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Attractors of Hamiltonian Nonlinear Partial Differential Equations

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In memory of Mark Vishik

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Preface

We present the theory of attractors of nonlinear Hamiltonian partial differential equations in infinite space. This is a new branch of the theory of attractors of PDEs initiated by one of the authors in 1990. This theory differs significantly from the case of dissipative systems. In particular, this theory has no analog for finite-dimensional Hamiltonian equations.

This book is the first monographic publication in this direction. Included are results on global attraction to stationary states, to solitons, and to stationary orbits; results on adiabatic effective dynamics of solitons and their asymptotic stability; and results on dispersive decay for linear Hamiltonian PDEs. The obtained results are generalized in the formulation of a new mathematical conjecture on global attractors of G-invariant nonlinear Hamiltonian partial differential equations.

We also describe the results of numerical simulations.

In conclusion, we discuss possible relations of this theory with the problem of mathematical interpretation of Bohr's transitions between quantum stationary states. The book is intended for

1. graduate and postgraduate students working with partial differential equations;
2. lecturers on PDEs;
3. mathematicians working in PDEs, mathematical physics, and mathematical problems of quantum theory.

On the Required Knowledge

All proofs are self-contained, and their overwhelming parts rely on traditional methods of analysis: ODEs, general theory of Hilbert and Banach spaces, distributions and their Fourier transform, Sobolev spaces, and definitions of Lie groups and Lie algebra and of their representations.

The key points of the proofs rely on a novel application of subtle methods of harmonic analysis: the Wiener Tauberian theorem, the Titchmarsh theorem on convolution, the theory of multipliers in the space of quasimeasures, and others. The applications are explained in detail and with exact references to the corresponding textbooks.

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