Part I

Money
Chapter 1
The Economy and the Planner’s Solution

Roadmap
In this chapter, our goal is to develop a model of the economy and then show how a make-believe, benevolent planner would choose the allocation. In other words, the planner would maximize the lifetime utility of all people living two periods subject to the resource constraint.

A model description has four main parts: (i) there is a description of the physical environment, which consists of things like how long the economy lasts, who lives there, how long each person lives, what goods are present, and what rules apply to when people can meet and trade; (ii) there is a description of how people get goods, such as things they are endowed with over their lifetime or the ways in which they can produce things; (iii) we need to know what kinds of goods people like and be able to compare different bundles of goods; and (iv) when we get to analyzing problems in which people act in their own self-interest, we need a way to combine the different actions that people want to take so that the quantity supplied is equal to quantity demanded. With all four pieces together, we have a model economy.

We begin by skipping the fourth part. Rather, we set up a planner who collects goods and redistributes them so that people are as well off as possible. The planner’s allocation is efficient, which means this is the best possible outcome that the economy could achieve. Because the planner is doing all the work, there are no trades to complicate things. To make things as easy as possible, the planner cares about the welfare of all people born in the first period of the economy.

The planner’s allocation is useful because it gives a reference point by which to compare the equilibrium outcomes that can be achieved when we do allow people to meet and execute mutually beneficial trades. At the end of the chapter, the student should understand the basic features of the model economy and why the planner’s allocation is efficient.
1 The Economy and the Planner’s Solution

1.1 Beginnings

In this book we will try to learn about monetary economies through the construction of a series of model economies that replicate essential features of actual monetary economies. All such models are simplifications of the complex economic reality in which we live. Model economies are descriptions of the physical environment in which people live, a description of the technology that produces goods and services, what people are endowed with by nature, and a description of what people want. Armed with these inputs, we use a planner to implement the quantities of goods that people will consume.

The first three chapters are essential to our explanation of trade. Chapter 1 gets the basics of the model economy and solves for quantities by allowing an omnipotent, omniscient planner to choose who gets what. Chapter 2 introduces trade, but with an accounting system – or blockchain – that keeps track of each trade a person undertakes. Chapter 3 gets rid of the accounting system and replaces it with money. What binds these three chapters together is the notion of efficiency. In other words, we learn the efficient allocation that the planner chooses. Then we ask whether the trades in an economy with the accounting system can implement the same quantities with the added feature of a pricing system. Then we get rid of the accounting system to see if money can implement the efficient allocation, solving for the price level.

Model economies are useful because they are able to illustrate key elements of the behavior of people who choose to trade. Value can be stored by letting the accounting system virtually store your choices or letting money tangibly record your choices. By mastering the basics, we can begin to see what accounts for things that we observe and even make predictions, based on the model economy, regarding reactions of important economic variables such as output, prices, government revenue, and public welfare to changes in policies that involve money. It will be clear that model economies are simplifications that throw out much of the rich set of choices that people can make. The model economy is not meant to be reality. One of the book’s primary goals is to match the observation that people value colored pieces of paper. Throughout the book we try to make our analysis more sophisticated by adding, one by one, important features to the model economy that are ignored in the first three chapters.

We concentrate on the overlapping generations. This model, introduced by Paul Samuelson (1958), has been applied to the study of a large number of topics in monetary theory and macroeconomic theory. Among its desirable features are the following:

- Overlapping generations models are easy to solve. Although they can be used to analyze quite complex issues, there are equilibria that are easy to characterize and to find. Many of their predictions may be described on a simple two-dimensional graph.
1.2 The Environment

- Overlapping generations models provide an elegantly parsimonious framework in which to introduce the existence of money. Money in overlapping generations models dramatically facilitates exchange between people who otherwise would be unable to trade.
- Overlapping generations models are dynamic. They demonstrate how behavior in the present can be affected by anticipated future events. They stand in marked contrast to static models, which assume that only current events affect behavior.

We begin this chapter with a very simple version of an overlapping generations model. As we proceed through the book, we introduce extensions to this basic model. These extensions allow us to analyze a variety of interesting issues.

Other model economies share the same three characteristics we identified in the bullet points. Our aim is not to be all encompassing and cover all of these alternatives. Rather, our approach is more topic driven. After building the basic framework, the extensions we introduce are tied to questions. By focusing on the overlapping generations model, we are able to utilize its flexibility. Over time, other model economies with the same three characteristics will likely exhibit the same flexibility, and coverage of the same broad set of topics will be made available.

