

## Index

- a priori bound, 18, 21, 46, 53, 55, 69, 75, 127  
 a priori estimate, 1, 32, 46, 58, 67, 77  
 Abraham, 58, 67, 169  
 absolutely continuous distribution, 96, 100  
 adiabatic asymptotics, 167, 168  
 adiabatic effective dynamics, 175  
 adiabatic invariant, 119  
 Agmon, 4, 178, 182  
 analytic continuation, 98, 132, 155  
 analytic function, 35–37, 44, 117, 132, 135, 159  
 analytic properties, 135  
 Ascoli–Arzelà theorem, 104  
 asymptotic completeness, 16, 19  
 asymptotic stability, 114, 120, 166  
 asymptotic stability of solitary manifold, 10, 12, 114, 122  
 asymptotic state, 19  
 asymptotics, 4–7, 9, 10, 16, 46, 60, 76, 80, 88, 89, 112, 115, 116, 118, 120–125, 130, 154, 168, 171, 174, 183, 195–197  
 attracting set, 37, 50, 65  
 attraction, 10, 11, 59, 65, 66, 94  
   in the mean, 50, 52, 56  
   to solitons, 95, 121  
   to stationary orbits, 91, 94, 95  
   to stationary states, 19, 50  
 attractor, ix, x, 1–3, 9, 14, 76, 192, 193  
  
 Bambusi, 79  
 Banach phase space, 2, 111  
 Banach space, ix, 6, 10, 127, 136  
 baryon, 10, 112, 113  
 Berestycki, 8  
  
 Berezin, 68  
 Bethe, 68  
 Bohr, 192, 198  
 Bohr’s postulates, 11, 192–194, 197, 198  
 Bohr’s quantum jumps, ix, 1, 5, 11, 25, 192–195  
 Bohr’s rule, 198  
 Born, 193  
 Born probabilistic interpretation, 198  
 Born series, 184  
 bound component, 102, 103  
 boundary value problem, 45, 48  
 Buslaev, 4, 10, 116, 124  
  
 canonical form, 88, 167  
 canonical transformation, 77, 79, 80, 88  
 Cauchy data, 40, 46, 53, 55  
 Cauchy problem, 16–18, 21, 30, 43, 53, 70, 72, 73  
 Cauchy residue theorem, 101, 132, 134  
 Cauchy–Schwarz inequality, 22, 32, 40, 47, 63, 102  
 Cazenave, 3  
 charge conservation, 126, 138  
 charge density, 13, 56, 58, 67, 77, 169, 194  
 chemical reaction, 2  
 Cherenkov radiation, 122  
 Chern–Simon–Schrödinger system, 168  
 chromodynamics, 193  
 classical electrodynamics, 1, 5, 13, 59, 67, 77, 169  
 classical particle, 175  
 Comech, 1, 4  
 comoving frame, 6, 7, 77–79, 121  
 complexification, 117

