to semigroups, 365
  - representations, 15–17
  - restriction to subgroups, 13
  - rotational, 9, 97
  - Schoenflies notation for point groups, 83, 84
  - Schrödinger, 18, 19
  - semidirect product, 213–215
  - semisimple, 25, 44, 129
  - simple, 25, 43, 44, 585
  - simply reducible, 51, 109, 117, 125
  - $SO(3)$ , 105
  - $SO(3,1)$ , 212
  - space groups, 93
  - $SU(N)$ , 143
  - subgroups, 12, 13
  - symplectic, 21
  - translational, 9, 215
  - two-element group, 9
  - unitary, 20
- GUTs, *see* grand unified theories (GUTs)
- Haar measure, 103
- hadron, definition, 326
- Hamilton’s principle, 283
- Hartree–Fock approximation, 395, 396
- Hausdorff space, *see* topology
- Heine–Borel theorem, 425
- helical states in topological insulators, 510
- helicity
  - and chirality, 252, 253, 259
  - and conservation of parity, 249
  - and Dirac Hamiltonian, 252, 253
  - definition, 249
  - for massless particles, 215, 273
  - not mixed for massless fermions, 249
  - operator, 250
  - states for fermions, 250, 251
- Higgs mechanism
  - abelian Higgs model, 312–315
  - and long-range fields, 319
  - and longitudinal polarization, 320, 321
  - and vacuum screening currents, 315–318
  - circumventing the Goldstone theorem, 311, 312
  - effect on particle spectrum, 315
  - gauge invariance and mass, 315, 316
  - Higgs boson, 321, 322
  - physical understanding, 314, 315
  - Standard Electroweak Model, 329
- highest-weight algorithm, 51, 187
- Hill–Wheeler equation, *see* generator coordinate method
- Hofstadter butterfly, *see* quantum Hall effect
- holonomy
  - and anholonomy, 469
  - Berry phase, 469
  - classical, 469
  - definition, 469
  - for parallel transport, 469
  - geometrical, 469, 475, 476
  - quantum, 469
  - topological, 469, 475, 476
- holons, 501
- homeomorphism, *see* topology
- homomorphism
  - and factor groups, 28
  - and isomorphism, 13–15
  - and universal covering groups, 120
  - kernel of, 29, 120
  - maps, 13, 14

- homotopies
  - and topological solitons, 443, 444
  - definition, 429
  - equivalence classes of, 429, 430
  - first homotopy group, 430
  - fundamental group, 431
  - group structure, 430
  - higher homotopy groups, 433
  - homotopy classes as topological invariants, 429, 430
- hook rule, *see* Young diagrams
- Hubbard model, 395–397
- Hund's rules, 182
- IBM, *see* interacting boson model (IBM)
- incompressible states, 81, 485
- induced representations
  - for  $E_2$ , 215–218
  - for Poincaré group, 269
  - steps of method, 216–218
- induced transformation, 15
- inertial frames, 224
- instantons
  - and the QCD  $\theta$ -vacuum, 448
  - and the strong CP problem, 448
  - boundary conditions, 446, 447
  - finite-action euclidean solutions, 445, 447
  - physical interpretation, 447, 449
  - topological classification, 447
- insulator, 81, 82
- integer quantum Hall effect, *see* quantum Hall effect
- interacting boson model (IBM)
  - and nuclear structure physics, 554
  - and Pauli effect, 555
  - general assumptions, 181
- intrinsic coordinate system, 377, 378
- intrinsic states, 185, 377, 378, 383
- invariant group integration
  - Haar measure, 103
  - integration measure, 103, 104
  - Lorentz-invariant measure, 262
  - orthogonality and completeness, 104
  - rearrangement lemma, 9, 103
  - weight function, 103
- invariant subgroups
  - abelian, 218
  - and conjugate subgroups, 24
  - and method of induced representations, 218
  - for cyclic group  $C_4$ , 25
  - translation group, 215
  - trivial and non-trivial, 24
- Ising model, 404, 408–411
- isobaric spin, *see* isospin
- isometries
  - and Killing vectors, 267
  - generators of, 267
  - isometry group of Minkowski space, 267
  - isometry group of sphere, 267
  - isometry groups, 267
- isomorphism, 13–17
- isoscalar factors, 195–197, 199
- isospin
  - adjoint representation, 58
  - analogy with angular momentum, 55
  - and isospace, 56
  - charge independence hypothesis, 56
  - group structure, 54, 55
  - multiplets, 57
  - neutron–proton system, 53, 57
  - pions, 58
- isotopic spin, *see* isospin
- Jacobi identity, 44
- Killing vectors, *see* isometries
- Klein–Gordon equation, 276, 439
- Kramers' theorem, 514
- Kronecker product, *see* representations
- laboratory coordinate system, 377, 378