Therefore, let us turn to the development of the basic overlapping generations model.

1.2 The Environment

You will quickly see why we call this an overlapping generations economy. Time is divided into equal-sized bits, which we will call periods. For simplicity, we can always call the starting period 1, the next period 2, and so on. When needed, we use the notation \( t \) to stand for the time period.

People in this economy, however, do not live forever. Indeed, they live for two periods. Anyone born in period \( t = 1 \) lives in period 1 and period 2, a person born in period \( t = 2 \) lives in periods 2 and 3, and so on. Or, put another way: the number of old people in any date \( t \) is the same as the number of young people born at date \( t - 1 \). Generally speaking, anyone born in period \( t \geq 1 \), is “young” in period \( t \) and “old” in period \( t + 1 \). In each period \( t \geq 1 \), \( N_t \) people are born. Note that we index time with a subscript. For example, in period \( t = 2 \), \( N_2 \) is our notation for the number of people born in period 2. The people born in periods \( t = 1, 2, 3, \ldots \) are called the “future generations” of the economy. In addition, in period 1, there are \( N_0 \) people that live for just one period. These people are called the “initial old.”

Next, we describe the population living in each period. In each period \( t \geq 1 \), there are \( N_t \) young people who were just born and there are \( N_{t-1} \) old people. It is the fact that two generations coexist that gives rise to the name overlapping generations. For example, in period \( t \), there are \( N_{t-1} \) old people and \( N_t \) young people living. Two generations always overlap with each other every period.
For simplicity, there is only one good in this economy. The good is perishable, meaning that it cannot be stored from one period to the next. In this basic setup, each person receives an endowment of the consumption good when young in the first period of life. The amount of this endowment is denoted as $y$. When old, no one receives any quantity of the consumption good. This pattern of endowments is illustrated in Table 1.1.

Of course you might think that this simplification is way too costly to help us understand the wide variety of goods and services available in today’s economy. But it is also easy to see that if you want to include work effort in this economy, it is easy. Suppose people are endowed with one unit of work time when young. Let people use that time productively, using a technology to transform effort into units of the consumption good. Now, we can interpret the endowment as an endowment of labor – the ability to work. By using this labor endowment (by working), each person is able to obtain a real income of $y$ units of the consumption good. To be even more concrete, consider an economy in which the only consumption good is coconuts. Each young person is capable of climbing the coconut tree and harvesting the edible nut. Old people, however, cannot climb the tree. You could imagine that young people would harvest nuts, storing the harvest in their hut. Unfortunately, coconuts are perishable, going bad before an old person can eat stored nuts.

### 1.3 Preferences

People consume the economy’s sole commodity and obtain satisfaction, or, in the economist’s jargon, from having done so.

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Table 1.1 The pattern of endowments.

<table>
<thead>
<tr>
<th>Generation</th>
<th>Period</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
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<td>Future generations</td>
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</tbody>
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*Notes:* The pattern of endowments. In each period $t$, generation $t$ is born. Each person lives for two periods. People are endowed with $y$ units of the consumption good when young and 0 units when old. In any given period, one generation of young people and one generation of old people are alive. The name of this model, the overlapping generations model, follows from this generational structure.
1.3 Preferences

1.3.1 Future Generations

Members of future generations in an overlapping generations model consume both when young and when old. Each person’s utility therefore depends on the combination, or bundle, of personal consumption when young and when old. We make the following four assumptions about a person’s preferences regarding consumption. The first two assumptions allow you to see that we can assign a consistent numerical value to a bundle of consumption. The second pair of assumptions helps us to draw a picture of a young person’s preferences over consumption when young and consumption when old. Based on the consistent numerical value associated with each bundle, the picture is a great device for characterizing the solution to each person’s lifetime decision problem.

It will be useful to have some notation. We denote the amount of the good that is consumed in the first period of life by a person born in period $t$ with the notation $c_{1,t}$. Similarly, $c_{2,t+1}$ denotes the amount the same person consumes in the second period of life. It is important to note that $c_{2,t+1}$ is consumption that actually occurs in period $t+1$, when the person born at time $t$ is old. When the time period is not crucial to the discussion, we denote first- and second-period consumption as $c_1$ and $c_2$. Let $(c_{a1},c_{a2})$ stand for a bundle of lifetime consumption referred to as Bundle A. Similarly, let $(c_{b1},c_{b2})$ stand for a bundle of lifetime consumption referred to as Bundle B.