- concentrated nonlinearity, 8, 13, 68, 91, 94  
concentration compactness, 123  
confining potential, 44, 57, 67, 68  
conjecture on attractors, ix, 4, 10, 111–113, 169, 192, 195  
continuous spectrum, 9, 59, 96, 99, 102, 108–110, 115, 118, 121, 122, 125, 134, 141, 147, 165, 173, 181, 197, 198  
convex hull, 106  
convolution, 59, 64, 104, 107, 109  
convolution representation, 63, 67  
Coulomb potential, 9, 58, 196  
critical energy, 123  
critical focusing nonlinear Schrödinger equation, 123  
critical point, 9, 79, 81, 82  
current density, 67, 194, 197
- d'Alembert, 17, 27  
equation, 6, 13, 26  
formula, 14, 20, 40, 41, 47  
operator, 194  
representation, 20, 30, 32, 40, 41, 52
- de Broglie, 193  
defocusing nonlinearity, 3  
determinant, 159  
difference scheme, 8, 91, 94, 174  
diffraction theory, 98  
Dirac, 68, 195  
distribution, 134, 136, 139  
equation, 8, 91, 94, 111, 121, 178  
matrices, 94  
discrete equation, 178  
discrete set, 19, 22, 25, 31, 34, 36, 37, 44, 46, 48, 59, 66, 76, 176  
discrete spectrum, 8, 9, 91, 114, 118, 129, 133, 136, 147, 161, 162, 173, 198  
dispersion relation, 173  
dispersive component, 102  
dispersive decay, ix, 11, 90, 178–181  
dispersive mechanism, 108  
dispersive radiation, 108, 111  
dispersive wave, 7, 9, 16, 73, 89, 90, 102, 109, 123, 171, 173–177, 196  
dissipative system, 1, 2, 76  
distribution, x, 14, 17, 18, 30, 97, 102, 104, 107, 124, 129, 132, 136, 182, 191  
Duhamel representation, 47, 90, 139, 149  
dynamical group, 16, 19, 47, 54, 118, 124, 129, 132, 138, 139, 152, 153  
dynamical interpretation, 194  
dynamical system, 43  
dynamical treatment of Bohr's postulates, 192  
dynamics, 10, 16, 31, 46, 115, 125, 194  
Dynkin scheme, 113
- effective dispersion relation, 168  
effective dynamics, ix, 11, 166–168, 170, 174–176  
effective Hamiltonian equation, 167  
effective Hamiltonian functional, 166, 167  
effective potential, 116  
eigenfunction, 4, 9, 59, 179, 193, 197  
eigenfunction expansion, 119, 122  
eigenvalue, 8, 10, 114, 118, 122, 129, 171, 173, 178–182  
eightfold way, 113  
Einstein, 168, 169, 198  
Einstein equations, 123  
Einstein–Dirac system, 168  
elementary particle, 111, 113, 198  
embedded eigenvalues, 8, 91, 96, 102  
energy, 46, 47, 57, 62, 72, 87, 88, 123, 168, 169, 179  
absorption, 76  
conservation, 2, 3, 5, 6, 18, 21, 31, 32, 44, 46, 58, 69, 70, 74, 76, 79, 83, 92, 116, 138  
density, 177  
dissipation, 2  
flow, 42  
functional, 42, 52  
norm, 16  
radiation, 2–6, 13, 21, 25, 41, 42, 45, 52, 59, 62, 63, 67, 76, 108–110, 118, 176  
transfer, 108, 109  
entire function, 106  
equations with delay, 2, 28  
equations with memory, 2  
Esteban, 8, 196  
example, 4, 6, 13, 19, 23, 27, 29, 35–37, 46, 60, 112, 119, 121–123, 137, 138, 173, 180, 183, 188, 197  
exceptional equation, 112  
extended electron, 58, 67, 169  
external potential, 57, 166, 194–196
- Faddeev, 68  
Fermi, 68

- Fermi Golden Rule, 59, 118, 120, 122  
 finite-energy solution, 3, 4, 9, 16, 26, 28, 58,  
   66, 67, 116, 123–126, 170, 171, 194,  
   195, 197  
 finite-energy state, 16, 29, 67, 87, 118,  
   124, 173  
 form-factor, 59  
 Fourier integral, 100  
 Fourier representation, 101, 129  
 Fourier transform, x, 58, 78, 96–99, 104, 106,  
   109, 154, 180  
 Fourier–Laplace transform, 97, 102, 108  
 free Klein–Gordon equation, 179  
 free Schrödinger equation, 118, 124, 129,  
   152, 196  
 free Schrödinger operator, 182  
 free wave equation, 16, 19, 61, 69, 74, 87,  
   89, 90  
 friction, 1, 2, 24, 25  
 fundamental solution, 109  
  