- Lagrange's theorem, 27
- Lagrangian density
  - and the classical action, 280
  - and the Euler–Lagrange equation, 283
  - and the Lagrangian, 280
  - for complex scalar or pseudoscalar field, 282
  - for Dirac field, 281, 282
  - for free fields, 281
  - for linear  $\sigma$ -model, 580
  - for massive vector field, 283
  - for massless vector field, 283
  - for scalar or pseudoscalar field, 282
  - mass terms, 282
- Landau levels
  - clean limit, 482, 487
  - cyclotron frequency, 481
  - cyclotron radius, 482
  - degeneracy, 482
  - density of states, 482
  - energy gaps, 482
  - filling factor, 482
  - for non-relativistic electrons, 480
  - gauge choice, 481
  - guiding center, 482
  - magnetic length, 481
  - non-relativistic Schrödinger equation, 481
  - physical origin, 482
  - quantization condition, 482
  - with impurity scattering, 482, 487
- Landau paradigm in condensed matter
  - and topology, 506
  - broken by fractional quantum Hall effect, 506
  - definition, 506
- lattice gauge theory, 459



- Laughlin gauge argument, *see* quantum Hall effect
- Legendre transformation, 440, 449
- lepton, definition, 326
- Lie algebras
  - adjoint representation, 44
  - alpha-string, 136
  - and Lie groups, 43
  - beta-string, 136
  - Cartan condition for semisimplicity, 129
  - Cartan subalgebra, 126
  - Cartan–Dynkin method, 126
  - Cartan–Weyl basis, 129
  - classification, 126–141, 608
  - compact, 130
  - compact algebras are semisimple, 130
  - definition, 42
  - Dynkin diagrams, 137–141, 608
  - invariant subalgebras, 43
  - Jacobi identity, 44
  - metric tensor (Cartan–Killing form), 129
  - rank, 47
  - rank of abelian, 47
  - rank of  $SU(N)$ , 47
  - root diagrams, 132
  - root space, 130, 131
  - root vectors, 131, 132
  - roots, 126, 129
  - semisimple, 44, 129
  - simple, 43, 44
  - structure constants, 42, 129
  - $SU(2)$  Cartan–Weyl basis, 129
  - using same symbols as for Lie groups, 43
  - weights, 126
- Lie bracket, 42, 462
- Lie groups
  - and Lie algebras, 43
  - compact, 42
  - definition, 40
  - fundamental representation, 45
  - generators, 40, 42
  - importance in physics, 58
  - Lie group universality, 414
  - unitary representations, 42
  - using same symbols as for Lie algebras, 43
- lightcone
  - and null world lines, 228
  - and spacelike world lines, 228
  - and timelike world lines, 228
  - classification of Minkowski vectors, 228
- linear  $\sigma$ -model
  - and PCAC, 580
  - explicit breaking of chiral symmetry, 582
  - Lagrangian density, 580–582
  - particle spectrum, 580–582
  - symmetry breaking in Goldstone mode, 580–582
  - symmetry breaking in Wigner mode, 580, 581
- little group, 217, 269, 272, 273
- Lorentz force, 287, 465, 478, 487, 509
- Lorentz group
  - and  $SL(2, C)$ , 230
  - and  $SO(3, 1)$ , 220
  - and space inversion (parity), 233–236
  - commutation algebra, 226
  - Dirac representation, 237
  - general properties, 229
  - higher-dimensional representations, 237
  - invariant integration measure, 262
  - leaves invariant  $t^2 - x^2 - y^2 - z^2$ , 212
  - Lorentz covariance of Maxwell equations, 240, 241
  - Lorentz transformations, 220, 224
  - Lorentz-covariant fields, 240
  - non-unitary representations, 238
  - parity and 4-spinors, 236
  - Poincaré and Lorentz representations, 274–277
  - self-conjugate representations, 235, 236
  - $SU(2) \times SU(2)$  representations, 232
  - two inequivalent spinor representations, 232
  - Weyl representations, 233
- Lorentz transformations
  - and spinors, 231
  - and Thomas precession, 230
  - and Thomas spin–orbit coupling, 230
  - boost generators, 225
  - boosts, 224, 225
  - classification, 227
  - covariant notation, 220
  - homogeneous, 227
  - improper, 228
  - inhomogeneous, 227
  - orthochronous, 227
  - proper, 227
  - rotation generators, 225
  - rotations, 225
  - tensors and transformation laws, 222
- Luttinger liquids
  - distinction from Fermi liquids, 501
  - fractionalization of quantum numbers, 501, 522
  - left movers, 501
  - right movers, 501
- M-theory, 588
- Möbius strip
  - and topological holonomy, 476
  - connectedness, 476
  - fiber bundle, 457, 458
  - global nature of topology, 506
  - orientability, 476
- magnetic charge
  - in emergent states, 474
  - in Maxwell equations, 474