**Assumption 1: Preferences are complete.** When facing two bundles, a person can provide a valid response to two statements. A valid response is either true or false. The two statements are: (1) I get at least as much happiness from Bundle A as I get from Bundle B and (2) I get at least as much happiness from Bundle B as I get from Bundle A.

What do these two true/false answers tell us? If a person says Statement 1 is true and Statement 2 is false, then I can tell that this person gets more happiness from Bundle A than from Bundle B. If the person says Statement 1 is false and Statement 2 is true, then I know that person gets more happiness from Bundle B than from Bundle A. If the person says Statement 1 and Statement 2 are both true, then I know that the person gets the same level of happiness from Bundle A and Bundle B. Thus, Assumption 1 offers a complete description of the happiness obtained from any two bundles. There are three options: Bundle A is preferred to Bundle B, Bundle B is preferred to Bundle A, or the person is indifferent between Bundle A and Bundle B.

**Assumption 2: Preferences are transitive.** To illustrate this assumption, I create a third bundle. Let Bundle D be $(c_{d1},c_{d2})$. Transitivity is just an assumption to guarantee consistency. We ask a person to provide valid responses to Statements 1 and 2 for Bundles A, B, and D. Suppose that Bundle A is preferred to Bundle B. Furthermore, suppose Bundle B is preferred to Bundle D. We can ensure that nothing screwy happens insofar as a person satisfying Assumption 2 will prefer Bundle A to Bundle D.
Armed with Assumptions 1 and 2, we can define a relationship that assigns a numerical value to each bundle and that numerical value is consistent with the preference ranking obtained from valid responses to Statements 1 and 2. In other words, if Bundle A is preferred to Bundle B, the numerical value assigned to Bundle A – that is, its utility – is greater than the numerical value assigned to Bundle B. If a person is indifferent between Bundle A and Bundle B, for example, the numerical values assigned to each bundle must be equal. The relationship that assigns a numerical value to bundle is called a utility function.

**Assumption 3: More is preferred to less.** Suppose Bundle A and Bundle B are constructed so that $c_a^1 = c_b^1$ and $c_a^2 > c_b^2$. This person is comparing bundles with the same quantity of consumption when young, but when old, Bundle A gives a greater amount of consumption than does Bundle B. According to Assumption 3, this person will always prefer Bundle A to Bundle B.

**Assumption 4: Diminishing marginal utility.** The purpose of this assumption is to put some curvature into the relationship between bundles. You will know why this is so useful after everything is put together. The simple overview of Assumptions 3 and 4 is that each extra unit you get makes you happier, but extra happiness is getting smaller and smaller with each extra unit. Figure 1.1 graphically depicts the meaning of Assumptions 3 and 4. Figure 1.1 plots the utility value of each extra morsel of consumption when young. Hopefully, from your previous economics classes you remember that marginal utility is defined as the difference between the utility you receive from consuming two different quantities, holding everything else constant. For example, suppose you hold the quantity of consumption when old fixed – call it $\bar{c}_2$; then the marginal utility is the difference in utility value associated with consuming $c_a^1$ and that associated with consuming $c_b^1$, where $c_a^1 > c_b^1$.

![Figure 1.1](image-url)
1.3 Preferences

Figure 1.1 portrays two of the assumptions that we make in order to characterize a person’s preferences. First, there is a positive relationship between the quantity consumed by a young person and utility. In other words, the slope of the utility curve is positive, showing that an increase in consumption when young results in greater utility; more is preferred to less. Second, the slope is getting flatter and flatter as the quantity of $c_1$ increases. Diminishing marginal utility assumes that the marginal utility gain is decreasing as the quantity of the good increases.

With Assumptions 1 through 4, we are able to assign a numerical value to every bundle. The utility function is the mathematical representation of a person’s preferences over all the bundles. It will be extremely useful to portray a person’s preferences graphically. We do this by introducing an indifference curve. An indifference curve connects all the consumption bundles such that there is equal utility. In other words, our young person is saying that for every point on the indifference curve, she responds true to Statements 1 and 2. Figure 1.2 displays a typical indifference curve. Along any particular indifference curve, utility is constant. Here, the person is indifferent between bundles A, B, and C.

