

Contents

|   |                                            |                |
|---|--------------------------------------------|----------------|
|   | <i>Preface</i>                             | <i>page ix</i> |
| 1 | <b>Introduction</b>                        | 1              |
|   | 1.1 From Data Processing to Communication  | 1              |
|   | 1.2 From Data Types to Session Types       | 2              |
|   | 1.3 Assumptions about Communication        | 4              |
|   | 1.4 Session Types in Programming Languages | 5              |
|   | 1.5 The Safety Guarantees of Session Types | 5              |
|   | 1.6 Binary and Multiparty Session Types    | 6              |
| 2 | <b>Basic Concepts</b>                      | 7              |
|   | 2.1 Session Types                          | 7              |
|   | 2.2 The Pi Calculus with Sessions          | 9              |
|   | 2.3 Statics                                | 12             |
|   | 2.4 Dynamics                               | 17             |
|   | 2.5 Choice                                 | 22             |
|   | 2.6 Runtime Errors                         | 26             |
|   | 2.7 Type Safety                            | 30             |
|   | 2.8 Sending a Channel on Itself            | 39             |
|   | 2.9 Linear Types and Affine Types          | 42             |
|   | 2.10 Notes                                 | 43             |
| 3 | <b>Infinite Behaviour</b>                  | 45             |
|   | 3.1 Infinite Session Types                 | 46             |
|   | 3.2 Recursive Types and Coalgebras         | 50             |
|   | 3.3 Equivalence and Duality                | 56             |
|   | 3.4 Processes                              | 64             |
|   | 3.5 Statics                                | 66             |
|   | 3.6 Dynamics                               | 68             |
|   | 3.7 Type Safety                            | 69             |
|   | 3.8 Notes                                  | 70             |
| 4 | <b>Sharing</b>                             | 72             |
|   | 4.1 Shared Channels                        | 72             |

|          |                                                       |     |
|----------|-------------------------------------------------------|-----|
| vi       | <b>Contents</b>                                       |     |
|          | 4.2 Unrestricted Types                                | 78  |
|          | 4.3 Statics                                           | 80  |
|          | 4.4 Replicated Input                                  | 85  |
|          | 4.5 Type Safety                                       | 89  |
|          | 4.6 The Unrestricted end Type                         | 96  |
|          | 4.7 Sending a Channel on Itself                       | 97  |
|          | 4.8 Notes                                             | 98  |
| <b>5</b> | <b>Subtyping</b>                                      | 99  |
|          | 5.1 Subtyping Session Types                           | 99  |
|          | 5.2 Properties of Subtyping                           | 105 |
|          | 5.3 Statics                                           | 107 |
|          | 5.4 Type Safety                                       | 110 |
|          | 5.5 Subtyping, Linearity and Sharing                  | 113 |
|          | 5.6 Notes                                             | 116 |
| <b>6</b> | <b>Algorithmic Typing</b>                             | 117 |
|          | 6.1 From Declarative to Algorithmic Type Checking     | 117 |
|          | 6.2 Algorithmic Relations and Predicates on Types     | 118 |
|          | 6.3 A Typechecking Algorithm                          | 120 |
|          | 6.4 Correctness of the Algorithmic Relations on Types | 128 |
|          | 6.5 Correctness of Algorithmic Typechecking           | 131 |
|          | 6.6 Notes                                             | 140 |
| <b>7</b> | <b>Functional Programming</b>                         | 141 |
|          | 7.1 Example: Online Shopping                          | 141 |
|          | 7.2 Types                                             | 147 |
|          | 7.3 Expressions                                       | 149 |
|          | 7.4 Dynamics                                          | 152 |
|          | 7.5 Statics                                           | 159 |
|          | 7.6 Type Safety                                       | 164 |
|          | 7.7 Notes                                             | 174 |
| <b>8</b> | <b>Linear Pi Calculus with Values</b>                 | 176 |
|          | 8.1 Processes                                         | 176 |
|          | 8.2 Dynamics                                          | 179 |
|          | 8.3 Types                                             | 182 |
|          | 8.4 Statics                                           | 183 |
|          | 8.5 Type Safety                                       | 185 |
|          | 8.6 Encoding Session Types                            | 188 |
|          | 8.7 Notes                                             | 196 |
| <b>9</b> | <b>Propositions as Sessions</b>                       | 197 |
|          | 9.1 Processes                                         | 197 |

|     | Contents                                     | vii |
|-----|----------------------------------------------|-----|
| 9.2 | Statics                                      | 200 |
| 9.3 | Dynamics                                     | 205 |
| 9.4 | Exponentials and Sharing                     | 207 |
| 9.5 | Clients in Parallel                          | 214 |
| 9.6 | Results                                      | 215 |
| 9.7 | The Relationship with Classical Linear Logic | 223 |
| 9.8 | Notes                                        | 225 |
|     | <i>References</i>                            | 227 |
|     | <i>Index</i>                                 | 232 |