- magnetic length, 481
- magnetic monopoles
  - as emergent states, 474
  - in Maxwell equations, 474



- Majorana equation
  - definition, 254
  - Majorana representation, 254
- Majorana mass, *see* neutrinos
- Majorana particles
  - and electron fractionalization, 524
  - and linear dispersion, 524
  - and topological protection, 524
  - for condensed matter quasiparticles, 523
  - in relativistic quantum field theory, 254, 255
  - in superconductors, 523, 524
  - in topological matter, 522, 524
- Majorana representation, *see* Majorana equation
- manifolds
  - are locally euclidean, 435
  - atlas, 435, 494
  - chart, 435, 494
  - compactness, 424, 425
  - connectedness, 426
  - definition, 435
  - differentiable, 434, 435
  - global versus local properties, 419
  - metric spaces, 434, 437
  - orientable, 475, 494
  - smoothness, 434
- maps
  - bijective, 14
  - codomain, 14
  - definition, 14
  - domain, 14
  - homomorphic, 13–15
  - image, 14
  - injective (“into”), 14
  - inverse image, 14
  - invertible, 14
  - isomorphic, 13–15
  - notation, 14
  - relationship to functions, 14
  - surjective (“onto”), 14
- matrix representations
  - ammonia molecule point group, 87, 88
  - and isomorphism, 16, 17
  - and linear vector spaces, 17
  - character, 24, 31–35, 108
  - character tables, 34, 35
  - determinant, 24
  - dimensionality, 16, 17
  - eigenvalues, 24
  - faithful, 16, 45, 59, 92, 101
  - homomorphism to abstract group, 15
  - invariants, 24
  - similarity transforms, 18, 19
  - special (unit determinant) matrices, 97
- Maxwell equations
  - gauge transformations, 241, 242
  - Lorentz covariance, 240, 241
  - manifestly covariant form, 242, 243
  - scalar potential, 241
  - vector potential, 241
- Maxwell wave equation, 276
- Meissner effect, *see* vacuum screening currents
- metal, 81
- metric
  - indefinite, 210, 211
  - negative definite, 210
  - positive definite, 210, 211
  - signature, 210
- metric spaces, *see* manifolds
- minimal substitution, *see* gauge invariance
- modular arithmetic, 38
- Mott insulators, 81, 559, 560
- Nambu–Goldstone mode, *see* spontaneous symmetry breaking
- natural units, 48, 64, 603, 604
- neutrinos
  - Dirac, 255
  - flavor oscillations, 255
  - flavors in Standard Model, 326
  - Majorana, 255
  - mass of, 255, 258, 330
  - seesaw mechanism, 258
  - Weyl, 255
- Noether’s theorem, 255, 284–286
- non-abelian statistics, 524
- non-compact groups
  - Lorentz group, 220, 229, 238
  - parameter space, 211
  - Poincaré group, 264
  - $SO(l, m)$ , 211
  - $SU(1, 1)$ , 210, 211
- normal subgroup, *see* invariant subgroups
- occupation number representation, *see* second quantization
- Ohm’s law, *see* resistivity
- orientable, *see* manifolds
- parallel transport
  - and curvature, 450, 490
  - in charge space, 455
  - on curved surfaces, 450, 490
- parity (P)
  - and Lorentz representations, 228, 234
  - and  $O(3)$  space inversion, 98
  - broken by octupole deformation, 393
  - commutation with Hamiltonian, 178, 394
  - for spinors, 257, 259
  - intrinsic, 235, 236
  - of the vacuum, 581
  - projection of, 392–395
  - spontaneously broken, 393–395
  - violation in weak interactions, 234, 259

- partition function
  - definition, 406
  - factorization of classical, 406
  - relation to quantum propagator, 406
  - Wick rotation and euclidean space, 406
- passive transformation, 223
- Pauli matrices, 46
- PCAC, *see* current algebra
- periodic lattices
  - and symmetry, 75
  - Bloch's theorem, 79, 80, 92, 486
  - Bragg zones, 77, 78
  - Brevais lattice, 75, 76
  - Brillouin zones, 77, 78
  - direct lattice, 75
  - lattice vector, 76
  - reciprocal lattice, 76, 77
  - Wigner–Seitz cell, 76
- permutation groups
  - and nuclear SD shells, 182
  - and Young diagrams, 65–67
  - associate representations, 182
  - basis vectors for irreps, 69
  - direct (inner) product, 70
  - multiplication law, 11
  - outer product, 70
  - $S_3$ , 11, 12
- phase transitions
  - classical, 400, 401
  - classical versus quantum, 407–411
  - classification, 401–403
  - critical dynamical symmetry, 355, 359–361, 549, 567, 568, 575
  - critical exponent, 402–405
  - Curie point, 402
  - dimensional crossover, 407
  - first-order, 401, 402
  - interplay of classical and quantum, 405
