Mechanics of Fluids

Providing a modern mathematical approach to classical fluid mechanics, this textbook presents an accessible and rigorous introduction to the field, with a strong emphasis on mathematical exposition.

It includes:

- A consistent treatment of a broad range of fluid mechanics topics, including vortical, potential, compressible, viscous, unstable, and turbulent flows
- Enhanced coverage of geometry, coordinate transformations, kinematics, thermodynamics, heat transfer, and nonlinear dynamics, to round out student understanding
- Robust emphasis on theoretical fundamentals and rigorous mathematical exposition, enabling students to gain confidence and develop a solid framework for further study
- 180 end-of-chapter problems, with full solutions and sample course syllabi available for instructors

With sufficient coverage for a one- or two-semester sequence, this textbook provides an ideal flexible teaching pathway for graduate students in aerospace, mechanical, chemical, and civil engineering, and applied mathematics.

Joseph M. Powers is a professor of Aerospace and Mechanical Engineering at the University of Notre Dame. His research uses computational science to elucidate the dynamics of high-speed reactive fluids as it applies to verification and validation of multiscale systems. He is the Editor-in-Chief of the *Journal of Propulsion and Power* and has previously published *Mathematical Methods in Engineering* (2015) and *Combustion Thermodynamics and Dynamics* (2016).

"An excellent first-level graduate textbook on fundamentals of fluid mechanics. By starting the book from the very basic vector notation, Professor Powers has made the book accessible to a large number of students who need to strengthen their mathematical background as well. This book will take the students all the way through rigorous understanding of hydrodynamic instabilities and turbulence. This is an excellent comprehensive book."

Bala Balachandar, *University of Florida*

"A rigorous mathematical treatise on the mechanics of fluids, in the spirit of Batchelor and Truesdell, something rarely seen today, and an exceptional counterpart to the many ad hoc books on this subject. For well-prepared students, this is a deeply technical introduction to this centrally important subject of physics and engineering."

Werner J. A. Dahm, *Arizona State University*

"A beautiful book on fluid mechanics: clear, insightful, comprehensive, rigorous, and detailed, with no stones unturned in derivations. This book will be a classic, one that I will often refer to when I need clarity and precision."

Tom Shih, *Purdue University*

"An enlightening 21st-century textbook in fluid mechanics which captures all the essence from the fundamentals of mechanics to the application of fluid dynamics. It comprehensively describes the intricate relationship between mathematics of statistical mechanics and physical observations of Newtonian fluids. This is a unique book which seamlessly relates 20th-century analytical mathematics-based fluid dynamics to 21st-century physics - and CFD-based fluid dynamics. I strongly recommend this book for an advanced undergraduate, or an introductory graduate-level, fluid dynamics course."

Chelakara S. Subramanian, *Florida Institute of Technology*

"This book achieves a rare combination of accessibility and mathematical rigor. It can provide a point of entry into contemporary fluid dynamics for the beginning graduate student while revealing fresh aspects of the subject to the seasoned researcher. I look forward to teaching from it."

William Eric Uspal, *University of Hawaiʻi at Mānoa*

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Preface

This book considers the mechanics of fluids with a focus on topics seen in entry level graduate courses in aerospace and mechanical engineering. As reflected in its title and structure, it is first a presentation of mechanics, followed by application to fluids. It provides a rigorous presentation of standard topics with a more complete exposition of underlying mathematical details than is typically found. In an era in which students perhaps too hastily turn to experimental or computational methods to understand fluid behavior, this book provides a detailed presentation of the underlying theoretical foundations in a fashion designed to provide missing links in analysis that students often need to gain confidence in their understanding. An additional reason to include details is so that results may be reproduced by others, an important feature that should distinguish deterministic science from less quantifiable disciplines. The bulk of each chapter has a standard presentation; however, many chapters are augmented with material not always found in common texts. Some topics that receive enhanced emphasis include geometry, coordinate transformations, kinematics, thermodynamics, heat transfer, and nonlinear dynamics.

The book is built on lecture notes for two courses developed over three decades in the Department of Aerospace and Mechanical Engineering of the University of Notre Dame. The first is AME 60635, Intermediate Fluid Mechanics, and the second is AME 70731, Viscous Flow Theory. Additional topics have been included, and so there should be more than enough material available for a two-course sequence in introductory graduate fluid mechanics. Many of the later chapters are self-contained, and some can be omitted to suit instructor needs. The notes from which this book was drawn were themselves initially guided by the fine text of Panton (2013);[1] the reader will notice some similarities in choice of notation and ordering of a few topics. The influence of many other expositions on this subject, listed in an extensive bibliography, is evident as well.

