

## NONLINEAR VALUATION AND NON-GAUSSIAN RISKS IN FINANCE

What happens to risk as the economic horizon goes to zero and risk is seen as an exposure to a change in state that may occur instantaneously at any time? All activities that have been undertaken statically at a fixed finite horizon can now be reconsidered dynamically at a zero time horizon, with arrival rates at the core of the modeling.

This book, aimed at practitioners and researchers in financial risk, delivers the theoretical framework and various applications of the newly established dynamic conic finance theory. The result is a nonlinear non-Gaussian valuation framework for risk management in finance. Risk-free assets disappear and low-risk portfolios must pay for their risk reduction with negative expected returns. Hedges may be constructed to enhance value by exploiting risk interactions. Dynamic trading mechanisms are synthesized by machine learning algorithms. Optimal exposures are designed for option positioning simultaneously across all strikes and maturities.

DILIP B. MADAN is Professor Emeritus at the Robert H. Smith School of Business at the University of Maryland. He has been a consultant to Morgan Stanley since 1996 and to Norges Bank Investment Management since 2012. He is a founding member and past president of the Bachelier Finance Society. He was a Humboldt Awardee in 2006, was named Quant of the Year in 2008, and was inducted into the University of Maryland's Circle of Discovery in 2014. He is the cocreator of the variance gamma model (1990, 1998) and of conic finance. He coauthored, with Wim Schoutens, *Applied Conic Finance* (Cambridge University Press, 2016).

WIM SCHOUTENS is a professor at KU Leuven, Belgium. He has extensive practical experience of model implementation and is well known for his consulting work to the banking industry and other institutions. He served as an expert witness for the General Court of the European Union, Luxembourg, and has worked as an expert for the International Monetary Fund and for the European Commission. In 2012, he was awarded the John von Neumann Visiting Professorship of the Technical University of Munich. He has authored several books on financial mathematics and is a regular lecturer to the financial industry. Finally, he is a member of the Belgium CPI commission.

Cambridge University Press

978-1-316-51809-0 — Nonlinear Valuation and Non-Gaussian Risks in Finance

Dilip B. Madan , Wim Schoutens

Frontmatter

[More Information](#)

---

# NONLINEAR VALUATION AND NON-GAUSSIAN RISKS IN FINANCE

DILIP B. MADAN

*Robert H. Smith School of Business, University of Maryland*

WIM SCHOUTENS

*KU Leuven*



**CAMBRIDGE**  
UNIVERSITY PRESS

Cambridge University Press  
978-1-316-51809-0 — Nonlinear Valuation and Non-Gaussian Risks in Finance  
Dilip B. Madan , Wim Schoutens  
Frontmatter  
[More Information](#)

CAMBRIDGE  
UNIVERSITY PRESS

University Printing House, Cambridge CB2 8BS, United Kingdom  
One Liberty Plaza, 20th Floor, New York, NY 10006, USA  
477 Williamstown Road, Port Melbourne, VIC 3207, Australia  
314–321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi – 110025, India  
103 Penang Road, #05–06/07, Visioncrest Commercial, Singapore 238467

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning, and research at the highest international levels of excellence.

[www.cambridge.org](http://www.cambridge.org)  
Information on this title: [www.cambridge.org/9781316518090](http://www.cambridge.org/9781316518090)  
DOI: 10.1017/9781108993876

© Dilip B. Madan and Wim Schoutens 2022

This publication is in copyright. Subject to statutory exception and to the provisions of relevant collective licensing agreements, no reproduction of any part may take place without the written permission of Cambridge University Press.

First published 2022

*A catalogue record for this publication is available from the British Library.*

ISBN 978-1-316-51809-0 Hardback

Cambridge University Press has no responsibility for the persistence or accuracy of URLs for external or third-party internet websites referred to in this publication and does not guarantee that any content on such websites is, or will remain, accurate or appropriate.