  - order parameter, 506
  - quantum, 400, 401
  - quantum critical point, 411, 412
  - scaling hypothesis, 403–405
  - second-order, 401, 402
  - superconducting, 505
  - universality, 403–405
- Poincaré group
  - Casimir operators, 266, 268
  - classification of states, 268, 269
  - commutators, 266
  - generators, 265, 266
  - group multiplication rule, 264, 265
  - little group, 269, 272, 273
  - Pauli–Lubanski pseudovector, 268
  - Poincaré and Lorentz representations, 274–277
  - representation theory, 266
  - representations for massive particles, 269–271, 274
  - representations for massless particles, 272–274
- point groups, *see* groups
- Pontryagin index, *see* solitons
- Proca (massive vector) field, 276
- projected shell model, 552
- projection
  - of angular momentum, 381–385
  - of chirality, 252
  - of electron momentum, 395–397
  - of electron spin, 395–397
  - of parity, 392–395
  - of particle number, 385–392
  - operators (properties of), 252
  - to restore symmetry, 377
- QCD
  - and color singlet states, 333
  - and quark masses, 334, 335
  - and SU(3) color symmetry, 162, 295, 332, 333
  - asymptotic freedom, 336, 337
  - chiral symmetry, 334, 335
  - color confinement, 333, 336, 337
  - color degree of freedom, 332
  - comparison with QED, 336, 337
  - constituent quark masses, 334, 335
  - coupling strength, 295
  - current quark masses, 334, 335
  - exotic hadrons, 336
  - gauge bosons, 295, 332, 333
  - gauge coupling strength, 335–337
  - glueballs, 336
  - gluons, 332, 333
  - Lagrangian density, 333, 334
  - Pauli problems without color symmetry, 332
  - running coupling, 335–337
  - scale parameter  $\Lambda$ , 335, 336
  - symmetries of Lagrangian density, 334
  - vacuum polarization, 337
- QED
  - abelian U(1) gauge symmetry, 291
  - comparison with QCD, 336, 337
  - extension to non-abelian symmetry, 291
  - gauge coupling strength, 336, 337
  - running coupling, 335–337
  - vacuum polarization, 337
- quantum 3D oscillator
  - and nuclear shell model, 178–180
  - eigenvalues, 174
  - group structure, 178
  - Hamiltonian, 174
  - many-body operators, 178
  - reduced matrix elements, 202, 204
  - unitary symmetry, 175, 176, 178
  - wavefunctions, 175
- quantum chromodynamics, *see* QCD
- quantum computers
  - basic principles, 528
  - decoherence problem, 530

- error correction, 530, 531
- exponentially more powerful than classical, 530
- fault tolerance, 530, 531
- qubits, 528–530
- topological protection, 531
- quantum critical point, *see* quantum phase transitions
- quantum cryptography, 528
- quantum electrodynamics, *see* QED
- quantum field theory
  - and gauge invariance, 280
  - anomalies, 301, 585
  - conserved currents and charges, 255, 284–286
  - covariant derivatives, 292
  - Euler–Lagrange equations, 283
  - Feynman slash notation, 283
  - gauge bosons, 290
  - global gauge invariance, 289, 290
  - Lagrangian densities, 281–283
  - local gauge invariance, 289, 290
  - Noether’s theorem, 255, 284–286
  - partially conserved currents, 287
  - quantization of classical fields, 280, 281
  - quantum electrodynamics, 289, 290
  - relativistic quantum fields, 280
  - spontaneous symmetry breaking, 295
  - symmetries for interacting fields, 286
  - the classical action, 280, 281
  - Yang–Mills fields, 291–295
- quantum Hall effect
  - and Berry phases, 491–493, 495
  - and topology, 489
  - anyons, 500
  - Chern numbers, 491–493, 495, 497
  - Chern–Simons theory, 500
  - classical Hall effect, 478, 479
  - edge states and conduction, 487, 489, 500
  - extended states, 485
  - for graphene, 346, 348
  - fractional, 478, 479
  - Hofstadter butterfly, 497
  - incompressible states, 484, 485
  - integer, 478, 479, 482–485
  - Laughlin gauge argument, 490
  - Laughlin liquid quasiparticles, 500
  - level crossings in the IQHE, 495
  - localized states, 485
  - maintenance of the resistance standard, 484
  - measurement of the fine-structure constant, 484
  - mobility gap, 485
  - phases of, 497
  - quantum Hall ferromagnetism in graphene, 348
  - TKNN invariant, 489, 512
- quantum information processing, 529
- quantum phase transitions
  - and dynamical symmetries, 412–414
  - and quantum critical behavior, 400, 401, 405, 406