 G-invariance, 111  
 G-invariant equation, 8, 111, 112, 195  
 Galgani, 79  
 Galilean transformation, 117  
 Gell-Mann, 10, 112  
 generator, 10, 100, 109, 112, 113, 118,  
   119, 125  
 generic equations, 4, 9, 11, 76, 111, 112  
 Georgiev, 8, 9, 196  
 Ginzburg–Landau equation, 12, 120, 173  
 Ginzburg–Landau potential, 23, 93, 122, 170,  
   173  
 global attraction, 2, 3, 10, 11  
   to  $N$ -frequency trajectories, 9  
   to solitary manifold, 7  
   to solitons, ix, 6, 7, 11, 77, 90, 170  
   to stationary orbits, ix, 8, 91, 193, 194  
   to “stationary  $SO(3)$ -orbits”, 9  
   to stationary states, ix, 2, 4, 6, 13–15, 19,  
   28, 37, 43, 45, 56, 66–68, 76  
 global attractor, ix, 1–3, 7, 15, 18, 24, 31, 45,  
   76, 112, 192, 194, 199  
 global energy norm, 16, 19, 89, 196  
 global minimum, 79, 81  
 global norm, 5, 6, 76, 118, 124  
 Green function, 68  
 Grillakis, 9  
 Gronwall inequality, 55  
 ground state, 114, 115, 123  
 group, 38, 128, 133–135, 149, 179  
  
 group of translations, 112  
 group velocity, 173, 176  
  
 Hamiltonian dynamics, 166, 175  
 Hamiltonian equation, 167  
 Hamiltonian functional, 6, 14, 17, 30, 43, 57,  
   58, 67, 77, 79, 81, 83, 87, 88, 92, 114,  
   126, 128, 129, 167, 170, 174  
 Hamiltonian operator, 4, 119, 122  
 Hamiltonian PDEs, ix, x, 1–3  
 Hamiltonian structure, 88, 111, 167  
 Hamiltonian system, 3, 14, 15, 17, 24, 30, 67,  
   76, 77, 81, 87, 88, 92, 124, 126, 128, 167,  
   170, 174  
 Haraux, 3  
 harmonic, 110  
 harmonic analysis, x, 4, 106  
 harmonic source, 108, 109  
 Heisenberg, 193, 198  
 Helmholtz equation, 98  
 Hessian, 129  
 higher harmonics, 108, 111  
 Hilbert manifold, 69  
 Hilbert phase space, 2, 16, 17, 29, 30, 44, 57,  
   67, 78, 80, 81, 92, 111, 117–119,  
   124, 128  
 Hilbert space, ix, 6, 14, 16, 29, 54, 57, 102  
 Hopf, 3  
 hyperbolic PDEs, 3  
  
 implicit function theorem, 143  
 incident wave, 20, 41  
 inflation of spectrum, 109–111  
 initial data, 3, 6, 14, 42, 46, 53, 69, 71–74, 77,  
   79, 84, 86, 87, 115, 116, 118, 121,  
   142–144, 154, 173  
 integral kernel, 154  
  
 Jörgens, 3  
 Jacobian matrix, 143  
 Jensen, 4, 125, 178, 181, 182  
 Journé, 178  
  
 Kato, 4, 125, 181, 182  
 Kato theorem, 96  
 kernel, 154  
 Kerr black hole, 123  
 Kerr solutions, 123  
 kink, 123, 171–174  
 Kirchhoff, 61  
 Kirchhoff formula, 84