Cambridge University Press

978-1-316-51809-0 — Nonlinear Valuation and Non-Gaussian Risks in Finance

Dilip B. Madan , Wim Schoutens

Frontmatter

[More Information](#)

---

To our perennial supporters:

to Vimla, Meena, Maneka, Sherif, Shivali, Sabrina, and Maia

—Dilip

to Ethel, Jente, and Maitzanne

—Wim

Cambridge University Press

978-1-316-51809-0 — Nonlinear Valuation and Non-Gaussian Risks in Finance

Dilip B. Madan , Wim Schoutens

Frontmatter

[More Information](#)

---

## Contents

	<i>Preface</i>	<i>page xi</i>
	<i>Acknowledgments</i>	<i>xiii</i>
<b>1</b>	<b>Introduction</b>	<b>1</b>
<b>2</b>	<b>Univariate Risk Representation Using Arrival Rates</b>	<b>5</b>
	2.1 Pure Jump Finite Variation Probability Models	7
	2.2 Probability Densities and Arrival Rates	11
	2.3 The Complex Exponential Variation	16
	2.4 Evaluating Event Arrival Rates	20
	2.5 Variation Outcomes	22
	2.6 Drift, Volatility, Risk Dimensions, and Their Compensation	26
<b>3</b>	<b>Estimation of Univariate Arrival Rates from Time Series Data</b>	<b>30</b>
	3.1 Complex Exponential Variations and Data	30
	3.2 Digital Moment Estimation	31
	3.3 Variance Gamma, Bilateral Gamma, and Bilateral Double Gamma Estimation Results	33
	3.4 Assessing Parameter Contributions	35
<b>4</b>	<b>Estimation of Univariate Arrival Rates from Option Surface Data</b>	<b>39</b>
	4.1 Depreferencing Option Prices	39
	4.2 Estimation Results	43
<b>5</b>	<b>Multivariate Arrival Rates Associated with Prespecified Univariate Arrival Rates</b>	<b>48</b>
	5.1 Multivariate Model for Bilateral Gamma Marginals	49
	5.2 The Role of Dependency Parameters in the Multivariate Bilateral Gamma Model	52
	5.3 Multivariate Bilateral Gamma Lévy Copulas	53
	5.4 Multivariate Model for Bilateral Double Gamma Marginals	55
	5.5 Simulated Count of Multivariate Event Arrival Rates	56
<b>6</b>	<b>The Measure-Distorted Valuation As a Financial Objective</b>	<b>59</b>
	6.1 Linear Valuation Issues	61

viii	<i>Contents</i>	
	6.2 Modeling Risk Acceptability	63
	6.3 Nonlinear Conservative Valuation	65
	6.4 Risk Reward Decompositions of Value	66
	6.5 Remarks on Modigliani–Miller Considerations	67
	6.6 Probability Distortions	67
	6.7 Measure Distortions Proper	73
	6.8 Dual Formulation of Measure Distortions	78
	6.9 Explicit Representation of Dual Distortions $\Phi, \tilde{\Phi}$ .	82
	6.10 Generic Considerations in the Maximization of Market Valuations	84
<b>7</b>	<b>Representing Market Realities</b>	<b>85</b>
	7.1 Risk Charges and the Measure Distortion Parameters	86
	7.2 Measure Distortions and Option Prices	87
	7.3 Measure-Distorted Value-Maximizing Hedges for a Short Gamma Target	90
	7.4 Measure Distortions Implied by Hedges for a Long Gamma Target	95
<b>8</b>	<b>Measure-Distorted Value-Maximizing Hedges in Practice</b>	<b>98</b>
	8.1 Hedging Overview	99
	8.2 The Hedge-Implementing Enterprise	100
	8.3 Summarizing Option Surfaces Using Gaussian Process Regression	101
	8.4 Selecting the Hedging Arrival Rates	104
	8.5 Approximating Variation Exposures	105
	8.6 Measure Distortion Parameters	106
	8.7 Backtest Hedging Results for Multiple Strangles on SPX	108
<b>9</b>	<b>Conic Hedging Contributions and Comparisons</b>	<b>110</b>
	9.1 Univariate Exposure Hedging Study	112
	9.2 Distorted Least Squares	113
	9.3 Example Illustrating Distorted Least-Squares Hedges	116
	9.4 Incorporating Weightings	118
	9.5 Measure-Distorted Value Maximization	119
	9.6 Greek Hedging	120
	9.7 Theta Issues in Exposure Design	120
	9.8 Incorporating Spreads	122
	9.9 No Spread Access and Theta Considerations	124
<b>10</b>	<b>Designing Optimal Univariate Exposures</b>	<b>126</b>
	10.1 Exposure Design Objectives	127
	10.2 Exposure Design Constraints	128
	10.3 Exposure Design Problem	128
	10.4 Lagrangean Analysis of the Design Problem	129
	10.5 Discretization and Solution	130
	10.6 Details Related to Lévy Measure Singularities at Zero	131
	10.7 Sample Optimal Exposure Designs	131
	10.8 Further Details about Some Particular Cases	132