  - and quantum critical points, 400, 401, 405, 406
  - contrasted with classical phase transitions, 400, 401
  - dynamical critical exponent, 406
  - for graphene in magnetic field, 359–361
  - Ising spins in a transverse field, 408–411
- quantum spin Hall effect, *see* topological matter
- quantum statistics
  - anyons, 525
  - Bose–Einstein, 525, 526
  - exotic in topological matter, 525
  - Fermi–Dirac, 525, 526
  - fractional, 525
- quantum teleportation, 528
- quarks
  - and isospin symmetry, 58, 162, 170
  - and SU(3) flavor symmetry, 161–164
  - and SU(6) flavor–spin symmetry, 171
  - color symmetry, 162, 332
  - flavors in Standard Model, 326
  - in mesons and baryons, 58, 165
  - mass of, 162, 170
  - quantum numbers, 162, 163
  - SU(3) weights, 165
- quasiparticles
  - and Fermi liquids, 557
  - as dressed particles, 387, 389
  - Bogoliubov quasiparticles, 387, 389, 390
  - Ising model with transverse field, 409
  - Majoranas as Bogoliubov quasiparticles, 524
- quasispin model, 181, 555, 556
- quaternions, 248, 249
- qubits, *see* quantum computers
- Racah coefficients, 536, 537
- Racah factorization lemma, 197
- Racah seniority, *see* seniority
- rearrangement lemma, *see* invariant group integration
- reduced matrix elements, 116–118
- renormalization, 291
- renormalization group, 365
- representations
  - adjoint, 45
  - completely reducible, 18
  - direct product, 31
  - direct sum, 19
  - fundamental, 45
  - in function spaces, 15
  - irreducible, 18, 33, 34
  - irreps of SO(3), 111, 112
  - preserve group multiplication, 15
  - real and complex, 149, 585
  - reducible, 18, 33, 34
- resistivity
  - and Ohm’s law, 478, 480
  - and resistance in Hall experiment, 480
- rotational invariance
  - and conservation of angular momentum, 6
  - restoration by projection, 381–385

- rotations
  - $D$ -matrices, 106–108, 605
  - 3D, 97
  - active, 105
  - and  $SO(3)$ , 105–107, 605
  - and tensors, 114, 116
  - Euler angles, 105, 106
  - matrix elements of rotation operator, 105–108, 605
  - passive, 105
- running coupling constants, 332, 335–337
- Schoenflies notation, *see* groups
- Schur’s lemma, 47, 56
- second quantization
  - basis transformations, 598, 599
  - creation and annihilation operators, 597, 598
  - Dirac notation, 593–596
  - motivation, 591
  - occupation number representation, 595–602
  - one-body operators, 600, 601
  - particle number operator, 599, 600
  - two-body operators, 601
- semiconductor, 81
- semidirect product groups, 213–215, 264, 265
- semigroup, 365
- semimetal, 81, 344
- semisimple groups, *see* groups
- seniority, 556
- sets
  - cardinality, 7
  - closed, 7, 422
  - definition of, 7
  - equivalence of, 7
  - intersection of, 7
  - membership in, 7
  - null set, 7
  - open, 7, 422
  - set-builder notation, 7
  - subsets, 7
  - union of, 7
  - Venn diagrams, 7
- shell model (nuclear)
  - and collective motion in the SD shell, 187
  - and dynamical symmetry approximations, 545
  - and Elliott model, 179–185
  - and  $SU(3)$  symmetry, 178, 179
  - classification of SD shell states, 179–185
  - magic numbers (shell gaps), 180, 544
  - normal- and abnormal-parity orbits, 545
  - orbital symmetry, 181
  - spin–isospin symmetry, 181
- similarity transform, 18
- simple groups, *see* groups
- simply reducible, *see* groups
- $SO(2)$ 
  - and Fourier series, 104
  - expansion of functions in irreps, 104
  - generators, 98, 99
  - group manifold, 99–101
  - irreducible representations, 104
  - subgroup of angular momentum  $SO(3)$ , 98
  - topology of manifold, 100, 101
- $SO(3)$ 
  - $D$ -matrices, 106–108, 605
  - and Euler angles, 105, 106
  - and  $O(3)$ , 98
  - axis–angle parameterization, 119
  - basis and rotation matrix elements, 105, 106
  - characters, 108
  - direct product of representations, 109
  - generators, 105
  - highest-weight algorithm, 186
  - irreducible multiplets, 111, 112, 605
  - relationship with  $SU(2)$ , 119, 120
  - subgroups of Elliott  $SU(3)$ , 186
  - tensor operators, 197
  - topology of manifold, 120
  - vector-coupling coefficients, 109–111
- solitons
  - and dispersion, 439
  - instantons, 445–447, 449
  - kink solution, 442
  - magnetic monopoles, 444
  - Pontryagin index, 444
  - superconducting vortex solutions, 444
