NONLINEAR VALUATION AND NON-GAUSSIAN RISKS IN FINANCE

What happens to risk as the economic horizon goes to zero and risk is seen as an exposure to a change in state that may occur instantaneously at any time? All activities that have been undertaken statically at a fixed finite horizon can now be reconsidered dynamically at a zero time horizon, with arrival rates at the core of the modeling.

This book, aimed at practitioners and researchers in financial risk, delivers the theoretical framework and various applications of the newly established dynamic conic finance theory. The result is a nonlinear non-Gaussian valuation framework for risk management in finance. Risk-free assets disappear and low-risk portfolios must pay for their risk reduction with negative expected returns. Hedges may be constructed to enhance value by exploiting risk interactions. Dynamic trading mechanisms are synthesized by machine learning algorithms. Optimal exposures are designed for option positioning simultaneously across all strikes and maturities.

DILIP B. MADAN is Professor Emeritus at the Robert H. Smith School of Business at the University of Maryland. He has been a consultant to Morgan Stanley since 1996 and to Norges Bank Investment Management since 2012. He is a founding member and past president of the Bachelier Finance Society. He was a Humboldt Awardee in 2006, was named Quant of the Year in 2008, and was inducted into the University of Maryland's Circle of Discovery in 2014. He is the cocreator of the variance gamma model (1990, 1998) and of conic finance. He coauthored, with Wim Schoutens, *Applied Conic Finance* (Cambridge University Press, 2016).

WIM SCHOUTENS is a professor at KU Leuven, Belgium. He has extensive practical experience of model implementation and is well known for his consulting work to the banking industry and other institutions. He served as an expert witness for the General Court of the European Union, Luxembourg, and has worked as an expert for the International Monetary Fund and for the European Commission. In 2012, he was awarded the John von Neumann Visiting Professorship of the Technical University of Munich. He has authored several books on financial mathematics and is a regular lecturer to the financial industry. Finally, he is a member of the Belgium CPI commission.

NONLINEAR VALUATION AND NON-GAUSSIAN RISKS IN FINANCE

DILIP B. MADAN
Robert H. Smith School of Business, University of Maryland

WIM SCHOUTENS
KU Leuven

CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE
UNIVERSITY PRESS

University Printing House, Cambridge CB2 8BS, United Kingdom

One Liberty Plaza, 20th Floor, New York, NY 10006, USA

477 Williamstown Road, Port Melbourne, VIC 3207, Australia

314–321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi – 110025, India

103 Penang Road, #05–06/07, Visioncrest Commercial, Singapore 238467

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning, and research at the highest international levels of excellence.

www.cambridge.org
Information on this title: www.cambridge.org/9781316518090
DOI: 10.1017/9781108993876

First published 2022

A catalogue record for this publication is available from the British Library.

ISBN 978-1-316-51809-0 Hardback

To our perennial supporters:
to Vimla, Meena, Maneka, Sherif, Shivali, Sabrina, and Maia
—Dilip
to Ethel, Jente, and Maitzanne
—Wim

Contents

Preface

Financial valuation as an expectation with respect to a probability both ignores the naturally present uncertainty in the probability and makes value a linear function of the risk. Nonlinear valuation instead is based on a diverse set of probabilities and two nonlinear valuations naturally arise. The lower valuation takes the infimum of expectations over the set of probabilities while the upper valuation takes the supremum. The result is a concave conservative valuation for assets that may be maximized and a convex conservative valuation for liabilities that may be minimized. The classical linear valuation, by virtue of its linearity, is problematic as an optimization objective. The resulting new financial objectives can be applied to all aspects of financial decision-making. Madan and Schoutens (2016) investigated and reported on applying these new nonlinear objectives for financial analysis to many aspects of finance.

The classical view of risk as multiple future outcomes at some horizon described by their probabilities is here revised. Risk is here viewed as an exposure to instantaneous changes in states with the resulting time horizon being sent to zero. A form of continuity can be modeled by allowing for infinite small or minuscule changes in state. The changes are described by the time frequency of their arrival rates with the aggregate arrival across all possible changes potentially infinite. As a result there is not a probability for the changes in states, but just a measure given by the arrival rate. Normalization to a probability is not possible; the aggregate arrival rate is infinite.

Risk is described by the arrival rate measure. Expectations are replaced by variations that are integrals of state changes with respect to the arrival rate measure. Nonlinear valuations allow for sets of alternate measures incorporating natural uncertainties in the measure. The lower and upper valuations are then infima and suprema or variations over the sets of measures. Probability distortions encountered in Madan and Schoutens (2016) are replaced by measure distortions introduced in Madan et al. (2017a).

The objectives of financial analysis are then reformulated with risk described by multidimensional arrival rate measures and the nonlinear valuations of risk are based on measure-distorted variations. The first five chapters present this implementable reformulation of risk and its value. The rest of the book presents applications that include multidimensional hedging, portfolio allocation, and derivative positioning. All activities may be undertaken statically at the level of the instant, which is the limit of letting the horizon go to zero, or dynamically by solving nonlinear partial integro-differential equations backward from a finite or infinite terminal date. In the latter case the arrival rates may be adapted to evolving filtrations of information. The difficulty with dynamic modeling is the general inability to commit future actions to be consistent with plans generated earlier. Hence only the initial decision of a dynamic plan may see implementation. Be that as it may, both formulations are considered. The dynamic formulation here works with a probability at a terminal date or horizon. A generalization to an infinite measure at the terminal date that cannot be normalized to a probability is an interesting subject for subsequent research and beyond the scope of research accomplished to date.

All implementation requires the estimation of multidimensional arrival rates from data, and the first chapters present the details for undertaking this work. Upon completion one has the ability to estimate the count of loss and gain arrival rates of positions in financial instruments. The instruments considered are assets and liabilities undertaken relative to a numeraire accounting for time value considerations. All positions are always risky.

Acknowledgments

This book is the work of the authors, but without the support of our colleagues, PhD students, coauthors, people we met at seminars and conferences, and referees of the papers we published, it would not have been possible.

We would like to thank explicitly Peter Carr, José Manuel Corcuera, Freddy Delbaen, Jan De Spiegeleer, Ernst Eberlein, Robert Elliott, Pankaj Khandelwal, Ajay Khanna, Peter Leoni, Shige Peng, Martijn Pistorius, Sofie Reyners, Yazid Sharaiha, Mitja Stadje, Juan Jorge Vicente Alvarez, King Wang, and Marc Yor.

Our gratitude and respect also go to the team of Cambridge University Press for the enthusiasm and professionalism with which they have embraced this book project.