The book is directed towards beginning graduate students and advanced engineering undergraduates. They have typically completed at least one undergraduate fluids course as well as courses in thermodynamics, linear algebra, vector calculus, and differential equations. Additionally, they have experience with basic numerical methods and modern software tools for solving such problems as root-finding, matrix inversion, determination of eigenvalues and eigenvectors, integration of nonlinear systems of ordinary differential equations, and some partial differential equations. A basic knowledge of such material is both necessary and sufficient preparation for the topics presented here.

[1] Panton, R. L. (2013). *Incompressible Flow*, 4th ed. New York: John Wiley.

Most of these students will later specialize in either experimental or computational fluid mechanics and will take additional specialized course work to these ends. All can benefit from a thorough preparation in theoretical foundations. As such, this book goes farther than most to bolster the connection between basic mathematical concepts relevant to the mechanics of fluids in an engineering context. While the material is relevant to experimental and computational fluid mechanics, it has minimal discussion either of underlying experiments or computations. Both of these vast subjects are well treated in other texts. That said, most of the topics developed here have clear relevance to problems in nature; however, in a few portions of the text, some unusual limits are considered to allow a more digestible exposition. These may involve specially prescribed flow fields, limits in which some physics are neglected, or limits in which competing mechanisms are unusually scaled so as to be in balance. This is often manifested by posing problems in which fluid properties such as density or viscosity are either zero or unity. A special emphasis is placed on problems that bolster the students' often tenuous confidence in using vector calculus, which is the most efficient language to describe the mechanics of fluids.

The book provides a survey of continuum fluid mechanics. Part I gives an extensive development of the compressible Navier–Stokes equations. It includes a review and exposition of the essential mathematics of differential geometry, kinematics, evolution axioms, constitutive equations, and a delineation of many special limits of the governing equations. Part II focuses on their solution in various limits: vortical, potential, compressible, viscous, unstable, chaotic, and turbulent flows. Most chapters contain example problems; some are mathematically motivated, and others are focused on quantitative problems involving fluid physics. A course that is more oriented towards development of the equations of continuum fluid mechanics may concentrate on Part I. Alternatively, one could give the briefest of introductions to governing equations and move straight to any of the chapters of Part II, which generally are self-supporting.

The expansiveness of these topics is such that many of them are only lightly treated; the reader should turn to more specialized books for additional detail. The emphasis here is on fluid physics and the mathematics necessary to efficiently describe the physics. Each chapter is concluded with exercises appropriate for homework. A detailed solution manual is available for instructors. Some of the problems require numerical methods that are routine for modern and widely available software tools; background for such tools and methods may be found in other sources. Specific hallmarks found throughout the book include (1) consideration of complete thermo-fluid systems so as to enable determination of velocity and temperature fields in compressible, viscous flows, (2) attention to the formalities of coordinate and similarity transformations, and (3) presentation of much classical material as well as a few novel or neglected topics selected to illustrate analysis of fluid physics. Some important topics are considered only briefly, for example (1) non-Newtonian flow, (2) stability, and (3) turbulence. The bulk of the book is devoted to fluids problems that are well described by a set of deterministic model equations with solutions that are well-behaved and amenable to causal inference. In a few instances, these nonlinear deterministic equations are pushed into regimes where the solutions acquire a more chaotic and random behavior. But we do not explicitly model the stochastic nature of fluid behavior or the effect of stochastic uncertainties in constitutive models; nor do we consider non-axiomatic frameworks, such as given by some data-driven modeling approaches that employ machine learning and artificial neural networks. While some computational results are presented, there is no treatment of the methods of computational fluid dynamics. The book provides the foundation for later courses that address these and several more advanced topics that are not typically considered in introductory graduate fluids courses.

The book is intended to be used as an instructional tool for students and advanced professionals who need to understand the fundamental mechanics of fluids. While in places, it gives a modern treatment of old subjects drawing upon recent scholarship, most of its topic matter is correctly described as "classical." As such, the extensive bibliography focuses on scholarly books, both historical and modern; for selected specialized topics, the underlying journal literature is drawn upon.

The author's gratitude is due to many, including former teachers, colleagues who commented on various drafts, inspiring family members, as well as the University of Notre Dame for providing support to bring this work to completion. Of those many, I am especially grateful to my talented colleague at Notre Dame, Prof. Jonathan F. MacArt, who carefully reviewed and contributed to Chapter 14 and skillfully performed the numerical simulations of laminar and turbulent Rayleigh–Bénard convective flow that both illuminate the cover and close the book. That said, my deepest appreciation is given to those who motivated me to prepare this book: the dozens of undergraduate and graduate students of AME 60635 and 70731 who have pushed me for excellence as I have pushed them. Many have gone on to careers of distinction in fluid mechanics, and for what small part I played in that, I am proud. This book is written with the hope that it can reach others who will be similarly motivated to learn and apply this beautiful science.