  - topological, 439
  - topological charge, 441
- space groups
  - generic form, 93
  - glide plane, 93
  - screw axis, 93
- spherical harmonics
  - addition theorem as Clebsch–Gordan series, 50
  - and deformed shapes, 393
  - parity of, 394
  - proof of addition theorem, 123
  - reduced matrix element, 118, 123
  - relation to  $D$ -functions, 108
  - table of, 605
- spin
  - for massive particles, 269–271, 274
  - for massless particles, 272–274
- spin filtering (locking) in topological insulators, 510
- spin–orbit coupling
  - and Lorentz invariance, 230, 512
  - in Dirac equation, 230
  - in topological matter, 512
  - in Weyl semimetals, 520
- spin–statistics theorem, 526
- spinons, 501

- spinors
  - Dirac, 236, 244, 245, 257
  - Majorana, 257
  - SL(2,C) Lorentz, 231, 232
  - SO(3) Pauli, 45, 120, 232, 257
  - Weyl, 248, 257
- spintronics, 510
- spontaneous symmetry breaking
  - and dynamical symmetry, 343
  - and the vacuum, 302
  - continuous symmetry, 305–309
  - definition, 301
  - discrete symmetry, 303–305
  - Goldstone modes, 307–309
  - in the Standard Electroweak Model, 329
  - is actually hidden symmetry, 505
  - little group, 308
  - multiple fields, 308
  - of  $Z_2$ , 409
  - restoring symmetry by projection, 381–392
  - stability subgroup, 308, 309
  - superconducting transition, 505
- Standard Electroweak Model
  - gauge group, 326, 327
  - Higgs field, 329
  - Lagrangian density, 327
  - particle spectrum, 330
  - running coupling constants, 332
  - spontaneous symmetry breaking, 329
  - unitary gauge, 330
  - weak hypercharge, 327
  - weak isospin, 326
  - Weinberg angle, 295, 331, 332
  - Yukawa fermion coupling, 329
- Standard Model
  - and electroweak interactions, 324–332
  - and grand unification, 338, 584
  - and strong interactions, 332–334, 336, 338
  - gauge symmetry group, 338
  - generations, 326
  - particles of, 326
- statistics, *see* quantum statistics
- Stokes' theorem
  - and the Berry phase, 466, 494, 508
  - definition, 467
- structure constants, 42, 44, 144
- SU(2)
  - and angular momentum, 19
  - center  $Z_2$ , 120
  - conjugate representation, 149
  - relationship with SO(3), 119, 120
  - topology of manifold, 120
  - universal covering group, 120
- SU(3)
  - and 3D quantum oscillator, 174–176
  - and fundamental quark representations, 162
  - and harmonic oscillator potential, 178, 179
  - and nuclear shell model, 178, 179
- Casimir operators, 145
- classification of SD shell, 179–185
- Clebsch–Gordan coefficients, 191, 192, 194
- Clebsch–Gordan series, 156, 192
- color symmetry and QCD, 162, 332
- conjugate representation, 149
- dimensionality of irreps, 146
- flavor multiplets, 163–165
- flavor symmetry and quarks, 161–165
- generators, 143
- graphical construction of direct products, 158
- irreducible representations, 146
- irreps of direct products, 152
- isoscalar factors, 195–197
- isospin subgroups, 166, 167
- matrix elements, 191
- matrix elements for  $a^\dagger$  and  $a$ , 203
- matrix elements of generators, 195
- oscillator angular momentum subgroups, 176, 177
- product of fundamental and conjugate reps, 194
- product of fundamental reps, 192
- quark structure for mesons and baryons, 165
- raising and lowering operators, 145
- real and complex representations, 149
- reduced matrix elements, 199, 200
- spherical operators, 202
- structure constants, 144
- structure of matrix elements, 199
- SU(2) subgroups in Elliott model, 185
- weight diagrams, 148
- weight diagrams and subgroups, 167
- weight space, 50, 145
- weights from Young diagrams, 157
- Wigner–Eckart theorem, 198
- Young diagrams, 150, 151
- Young diagrams and subgroups, 168
- SU(4) model of superconductivity
  - algebra, 561–563
  - and dynamics, 560, 561
  - Casimir, 565, 566
  - coherent state energy surfaces, 571–573
  - collective subspace, 564, 565
  - dynamical symmetries, 566–569
  - gap and phase diagrams, 570, 571
  - generalized AF instability, 574
  - generalized Cooper instability, 573, 574
  - Hamiltonian, 565, 566
  - high critical temperatures, 574–576
  - no double occupancy, 569, 570
  - universality, 576
- SU(6) flavor–spin symmetry
  - classification of baryons and mesons, 171
  - Clebsch–Gordan series, 171

- SU(6) flavor–spin symmetry (Cont.)