- Klein–Gordon equation, 8, 10, 90, 91, 94, 96,  
 108, 109, 111, 121, 173, 178–180  
 Klein–Gordon–Dirac system, 9  
 Klein–Gordon–Maxwell system, 168  
 Kopylova, 1, 4  
 Krein theory, 4  
 Krein–Langer theory, 119, 122
- Lagrangian, 87  
 Lagrangian functional, 87  
 Lamb, 15  
 Landau, 1  
 Laplace representation, 135  
 Laplace transform, 131, 132  
 Larmor formula, 67  
 laser radiation, 193  
 Lax, 3  
 Legendre transformation, 87, 88  
 Liénard formula, 67  
 Liénard–Wiechert asymptotics, 60, 67  
 Liénard–Wiechert formulas, 59  
 Lie algebra, 4, 113  
 Lie group, 112  
 Lie symmetry group, 4, 111, 113  
 limiting absorption principle, 96, 98  
 limiting amplitude, 4, 109  
 limiting amplitude principle, 108  
 linear eigenvalue problem, 196  
 linear hyperbolic equation, 3  
 linear Schrödinger equation, 9, 198  
 linearization, 117, 122, 125, 130, 133, 173  
 linearized dynamics, 10, 115, 119, 122, 124  
 linearized equation, 11, 117, 122, 125, 134,  
 138, 139, 144, 154, 171, 173  
 linearized operator, 129  
 Lions J.-L., 3  
 Lions P.-L., 8  
 local attraction, 10, 114  
 local energy decay, 3  
 local energy norm, 195  
 local energy seminorm, 17, 109  
 local seminorm, 5, 7, 76  
 long-time asymptotics, 3, 4, 9, 11, 37, 50, 109,  
 113, 123, 198  
 long-time attraction, 115  
 long-time convergence, 122  
 long-time decay, 88  
 Lorentz contraction, 171  
 Lorentz transformation, 171  
 lower harmonics, 108  
 Lusternik–Schnirelmann theory, 9
- Lyapunov, 10, 11  
 Lyapunov function, 120
- majorant, 120, 124, 138, 151  
 mass–energy equivalence, 11, 166, 168, 169  
 Maxwell equations, 10, 67, 123, 194, 196  
 Maxwell field, 77  
 Maxwell potentials, 194  
 Maxwell–Dirac system, 8, 192, 195, 196, 199  
 Maxwell–Lorentz equations, 66, 79, 88, 90,  
 168, 169  
 Maxwell–Lorentz equations with rotating  
 charge, 9, 168  
 Maxwell–Schrödinger system, 9, 12, 192–194,  
 196–199  
 measure, 104  
 metastable tori, 122  
 method of compactness, 1, 3  
 metric, 17, 18, 51, 57, 59  
 modulation equations, 11, 115, 119, 125,  
 140–142  
 momentum, 80–83, 88, 166, 167  
 momentum conservation, 77, 79–81, 83  
 Morawetz, 3  
 multiphoton radiation, 121  
 multiplier, 180  
 multiplier of quasimeasures,  $x$ , 4, 8, 91,  
 99, 105
- Navier–Stokes equations, 1, 2  
 Ne’eman, 10, 112  
 Noja, 68  
 nondiscrete set, 35  
 nonlinear dynamics, 110  
 nonlinear eigenfunction, 124  
 nonlinear eigenvalue problem, 4, 93, 112, 126,  
 193, 195, 196  
 nonlinear Goursat problem, 53–55  
 nonlinear Hamiltonian equation, 3, 4, 11, 13,  
 14, 192  
 nonlinear Hartree equation, 168  
 nonlinear Helmholtz equation, 106  
 nonlinear Kato theorem, 8, 91, 99, 102  
 nonlinear Klein–Gordon equation, 90, 91,  
 115, 122  
 nonlinear Lamb system, 14, 15, 28  
 nonlinear oscillator, 8, 10, 14, 25, 28, 29, 68,  
 91, 94, 114, 121, 124  
 nonlinear radiative mechanism, 91, 107, 111  
 nonlinear scattering, 15

