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A Course in Stochastic Game Theory

EILON SOLAN
Tel-Aviv University



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The book is dedicated to the people who shared my life, my work, and my love of games in the last thirty years. To Abraham Neyman, who introduced me to game theory and to stochastic games; to Nicolas Vieille and Dinah Rosenberg, with whom I spent countless fun weeks of studying stochastic games; to Ehud Lehrer, who has been my colleague and partner at Tel Aviv University for the past 20 years; to my parents, Chaim and Zafrira; and to my two sons, Omri and Ron, who listened to game-theoretic problems since birth and eventually became coauthors.

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