  - motivation, 171
  - shortcomings of SU(6) model, 172
- subgroups, 12, 13
- summation convention (repeated index), 40, 220
- superconductivity
  - and dynamical symmetry, 557
  - and screening currents, 319
  - and superfluidity, 385
  - BCS theory, 385–390, 557
  - conventional, 557
  - Cooper instability, 557, 559
  - Cooper pairs, 318
  - Meissner effect, 318, 319
  - SU(4) model, 412–414, 560–576
  - unconventional, 557
- superfluidity, *see* superconductivity
- superstrings, 588
- supersymmetry
  - and dark matter, 588
  - and the gauge hierarchy problem, 588
  - broken, 589
  - evidence for, 588, 589
  - fermion–boson partners, 589
  - in GUTs, 588
  - in string theories, 588
  - introduction, 589
- symmetry
  - and conservation laws, 5
  - and invariance, 5
  - and quantum operators, 5
  - breaking by anomalies, 301
  - breaking explicitly (Wigner mode), 301–303
  - breaking in the Higgs mode, 301, 302
  - breaking in the Wigner mode, 301
  - covering operations, 5, 82
  - dynamical symmetries, 59, 60, 277, 301, 343
  - essence of, 5
  - non-abelian gauge, 162
  - of the classical action, 281
  - restoration by projection, 377
  - spontaneously broken, 185, 301–309, 343
  - SU(3) flavor, 161–165
- tangent bundle, *see* fiber bundle
- tensors
  - as geometrical objects, 223
  - for the rotation group, 114–116
  - Lorentz, 222, 223
  - matrix element of scalar product, 540, 541
  - matrix elements of tensor products, 537, 539
  - raising and lowering indices, 222
  - reduced matrix elements, 116–118
  - SO(3) tensor products, 114, 116
  - spacetime, 223
  - spherical, 114, 116
  - tensor fields, 223
  - under group transformations, 112–114
  - Wigner–Eckart theorem, 116–118
- Thomas precession, 230
- time-reversal invariant momenta (TRIM), 514
- time-reversal symmetry (T)
  - and Chern numbers, 509
  - and Kramers’ theorem, 514
  - and magnetic fields, 509
  - antiunitary, 514
  - conserved by  $V - A$  interaction, 260
  - for Lorentz group, 233
  - implemented by antilinear operators, 233
  - in weak interactions, 260
  - Kramers doublets, 514
- TKNN invariant, *see* quantum Hall effect
- topological defects
  - disclinations, 489
  - dislocations, 489
  - vortices, 489
- topological matter, 504
  - and discrete symmetries, 508, 509
  - and quantum computers, 528
  - and spintronics, 510
  - and time-reversal symmetry, 508, 509
  - bulk–boundary correspondence, 512
  - helical states, 510
  - Majorana particles, 522, 524
  - quantum Hall effect, 489
  - quantum spin Hall effect, 509–512
  - spin filtering (locking), 510
  - topological insulators, 509–512
  - topological protection, 491, 504
  - topological superconductors, 524
  - Weyl points (Weyl nodes), 518, 520
  - Weyl semimetals, 518, 520
  - $Z_2$  classification, 512
- topological protection
  - and Majorana particles, 524
  - and topological matter, 504
  - for quantum computers, 531
- topology
  - Aharonov–Bohm effect, 468
  - and the Brillouin zone, 495, 496, 507, 508
  - and the condensed matter paradigm, 504
  - basic concepts, 419
  - Chern theorem, 493
  - closed balls, 422
  - coffee cups and doughnuts, 433
  - compactness, 424, 425
  - compactness as topological invariant, 427
  - connectedness, 101, 426
  - connectedness as topological invariant, 427
  - continuity, 421, 423, 434
  - continuous map, 423
  - definition for set, 423
  - difference for SU(2) and SO(3), 101, 119, 120
  - dimensionality as topological invariant, 427, 428

- discrete nature, 419
- equivalence classes, 426, 427
- Euler characteristic, 492
- first homotopy (fundamental) group, 430–433
- Gauss–Bonnet theorem, 492
- genus, 420, 492
- global versus local properties, 419
- Hausdorff space, 423, 424
- higher homotopy groups, 433, 434