- nonlinear Schrödinger equation, 94, 115, 122, 124, 168  
 nonlinear wave equation, 13, 28, 43, 53, 79, 115, 122, 123, 168, 170, 174  
 nonlinearity, 19, 43, 70, 117, 125, 173  
 nonlocal interaction, 94  
 nonresonance conditions, 122  
 Nöther theorem, 126  
 nuclear reactions, 199  
 nuclei, 199  
 nuclei classification, 199  
 numerical experiment, 111  
 numerical simulation, ix, 11, 168, 170, 171, 173, 175, 193
- omega-compact trajectory, 95, 96  
 omega-compactness, 95, 96, 103  
 omega-limit point, 45  
 omega-limit trajectory, 8, 13, 45, 91, 95, 96, 105, 109, 110  
 omega-set, 38  
 orbit, 7, 23, 38, 50, 128, 192  
 orbital stability, 9, 79, 80, 83, 84, 120, 125, 129  
 orthogonal symmetry group, 4, 9  
 orthogonality condition, 115, 141, 142, 179  
 oscillator, 15, 20, 23, 25, 26, 124–126  
 oscillatory integral, 135
- Paley–Wiener estimates, 100  
 Parseval–Plancherel identity, 82, 100, 104, 153  
 particle, 15, 56, 58, 59, 67, 113, 121, 169  
 particle momentum, 167  
 Peierls, 68  
 Perelman, 10, 116, 124  
 perturbation, 59, 123  
 perturbation theory, 118, 121, 141, 197–199  
 Phillips, 3  
 Poincaré, 120  
 Poincaré normal form, 120  
 point nonlinearity, 14  
 point particle, 67  
 polynomial, 35, 127  
 polynomial nonlinearity, 170, 173  
 polynomial potential, 174  
 Posilicano, 68  
 potential, 9, 23, 32, 35–37, 46, 57, 67, 68, 70, 77, 84–87, 114–117, 122, 125, 126, 170, 172, 173, 175, 177–181, 194, 197  
 potential energy, 32, 44  
 probabilistic interpretation, 193  
 projection operator, 141
- quantum  
 amplifier, 193  
 chromodynamics, 113  
 field theory, 193, 199  
 jumps, 194  
 mechanics, 192  
 postulate, 192, 193, 198  
 stationary orbit, 193  
 stationary state, ix, 124, 192, 193, 199  
 theory, ix  
 quantum postulate, 2  
 quantum stationary state, 11  
 quantum theory, 2, 193, 198, 199  
 quarks, 113  
 quasi-periodic function, 10  
 quasimeasure, 96, 97, 99, 103, 104
- radiation, 26, 109, 175–177, 192, 198  
 radiation damping, 1, 5, 13, 59, 66, 67, 77  
 radiation integral, 62  
 radiation power, 67  
 radiative mechanism, 173  
 real-analytic function, 31, 34, 44, 49  
 reduced equation, 20, 22, 23, 25  
 reduction of spectrum, 96, 104, 105  
 reflected wave, 20  
 relative equilibria, 120  
 relativistic equation, 7, 8, 11, 12, 90, 95, 111, 120, 122, 168, 170, 174, 196  
 relativistic kinetic energy, 56  
 relativistic particle, 7, 10, 13, 56, 66, 77  
 Relativity General Theory, 123  
 Relativity Special Theory, 169  
 relaxation, 2, 22, 28, 39, 40, 42, 43  
 relaxation of acceleration, 59, 60, 63, 64, 67, 79, 80, 84, 86  
 resolvent, 109, 125, 132, 134, 135, 154, 158, 159, 182, 183  
 resonance, 114, 118, 122, 178–181  
 retarded potential, 59, 60, 67, 86  
 Reynolds number, 1  
 Riemann–Lebesgue theorem, 102, 105, 154  
 Riesz interpolation theorem, 179, 180  
 Riesz projection, 132, 181  
 rotation group, 124, 126  
 rotation symmetry, 126