*Contents*

ix

<b>11</b>	<b>Multivariate Static Hedge Designs Using Measure-Distorted Valuations</b>	135
11.1	A Two-Dimensional Example	136
11.2	A 10-Dimensional Example	142
<b>12</b>	<b>Static Portfolio Allocation Theory for Measure-Distorted Valuations</b>	150
12.1	Measure Integrals by Simulation	152
12.2	Dual Formulation of Portfolio Problem	152
12.3	Approximation by Probability Distortion	154
12.4	Implementation of Portfolio Allocation Problems	154
12.5	Mean Risk Charge Efficient Frontiers	156
12.6	Sensitivity of Required Returns to Choice of Points on Frontiers	163
12.7	Conic Alpha Construction Based on Arrival Rates	164
12.8	Fixed Income Asset Efficient Exposure Frontiers	165
<b>13</b>	<b>Dynamic Valuation via Nonlinear Martingales and Associated Backward Stochastic Partial Integro-Differential Equations</b>	171
13.1	Backward Stochastic Partial Integro-Differential Equations and Valuations	173
13.2	Nonlinear Valuations and BSPIDE	175
13.3	Spatially Inhomogeneous Bilateral Gamma	176
13.4	Dynamic Implementation of Hedging Problems	179
<b>14</b>	<b>Dynamic Portfolio Theory</b>	184
14.1	The Dynamic Law of Motion	185
14.2	Relativity Dynamics	187
14.3	The Full Sample	188
14.4	Portfolio Construction	188
14.5	Stationary Exposure Valuation	191
14.6	Stationary Value and Policy Results	192
14.7	Building Neural Net Policy Functions and Simulating Trades	192
<b>15</b>	<b>Enterprise Valuation Using Infinite and Finite Horizon Valuation of Terminal Liquidation</b>	195
15.1	Bilateral Gamma Enterprise Returns	197
15.2	Prudential Capital for Bilateral Gamma Returns	201
15.3	Regulatory Risk Capital for Enterprises with Bilateral Gamma Returns	209
15.4	Calibration of Measure-Distortion Parameters	210
15.5	Results for Equity Enterprises	216
15.6	Results for Treasury Bond Investments	216
15.7	Results for Hedge Fund Enterprises	217
15.8	Short Position Capital Requirements	220
15.9	Equity versus Leveraged Equity	220

<b>16</b>	<b>Economic Acceptability</b>	223
16.1	Interplay between Equity Markets and Regulators	224
16.2	Candidate Physical Laws of Motion	225
16.3	Adapted Measure Distortions	226
16.4	Equity and Regulatory Capital Constructions	228
16.5	Financial Sector Capital during and after the Financial Crisis	230
<b>17</b>	<b>Trading Markovian Models</b>	235
17.1	Return Dependence on States	237
17.2	Markovian State Dynamics	239
17.3	Formulation and Solution of Market Value Maximization	240
17.4	Results on Policy Functions for 10 Stocks	242
17.5	Results for Sector ETFs and SPY	243
<b>18</b>	<b>Market-Implied Measure-Distortion Parameters</b>	245
18.1	Designing the Time Series Estimation of Measure-Distortion Parameters	245
18.2	Estimation Results	247
18.3	Distribution of Measure-Distorted Valuations for Equity Underliers	247
18.4	Structure of Measure-Distorted Valuation-Level Curves	249
18.5	Valuation Frontiers	250
18.6	Acceptability Indices	251
18.7	Acceptability-Level Curves	252
18.8	Equilibrium Return Distributions	253
18.9	Empirical Construction of Return Distribution Equilibria and Their Properties	254
	<i>References</i>	257
	<i>Index</i>	265

