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Excerpt
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Introduction

Risk is often defined by the probabilities of possible outcomes, be they the tossing of coins, the rolling of dice, or the prices of assets at some future date. Uncertainty exists as the possible outcomes are many and the actual outcome is not known. The risk may be valued statistically at its expected value or in a market at the current price to be paid or received for acquiring or delivering a unit of currency on the resolution of the risk. The market value is also understood to be a discounted expected value under altered probabilities that reflect prices of events as opposed to their real probabilities. By construction the value of a risk is hence a linear function on the space of risks with the value of a combination being equal to an equivalent combination of values. As a consequence value maximization is not possible as nonconstant linear functions have no maximal values. Optimization becomes possible only after introducing constraints that limit the set of possibilities.

This view of risk, as probabilities for multiple outcomes, and its valuation, as an expectation, is reformulated here, with important consequences for the practice of financial decision-making and analysis. Risk is viewed in a substantially different way with the principles employed in valuing risk also suitably revised.

Risk is seen as an exposure to a change in state that may occur instantaneously at any time. The sets of alternate current states, like possible outcomes, are many. But the probabilities of outcomes are replaced by the arrival rates of possible state changes. However, unlike probabilities that sum to or integrate to unity, aggregate arrival rates may be infinite. Indeed, in general, there could be many small and insignificant changes in state. The total measure across all possibilities could be infinite and the normalization of counts to unity is lost. Risk is then defined by an infinite measure on possible changes of state with all substantive state changes having a finite measure, but insignificant or small changes may occur infinitely often. The aggregate effect of the infinite number of small changes is, however, finite.

Apart from the reformulation of risk, we follow Peng (2019) in recognizing that the valuation of risk should not be based on a single probability or a single measure. Such an approach ignores our natural uncertainty about the probability or measure. In valuation, we allow for alternate probabilities or measures being relevant. The resulting relevant valuations then turn out to be infima and suprema of expectations over the set of all alternates, be they probabilities or measures. There are then naturally two valuations, one lower associated with the infima, the other upper associated with the suprema. The lower is concave and may be maximized while the upper is convex and may be minimized. Value optimization is thereby restored to some extent.

In this regard it may be noted that the maximization of market value has been a rallying cry for decision-making in capitalist economies. Milton Friedman argues for it in a classic article in the *New York Times*, September 13, 1970, entitled "The Social Responsibility of Business Is to Increase Its Profits." Yet another, more detailed advocacy is presented in Jensen (2002). However, it has also long been recognized in economics and finance that value maximization is problematic given its linearity. The linearity was observed, for example, as a consequence of the law of one price, that is, using a single probability, and the absence of arbitrage in Ross (1978). The value of a portfolio is then the sum of the component values and independent of how they are combined. Similarly a position hedged at zero cost has the same value no matter the hedge. Under linearity, value maximization just does not work, and many authors resort to personalized utility-based valuations. These are controversial as markets may disregard the particular utility being employed with serious consequences for the enterprise. A way out is to build a market-based nonlinear valuation function. This is precisely what the Peng (2019) theory of nonlinear expectations or martingales delivers. The purpose of this book, partially, is to bring these ideas to finance in a practical and implementable way.

An earlier work (Madan and Schoutens, 2016) dealt with nonlinear valuations associated with alternate probabilities of outcomes realized at some time horizon. The purpose here is to develop the results, with applications, in the context of infinite arrival rate measures for instantaneous exposures at a zero time horizon.

An important financial implication of a zero time horizon, encountered in Chapter 12, is the absence of any sense of a positive risk-free rate at a zero time horizon. Recognizing furthermore the perpetual presence of risk exposures, however assets are held, leads to an empirically observed negative return on exposures with a low risk level. The reference rate in an instantaneous exposure economy can become a negative rate. As a consequence required returns on many equity assets turn out to be negative. The assertion of a positive required return on equity turns out to be just an accidental or unintended consequence of assuming a static model with a horizon coupled with a positive risk-free rate over this horizon. Participants in actual economies exposed to substantial instantaneous or zero horizon risks have no such artificial horizon effects. Negative required returns for some equity assets are completely consistent with zero horizon economic risks.

Many financial situations are managed by a partial hedging of risks. Hedging targets the conditional expectation of the risk to hedge given the outcomes on the hedging assets. But for a liability, one would like bias positions to be a little over the liability while for an asset delivering on the hedge, one may want to be a little under the asset value. In the setting of hedging by least-squares minimization, the structure of upper and lower valuations leads to the concept of distorted least squares that accomplish parametrically these value-sensitive upper and under hedges. These hedging improvements are encountered in Chapters 9 and 11 in the univariate and multivariate contexts. The direct maximization of lower valuations and the minimization of upper valuations for hedging purposes are also studied in Chapters 7 and 8.

Aside from hedging considerations, risk exposures are occasionally designed in limited ways in portfolio theory and more aggressively in options markets, providing access to a richer class of exposures. Chapters 11 and 12 take up the question of portfolio theory. For option exposures, the question has been addressed by applying utility theory in a static, single-period context using nonlinear valuations in Madan and Schoutens (2016). Chapter 10 takes up the problem of constructing optimal arrival rates made possible by positions in a portfolio of options.

Instantaneous exposure design is an important part of financial management, but on occasion one has to take account of longer-term objectives in a dynamic setting. Nonlinear valuation in a dynamic setting takes us to the developing literature on backward stochastic partial integro-differential equations, addressed here in a Markovian setting. The dynamic hedging problem for nonlinear value optimization is addressed in Chapter 13 with the portfolio problem taken up in Chapter 14. For both problems, one needs to usefully represent value and asset allocations as functions of state variables in possibly high dimensions. Applications are made of numerous machine learning methods to synthesize the functions involved. For the value maximization, we use Gaussian process regression (GPR) while for portfolio allocations feedforward neural nets are used. A wealth accumulation comparison is made between a myopic investor and a long-term investor in a Markovian context. It is observed that the long-term investor delivers the superior performance.

A special case of critical importance in finance is the valuation of equity itself. The lower valuation of equity is the capital the equity markets safely provide economic enterprises, and it must be sufficient to cover the risks being taken. The latter are monitored and formulated by financial regulatory bodies in modern capitalist economies and the question arises as to whether equity markets are providing sufficient capital. The lower valuation is a prudent valuation and as such fits into the prudential framework. If so, one has an economically acceptable enterprise. These issues are formulated and empirically analyzed in Chapters 15 and 16.

An important trading strategy pursued in the financial markets is termed "pairs trading." Often this involves trading one stock against another, related stock. Higher-dimensional forms of such trading strategies are formulated and solved for a nonlinear dynamic valuation objective in Chapter 17. The methods are empirically illustrated in up to 10 dimensions.

Nonlinear valuation objectives are formulated from the perspective of determining a lower market value of a risk and not a personalized value. It is not about what you think something is worth but what you would get for it in the market. Parameters in formulating nonlinear valuation objectives must therefore be derived from market data. This issue is taken up in Chapter 18. It is followed up in Elliot et al. (2020).

Chapters 7 onward deliver the many financial applications of nonlinear valuation applied to finance. The construction of practical and implementable nonlinear valuation objective functions is the subject matter of Chapter 6. A prerequisite for this is the description of risk to which nonlinear valuation is applied. The application requires the full description of the risk with all parameters estimated. Chapters 2 through 5 accomplish these tasks in both a univariate and multivariate context from both the backward-looking time series data and the forward-looking data of option markets.