- scalar wave field, 77  
 scattering, 3, 4, 16, 123  
   asymptotics, 19  
   operator, 19  
   state, 16, 118, 124, 130  
   theory, 3  
 Schrödinger, 11, 194  
   equation, 11, 107, 112, 114, 117, 121, 122, 124, 130, 139, 178–181, 194, 195, 197  
   operator, 109, 114, 115, 173, 181, 182, 193, 197  
   theory, 193, 194, 197  
 Schwarzschild black hole, 123  
 Segal, 3  
 self-energy, 58, 169  
 seminorm, 29, 44, 52, 57, 59  
 separatrix, 2  
 Seré, 8, 196  
 Shatah, 9  
 Sigal, 118  
 single-frequency asymptotics, 7, 195  
 single-frequency spectrum, 109  
 slowly varying external potential, 166, 168, 174  
 Sobolev embedding theorem, 1, 30, 55, 74  
 Sobolev space,  $x$ , 39, 97, 127, 180  
 Soffer, 10, 11, 90, 114, 178  
 Sogge, 11, 178  
 solitary manifold, 8, 10, 11, 78, 93, 94, 115, 118, 119, 121, 124, 125, 133, 166, 167, 175  
 solitary wave, 126  
 soliton,  $ix$ , 10, 11, 78–84, 86, 90, 114, 117, 119–123, 125, 140, 142, 152, 166–171, 174–177, 196  
 soliton asymptotics, 84, 152, 174, 196  
 soliton-like asymptotics, 166  
 soliton-like solutions, 174, 175  
 spectral density, 96  
 spectral Fourier representation, 182  
 spectral gap, 93, 96, 104, 107  
 spectral gap infinite, 95  
 spectral inclusion, 105–108  
 spectral representation, 9, 96, 99, 125, 184  
 spectrum, 10, 96, 118, 165, 173  
 spectrum absolute continuous, 120  
 Spohn, 1, 4  
 spreading of spectrum, 108  
 stationary equation, 44, 48, 78, 116  
 stationary G-orbit, 8, 9, 112, 195, 196  
 stationary orbit, 7–9, 93, 95, 96, 114–117, 122, 124–130, 133, 142, 192–194  
 stationary solution, 6, 34, 36, 48, 68, 81, 123, 171, 173  
 stationary state, 2, 5, 16, 18, 19, 25, 31, 33, 44, 45, 48, 58, 59, 74, 76, 170  
 stochastic Lamb system, 15  
 Strauss, 3, 8, 9  
 Strichartz estimate, 179  
 strictly nonlinear equation, 8, 9, 91, 93  
 string, 14, 15, 23, 28, 29  
 strong Huygens principle, 6, 74, 79, 80, 84, 87, 88  
 strong interaction, 10, 113  
 Stuart, 1, 4, 10  
 Sulem, 10, 116  
 symmetry, 46, 129  
 symmetry group, 4, 9, 10, 111–113, 117, 128, 195  
 symmetry group of translations, 4, 6  
 symplectic form, 119, 128, 129, 134, 141  
 symplectic manifold, 119  
 symplectic projection, 116, 118, 120, 133, 134, 147  
 symplectic structure, 80, 82, 87  
 tangent space, 118, 119, 129, 133, 134  
 tempered distribution, 96, 97, 99, 100  
 threshold frequency, 109  
 time delation, 171, 173  
 Titchmarsh, 106  
 Titchmarsh convolution theorem,  $x$ , 1, 4, 8, 91, 94, 96, 106–108, 111  
 total momentum, 167  
 transition, 5, 25, 192  
 translation-invariant equation, 116, 118, 120, 196  
 transversal component, 11, 119, 120, 125, 142, 143  
 transversal dynamics, 142  
 transversal subspace, 119  
 trivial group, 112  
 trivial symmetry group, 4, 76  
 truncated resolvent, 181  
 Tsai, 10  
 U(1)-invariant oscillator, 121  
 ultraviolet divergence, 58

- unitary group, 112
- unitary symmetry group, 4, 7
  
- Vainberg, 1, 3, 4
- Van der Corput lemma, 186, 191
- variation, 33, 48
- variation equation, 33, 48
- Vinnichenko, 170
- von Neumann, 197
  
- wave equation, 2, 3, 6, 10, 13, 15, 46, 56, 58, 68, 78–80, 86, 91, 122, 123, 178, 179
- wave field, 108, 109
- wave mapping, 123
- wave packet, 173, 176
- wave–particle duality, 193, 198
- wave–particle system, 56, 77
- waveguide, 109
- weak compactness, 102
  
- weak convergence, 102
- weak coupling, 89
- weighted norm, 115, 121
- weighted Sobolev norm, 178
- Weinstein, 10, 114
- well-posedness, 3, 20, 32, 45, 46, 69, 70, 72, 123
- Wiener condition, 13, 59, 60, 64, 67, 76–78, 94, 118, 121
- Wiener Tauberian theorem, x, 1, 4, 13, 59, 64, 67, 77
- Wigner, 68
  
- Yafaev, 68
- Yau, 10
  
- Zeidler, 68
- Zeldovich, 68
- Zygmund lemma, 125, 135, 154