## Preface

Financial valuation as an expectation with respect to a probability both ignores the naturally present uncertainty in the probability and makes value a linear function of the risk. Nonlinear valuation instead is based on a diverse set of probabilities and two nonlinear valuations naturally arise. The lower valuation takes the infimum of expectations over the set of probabilities while the upper valuation takes the supremum. The result is a concave conservative valuation for assets that may be maximized and a convex conservative valuation for liabilities that may be minimized. The classical linear valuation, by virtue of its linearity, is problematic as an optimization objective. The resulting new financial objectives can be applied to all aspects of financial decision-making. Madan and Schoutens (2016) investigated and reported on applying these new nonlinear objectives for financial analysis to many aspects of finance.

The classical view of risk as multiple future outcomes at some horizon described by their probabilities is here revised. Risk is here viewed as an exposure to instantaneous changes in states with the resulting time horizon being sent to zero. A form of continuity can be modeled by allowing for infinite small or minuscule changes in state. The changes are described by the time frequency of their arrival rates with the aggregate arrival across all possible changes potentially infinite. As a result there is not a probability for the changes in states, but just a measure given by the arrival rate. Normalization to a probability is not possible; the aggregate arrival rate is infinite.

Risk is described by the arrival rate measure. Expectations are replaced by variations that are integrals of state changes with respect to the arrival rate measure. Nonlinear valuations allow for sets of alternate measures incorporating natural uncertainties in the measure. The lower and upper valuations are then infima and suprema or variations over the sets of measures. Probability distortions encountered in Madan and Schoutens (2016) are replaced by measure distortions introduced in Madan et al. (2017a).

The objectives of financial analysis are then reformulated with risk described by multidimensional arrival rate measures and the nonlinear valuations of risk are based on measure-distorted variations. The first five chapters present this implementable reformulation of risk and its value. The rest of the book presents applications that include multidimensional hedging, portfolio allocation, and derivative positioning. All activities may be undertaken statically at the level of the instant, which is the limit of letting the horizon go to zero,

or dynamically by solving nonlinear partial integro-differential equations backward from a finite or infinite terminal date. In the latter case the arrival rates may be adapted to evolving filtrations of information. The difficulty with dynamic modeling is the general inability to commit future actions to be consistent with plans generated earlier. Hence only the initial decision of a dynamic plan may see implementation. Be that as it may, both formulations are considered. The dynamic formulation here works with a probability at a terminal date or horizon. A generalization to an infinite measure at the terminal date that cannot be normalized to a probability is an interesting subject for subsequent research and beyond the scope of research accomplished to date.

All implementation requires the estimation of multidimensional arrival rates from data, and the first chapters present the details for undertaking this work. Upon completion one has the ability to estimate the count of loss and gain arrival rates of positions in financial instruments. The instruments considered are assets and liabilities undertaken relative to a numeraire accounting for time value considerations. All positions are always risky.

## Acknowledgments

This book is the work of the authors, but without the support of our colleagues, PhD students, coauthors, people we met at seminars and conferences, and referees of the papers we published, it would not have been possible.

We would like to thank explicitly Peter Carr, José Manuel Corcuera, Freddy Delbaen, Jan De Spiegeleer, Ernst Eberlein, Robert Elliott, Pankaj Khandelwal, Ajay Khanna, Peter Leoni, Shige Peng, Martijn Pistorius, Sofie Reyners, Yazid Sharaiha, Mitja Stadje, Juan Jorge Vicente Alvarez, King Wang, and Marc Yor.

Our gratitude and respect also go to the team of Cambridge University Press for the enthusiasm and professionalism with which they have embraced this book project.

Cambridge University Press

978-1-316-51809-0 — Nonlinear Valuation and Non-Gaussian Risks in Finance

Dilip B. Madan , Wim Schoutens

Frontmatter

[More Information](#)

---