- homeomorphism, 426, 427, 517
- homotopies, 429–434
- metric spaces, 437
- neighborhoods, 421
- of Möbius strip, 457, 475, 506
- of  $SO(2)$  manifold, 100, 101
- open balls, 421, 422
- open sets, 422, 423
- qualitative nature, 419
- relationship to geometry, 419
- simply connected spaces, 431
- simply-connected spaces, 431–433
- the discrete topology, 423
- the indiscrete topology, 423
- topological invariants, 427, 428
- topological matter, 504
- topological protection, 492, 504
- topological spaces, 421, 423
- winding number, 420, 432–434, 495
- translation group
  - and space groups, 93
  - as abelian invariant subgroup, 215
- translational invariance, 6, 9, 82
- TRIM, *see* time-reversal invariant momenta (TRIM)
- ungrade, 91
- unitary symmetry
  - conjugate representations, 154
  - dimensionality of representations, 155
  - direct product of  $SU(N)$  representations, 156, 157
  - fundamental representations, 154
  - relationship of  $SU(N)$  and  $U(N)$ , 152
  - review of compact group  $SU(N)$ , 209
  - Young diagrams, 150
- universal covering group
  - for  $SO(3)$ , 101
  - for  $SO(3,1)$ , 231
  - for  $SU(2)$ , 120
  - unique and singly connected, 101
- universality, 403–405
- vacuum polarization
  - Feynman diagrams, 336, 337
  - for abelian gauge theories, 336, 337
  - for non-abelian gauge theories, 336, 337
- vacuum screening currents
  - and effective mass, 316
  - and massive photons, 318
  - and the Higgs mechanism, 315
- diamagnetic atomic screening, 317, 318
- Meissner effect, 318
- valence band, 81, 82
- variational principle, 379, 380
- vector representation, *see* adjoint representation
- vector space, 41
- vector-coupling coefficients
  - $3J$  symbols, 111
  - Clebsch–Gordan coefficients for  $SO(3)$ , 110, 111
  - Condon–Shortley phase convention, 110
  - for  $SO(3)$ , 109–112
  - for  $SU(3)$ , 191, 192, 194
  - symmetries, 111
  - triangle inequality, 110
  - unitarity of transformation, 110
- weak hypercharge, 327
- weak interactions
  - $V - A$  current, 259, 325, 327
  - double  $\beta$ -decay, 256
  - Fermi current–current theory, 325
  - intermediate vector bosons, 325
  - neutral current, 327
  - neutrinoless double  $\beta$ -decay, 256
  - parity violation, 234, 259
  - violation of unitarity in Fermi theory, 325
- weak isospin, *see* Standard Electroweak Model
- Weinberg angle, *see* Standard Electroweak Model
- Weyl equations
  - definition, 248
  - describe massless fermions, 248, 521
  - for neutrinos, 255
  - in Weyl semimetals, 518
  - linear dispersion, 518
  - Weyl spinors, 248
- Weyl group, 133
- Weyl hyperplanes, 133
- Weyl points or Weyl nodes, *see* topological matter
- Weyl reflections, 133
- Weyl semimetals, *see* topological matter
- Wigner coefficients, *see* vector-coupling coefficients
- Wigner supermultiplet theory, 181
- Wigner–Eckart theorem
  - and selection rules, 118
  - for  $SO(3)$ , 117, 118
  - for  $SU(3)$ , 198
  - reduced matrix elements, 116
- Wigner–Seitz cell, 76
- winding number, *see* topology
- Yang–Mills fields
  - and anomalies, 295
  - and mass terms, 295
  - as generalization of QED, 293
  - instantons, 445–447, 449



Yang–Mills fields (Cont.)	dimensionality of $SU(N)$ irreps, 155
non-abelian gauge fields, 291–293	for many-particle states, 66
non-linearities of, 294	for two particles, 66
properties of, 293	hook rule, 69, 155
QCD, 295	partitions, 66
standard electroweak theory, 295	projection operators, 69
topology of euclidean vacuum, 445–447, 449	standard arrangement, 67
Young diagrams	$SU(N)$ , 151
and the permutation group, 65–67	two particles in three states, 154
and unitary symmetry, 150	two particles in two states, 153
and weights for $SU(3)$ irreps, 157	Young operator, 70
and Young tableau, 67	Yukawa coupling, 329