Symmetry, Broken Symmetry, and Topology in Modern Physics

A First Course

Written for use in teaching and for self-study, this book provides a comprehensive and pedagogical introduction to groups, algebras, geometry, and topology. It assimilates modern applications of these concepts, assuming only an advanced undergraduate preparation in physics. It provides a balanced view of group theory, Lie algebras, and topological concepts, while emphasizing a broad range of modern applications such as Lorentz and Poincaré invariance, coherent states, quantum phase transitions, the quantum Hall effect, topological matter, and Chern numbers, among many others. An example-based approach is adopted from the outset, and the book includes worked examples and informational boxes to illustrate and expand on key concepts. 344 homework problems are included, with full solutions available to instructors, and a subset of 172 of these problems have full solutions available to students.

Mike Guidry is Professor in Physics and Astronomy at the University of Tennessee. He is the author of more than 125 journal articles and six published textbooks. He has been the Lead Educational Technology Developer for several major college textbooks in introductory physics, astronomy, biology, genetics, and microbiology. During his career, he has won multiple teaching awards and has taken the lead in a variety of science outreach initiatives.

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Yang Sun dedicates this book to his wife Ping Zheng and his son Yan Sun.
Mike Guidry dedicates this book to his late father, Clifford Guidry, for instilling the virtues of critical thinking and innate curiosity.

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Preface

Undergraduate physics majors today take classes and participate in research experiences that expose them increasingly to more advanced mathematical topics that they will encounter in many research fields if they continue on to graduate school. For example, at the end of the twentieth century a significant introduction to general relativity was uncommon at the undergraduate level. Now the rudimentary mathematics underlying general relativity (differential geometry and associated tensor calculus) and the basics of physical applications in gravitational physics are taught routinely at the advanced undergraduate level in many universities.

However, there is one set of topics and associated mathematics of increasing importance for various research disciplines that undergraduates (and even many graduate students, truth be told) are seldom exposed to in any systematic fashion. We will organize these topics loosely under the rubric of applying group theory, Lie algebras, geometry, and topological concepts to physical systems. In fact, we would argue that in the landscape of undergraduate mathematical preparation for research at the next level these topics are the most important omission in the mathematical and conceptual preparation for many of our students.

Why is this so? Is it an essential problem generated by the topics being inherently too difficult to teach effectively before graduate school, or is it more a problem associated with the lack of pedagogical materials to enable them to be taught easily to advanced undergraduate physics majors? Although some mathematical concepts in these areas are not trivial, we would argue that – assuming students to have had a solid one-year course in quantum mechanics (and ideally a similar course in electromagnetism), as routinely taught at the advanced undergraduate level – they are no more difficult than the mathematics of general relativity, which also involves concepts and mathematical techniques to which many undergraduates have had little prior exposure. Therefore, we would argue that the major impediment to being able to teach these concepts systematically to advanced undergraduates is not that it is too difficult for the students, but rather it is the scarcity of pedagogical material for instructors that would make teaching such a course feasible within the usual undergraduate curriculum.

To be sure, there are many good book and journal article resources available on the general topics of groups, algebras, topology, and geometry as applied in modern physics (many in the bibliography of this book). However, few if any address these topics in a coherent and cross-disciplinary fashion that is deliberately pitched at an audience having advanced undergraduate preparation and broad scientific interests. This book is a step toward filling that gap. It is adapted from lectures taught initially over a number of years to graduate students, and from many of our own research articles on applying Lie algebras and Lie groups to a variety of many-body systems. However, the material has been systematically reorganized, rewritten, and supplemented by additional pedagogical material to make it intelligible to advanced undergraduate students.

It may be noted that there is historical precedent for this idea, already alluded to above. In our opinion, the reason that we now teach general relativity systematically to undergraduates was primarily the advent in the early 2000s of textbook and resource material that demonstrated explicitly a blueprint for teaching undergraduate general relativity (aided greatly by inherent student interest generated by remarkable discoveries in modern gravitational physics). Thus, one may view the present book as an attempt to do something similar to enable systematic teaching of groups, algebras, topology, geometry, and their modern physics applications to new generations of undergraduate students.

Student Preparation: The reader of this book is expected to have physics experience commensurate with that of a third or fourth year undergraduate physics major in a U.S. university, and to be familiar with the material typically covered in an advanced undergraduate quantum mechanics course, including quantum mechanics expressed in second-quantized Dirac notation (bras, kets, and creation and annihilation operators), and to be aware of basic concepts from special relativity as typically taught in introductory modern physics. While some familiarity with the Dirac equation and with relativistic quantum field theory is useful, we do not assume such preparation, and introduce these concepts as part of the presentation where needed. It is desirable that the student have an advanced undergraduate understanding of electromagnetism (in particular Maxwell's equations and gauge transformations), but this is not essential for the diligent student as we introduce the required concepts as an integral part of the presentation. We assume the reader to be familiar with basic algebra, geometry, calculus, differential equations, and linear algebra, but to have minimal prior knowledge of group theory, Lie algebras, differential geometry, and topology.

Examples and Boxes: To aid in comprehension, many worked examples and supplementary information boxes are scattered throughout each chapter. These serve two general functions: to illustrate how to do some essential tasks, like showing how to write a cyclic group as a direct product of smaller cyclic groups, or to set in context and provide broader perspective, as in a discussion of the general concept of equivalence classes as background for understanding the partition of group elements into conjugacy classes.

Problems and Solutions: A total of 344 problems of varying complexity and difficulty may be found at the ends of the chapters, each chosen to familiarize the reader with basic concepts, illustrate important points, fill in details, or prove assertions made in the text. The solutions for all 344 problems are available from the publisher as a PDF file in typeset book format for instructors, and a subset of 172 problem solutions is available to students in the same format. Those problems with solutions available to students are marked by the symbol *** at the end of the problem.