To illustrate the indifference curve, suppose we offer a person the following consumption choices:

- Bundle A consists of three units of the consumption good when a person is young and six units of the consumption good when a person is old. We denote this bundle as $c_1 = 3$ and $c_2 = 6$.
- Bundle B consists of five units of the consumption good when a person is young and four units of the consumption good when a person is old ($c_1 = 5$ and $c_2 = 4$).

By Assumptions 1 through 4, this person has assigned a numerical value to these bundles by the utility function. On this indifference curve, we show the two points A and B. We also illustrate a third point, Bundle C, representing the bundle $c_1 = 11$. 

![Indifference Curve](image)
and $c_2 = 2$. Because C lies on the same indifference curve as points A and B, point C yields the same level of utility as points A and B for the person.

Note some features of the indifference curve. The first is that the curve becomes flatter as we move from left to right. This is how indifference curves represent Assumption 4. Note that the slope of the indifference curve is called the marginal rate of substitution. Hence, the curvature of the indifference curve is called the “assumption of diminishing marginal rate of substitution” and diminishing marginal utility can explain this property. To illustrate this assumption, start at point A, where $c_1 = 3$ and $c_2 = 6$. Suppose we reduce the person’s second-period consumption by two units. The indifference curve tells us that, to keep the person’s utility constant, we must compensate him or her by providing two more units of first-period consumption. This places the person at point B on the indifference curve.

Now suppose we reduce second-period consumption by another two units. Our person will remain indifferent if six more units of first-period consumption are provided. In other words, we must compensate a person with ever-increasing amounts of first-period consumption as we successively cut second-period consumption. This should make intuitive sense; people are more reluctant to give up something when they only have a little.

Consider food and clothing as an example. A person who has a large amount of clothing and very little food would be willing to give up a fairly large amount of clothing for another unit of food. Conversely, this person would be willing to give up only a small amount of food to obtain another unit of clothing.

We demonstrate this assumption of diminishing marginal rate of substitution by drawing an indifference curve that becomes flatter as we move downward and to the right along the curve. We also assume that the indifference curves become infinitely steep as we approach the vertical axis and perfectly flat as we approach the horizontal axis. The curves never cross either axis. This might be justified by saying that consuming nothing in any one period would mean horrible starvation, to which consuming even a small amount is preferable. This is Assumption 3.

It is also important to keep in mind that the indifference curves are dense in the $(c_1, c_2)$ space. This means that if you pick a combination of first- and second-period consumption, there is an indifference curve running through that point. However, to avoid clutter, we normally show only a few of these indifference curves. A group of indifference curves shown on one graph is often called an “indifference map.” Figure 1.3 illustrates an indifference map that obeys our assumptions. Formally, an indifference map consists of a collection of indifference curves. In Figure 1.3, we consider an example in which there is a constant quantity of consumption when old. At this fixed level of old-age consumption, a person prefers a larger quantity of consumption when young. Bundle C is preferred to Bundle B and Bundle B is preferred to Bundle A. Utility increases in the direction of the arrow; indifference curves farther from the origin correspond to higher utility.
1.3 Preferences

Direction of Increasing utility

ABC

c1
c2
U0
U1
U2

Figure 1.3 An indifference map.

Note that utility is increasing in the direction of the arrow. How do we know this? Compare points A, B, and C. Each of these bundles gives the person the same amount of second-period consumption. However, moving from point A to B to C, the person receives more and more first-period consumption. Hence, the person will prefer point B to point A. Likewise, point C will be preferable to points A and B. This is Assumption 2.

It is often useful to draw an analogy between an indifference map and a contour map that shows elevation. On an indifference map, the curves represent points of constant utility; on a contour map, the curves represent points of constant elevation. Extending the analogy, if we think of traversing the indifference map in a northeasterly direction, we would be going uphill. In other words, utility would be increasing. In fact, an indifference map, like a contour map, is merely a handy way to illustrate a three-dimensional concept on a two-dimensional drawing. The three dimensions here are first-period consumption, second-period consumption, and utility.

One other important concept is that our person’s rankings of preferences are transitive. If a person prefers bundle B to bundle A and bundle C to bundle B, then that person must also prefer bundle C to bundle A. Graphically, this implies that indifference curves cannot cross. To do so would violate this property of transitivity and Assumption 2 (see Figure 1.4). This figure portrays two indifference curves that cross at point A. We know that indifference curves represent bundles that give a person the same level of utility. In other words, the person whose preferences are represented by Figure 1.4 is indifferent between Bundles A and B because they