Conventions: A broad variety of conventions may be found in books and journal literature related to the subject matter of this book. In choosing our conventions we have been guided by two main principles: (1) this is not a mathematics textbook but rather one about physics applications, and (2) a central goal is to equip advanced undergraduate students to use the available physics literature in fields covered by this book.

While parts of the book employ standard MKS or CGS units, professionals in many fields touched on by these topics routinely use natural units that are defined such that fundamental constants like the speed of light or Planck's constant take unit value. One of the purposes of the present material is to address the significance of these topics for cutting-edge research in a variety of fields, and to encourage students to use and explore the corresponding literature. Thus we have not shied away from using and explaining natural units where appropriate.

Mathematically one should distinguish groups from associated Lie algebras symbolically. But in the kinds of practical physics applications emphasized here it is usually clear from the context whether one means a group or an algebra, and it can become rather pedantic to distinguish formally. Therefore, we have generally used the same symbols for a Lie algebra and associated Lie groups, stating in words whether the symbol stands for a group or algebra if there is any chance of confusion.

Likewise, it is common in introductory quantum mechanics to distinguish operators from non-operator quantities by special notation, such as a special font or placing a caret over operators. Since the reader is assumed to understand enough quantum mechanics to know the difference between operator and non-operator quantities, in the interest of clean and compact notation we will usually not use special fonts or symbols to denote operators, letting the context dictate which quantities are operators. We deviate from this practice only if confusion might otherwise result.

Because it is common in many fields, we often use the Einstein convention for implied summation on repeated indices. Mathematically, one should distinguish upper and lower indices and employ them consistently, requiring summation on precisely one upper and one lower repeated index. However, the practical distinction between upper and lower tensor indices is important primarily for non-euclidean metrics and, since many physics problems are formulated explicitly or implicitly in euclidean manifolds, one finds a broad variety of adherence or non-adherence to this rule in the physics literature. Therefore we have adopted a loose overall summation convention that any repeated index implies a summation on that index, irrespective of vertical position unless stated otherwise, but have adhered to more rigorous mathematical conventions for non-euclidean metrics like for Minkowski space, where we distinguish explicitly vectors from dual vectors, and accordingly require clear distinction between upper and lower tensor indices in notation and summation convention. While less than desirable formally, we believe that this hybrid approach is of practical utility in preparing students to engage with real-world physics literature.

Resources for Teaching: For those wishing to teach from this book, two free resources are available from Cambridge University Press.

1. *Instructor Solutions Manual,* which is a PDF file typeset in the format of the book that presents the full solutions for all 344 problems at the ends of chapters. This manual is available only to instructors.
2. *Student Solutions Manual,* which is a PDF file typeset in the format of the book that contains the full solutions for a subset of 172 of the 344 problems at the ends of chapters.

This manual is available to students, instructors, and general readers. The problems contained in this solutions manual for students are marked by *** at the end of the problem in the text.

These resources may be found at the Cambridge University Press website for this book.

Sample Courses: For those teaching from this book, unless one has the luxury of a two-semester timeframe the material contained here is too much for a single course. This book should be viewed as an integrated resource from which one can tailor courses of varying length and emphasis. We give some examples to suggest possible directions to go, assuming a single-semester course.

1. *Overview Course:* A broad introduction to groups, algebras, topology, and geometry can be constructed from Chs. 2–6, 8–10, 12–18, 24, and 27–29 (add Ch. 7 if you wish to cover formal classification of Lie groups). Worked problems in the *Instructor Solutions Manual* 244. Worked problems in the *Student Solutions Manual* 121. Supplemental special topics: Chs. 30–34.
2. *Symmetry and Broken Symmetry Course:* Chs. 2–23. Worked problems in the *Instructor Solutions Manual* 273. Worked problems in the *Student Solutions Manual* 135. Supplemental special topics: Chs. 30–34.
3. *Topology in Physics Course* (including basic group theory, Bloch theorem and Brillouin zone, Dirac, Weyl, and Majorana equations, and Lorentz/Poincaré and gauge symmetry as background): Chs. 2–5, 13–16, 24–29. Worked problems in the *Instructor Solutions Manual* 187. Worked problems in the *Student Solutions Manual* 95.

Note that the 344 problems with complete solutions in the *Instructor Solutions Manual* are a resource that can be used to broaden and deepen the coverage of particular topics in these chapters.

Summary: This book provides a unified and pedagogical discussion of groups, algebras, geometry, and topology in modern physics that is tailored specifically for advanced undergraduate students, with integrated and comprehensive support material enabling the aspiring instructor to teach these topics at that level. It is our hope that this material will facilitate a systematic introduction to these concepts at the advanced undergraduate level, enabling our students to enter graduate school armed with the tools and understanding to begin participating immediately in many of the most interesting and challenging research topics in modern physics.

Acknowledgments: We would like to extend our thanks to the many students and colleagues whose questions and comments sharpened this presentation, to our long-time research collaborators Cheng-Li Wu, Da Hsuan Feng, Lian-Ao Wu, and Weimin Zhang in the development of dynamical symmetries for many-body systems that have been incorporated in this book, and to Nicolas Gibbons, Sarah Armstrong, Nicola Chapman, and Christian Green at Cambridge University Press, and Franklin Mathews Jabaraj at Straive for all their help in bringing this book to publication. This book was completed during the Covid-19 pandemic of 2020–2021. We are particularly grateful to our families, professional colleagues, support staff, and myriad food, medical, and service workers whose sacrifices and willingness to brave the virus kept us well, fed, as sane as could be expected, and reasonably functional during trying times.