

## Index

- Banach manifold, 24
- Bastiani calculus, 7, 203
- Boman's theorem, 202
- bornology, 202
- bump function, 195
- Butcher group, 185
- Butcher–Connes–Kreimer algebra, 182
  
- canonical manifold of mappings, 41, 219
- carrier of a function, 195
- chain rule, 11, 19, 21
- change of charts, 16
- character group of a Hopf algebra, 178
- character of an algebra, 177
- Chen's relation, 160, 167, 168
- Christoffel symbol, 91
- CIA
  - see* continuous inverse algebra, 49
- compact open  $C^\infty$ -topology, 4, 30, 192, 198, 221
- compact open topology, 13, 208
- continuous inverse algebra, 49
  - unit group, 49
- continuous projection, 194
- convenient calculus, 202
- convenient smooth map, 202
- coordinate projection, 11
- cotangent bundle, 21, 81
- current group, 72
- curvature, 96, 100
  - Bianchi identity, 101
  - sectional, 151
- curve
  - $c$ -parallel, 99
  - energy of  $a$ , 84, 222
  - length of  $a$ , 84
  - passing through a point, 19
  - piecewise  $C^1$ , 84
  - principal part of  $a$ , 99
  - smooth, 2
- derivation of an algebra, 227, 230
- derivative
  - covariant, 94
  - covariant, along a map, 109
  - directional, 7
  - iterated directional, 8
  - left logarithmic, 62, 238
  - Lie derivative, 227, 236
  - metric derivative, 97
  - of a manifold-valued curve, 22
- diffeomorphism group, 35, 50, 120
  - acting  $n$ -transitive, 121
  - canonical action, ix, 51
  - contactomorphisms, 52
  - stabiliser of a point, 121
  - symplectomorphisms, 52
  - volume-preserving, 52, 144, 242
- differential form, 231, 240
  - (locally) integrable, 237
  - Lie derivative, 236
  - preserved by a diffeomorphism, 52
  - pullback, 235
  - volume form, 241
- dual space, 4, 83, 193
  
- embedding, 23
  - set of, 34
- equation of Lie type, 60
- Euler equations
  - Eulerian form, 143
  - Lagrangian form, 145

- Euler–Arnold equation, 152
- Euler–Poincaré equation, 149
- evaluation map, 39, 210
- evolution map, 62
- exponential law, 39, 211
- exterior differential, 232
  
- fibre product (of manifolds), 27
- final topology, 206
- flip of the double tangent bundle, 215
- formal power series, 183
- Fréchet calculus, 14
- Fréchet space, 3, 198
- Fundamental theorem of calculus, 5
  
- geodesic, 88, 100, 139
  - of a spray, 92
  - of the  $L^2$ -metric, 112
- geodesic distance, 85
  - non-vanishing, 86
- geodesic equation, 92, 99
- gradient, 143, 242
- Grossman’s ellipsoid, 103
- group of bisections, 126
  - vertical bisections, 130
- group of gauge transformations, 77
  
- Hölder continuity, 169
- Hahn–Banach theorem, 4
- half-Lie group, 155
- Hilbert sphere, 16, 83
- homogeneous space, 77, 183
- Hopf algebra, 177
- Hopf–Rinow theorem, 102
- Hunter–Saxton equation, 153
  
- immersion, 23
  - naïve, 24
  - set of, 33
- infinitesimally injective (or surjective), 24
- initial topology, 206
  
- Kolmogorov’s normability criterion, 192
  
- Lie algebra, 55
  - associated to a Lie group, 57
  - current algebra, 72
  - Jacobi identity, 55
  - locally convex, 56
  - of (left-)invariant vector fields, 56
  - of divergence-free vector fields, 243
  - of vector fields, 228
- Lie bracket, 55
  - of vector fields, 56
  - trivial, 56
- Lie group, 48
  - Bourbaki construction principle, 70
  - exponential, 65
  - exponential coordinates, 66
  - left- (right-)translation, 48
  - Lie subgroup, 52
  - locally exponential, 66
  - regular (in the sense of Milnor), 62
  - semidirect product, 54
  - semiregular, 62
  - split exact sequence, 54
  - tangent Lie group, 53
- Lie group action, 51
  - adjoint action, 60
- Lie groupoid, 125
  - action groupoid, 127
  - bisection action groupoid, 134
  - current groupoid, 128
  - enough bisections, 135
  - gauge groupoid, 128
  - pair groupoid, 127
  - tangent groupoid, 132
  - unit groupoid, 127
  - vertex group, 125
- Lie polynomial, 164
- Lie series, 164
- Lie theorems, 60
- local addition, 43, 213
  - normalised, 218
- local-to-global argument, 196
- loop group, 74
- Lyons lift, 170
- Lyons’ lifting theorem, 170
  
- Mackey complete, 5, 187, 202
- manifold atlas, 16
- manifold chart, 16
- Maurer–Cartan form, 237
- mean value theorem, 9
- Minkowski functional, 83, 192
- multiplicative functional, 168, 179
  
- Neumann inversion formula, 161
  
- Omori’s theorem, ix, 155
- orbifold, 124, 127
  
- partition of unity, 195
- pre-shape space, 114

principal bundle, 76, 129  
   gauge group, 76  
   structure group, 76  
 pro-Lie group, 166  
 product manifold, 17  
 pullback, 31, 45, 209  
 pushforward, 13, 31, 43, 209

Riemann–Stieltjes sum, 157, 158  
 Riemannian metric  
    $H^1$ -metric, 113  
    $\dot{H}^1$ -semimetric, 153  
    $L^2$ -metric, 81, 108, 140  
   co-orthogonal structure, 102  
   elastic metric, 116  
   invariant  $L^2$ -metric, 83,  
     88, 89, 144  
   pullback metric, 116  
   right/left invariant, 87  
   robust, 101  
   strong, 81  
   weak, 80

rough norm, 197  
 rough path, 170  
   branched, 181, 183  
   Brownian motion, 171  
   set of weakly geometric, 170  
   weakly geometric, 170

rule on partial differentials, 9, 22

Schwartz' theorem, 8, 12  
 seminorm, 3  
   basis condition, 191  
   fundamental system of, 191  
   generating family of, 3  
   separating family of, 190

shape analysis, 106  
 shape space, 114  
 shuffle algebra, 176, 183  
 signature of a smooth path, 162  
 smooth variation, 138  
   right-shifted, 146

space  
    $C^k$ -paracompact, 196  
    $C^k$ -regular, 196  
   complemented subspace, 194  
   convenient vector space, 202  
   locally convex, 3, 192  
   metrisable, 3  
   of Hölder continuous  
     functions, 169  
   of smooth  $p$ -forms, 232

  sequentially closed, 12  
   short exact sequence, 25  
   topological vector space, 1, 186

spray, 91  
   associated covariant derivative, 95  
   connector, 96  
   metric, 92  
   metric spray, 97

square root velocity transform, 116  
 SRVT  
   *see* square root velocity  
     transform, 116

Stacey–Roberts Lemma, 45  
 standard simplex ( $\Delta$ ), 168  
 submanifold, 17  
   split, 17  
 submersion, 23  
   naïve, 24  
   set of, 33

support of a function, 195

tangent bundle, 19  
 tangent map, 21  
 tangent space, 19  
 tangent vector (geometric), 19  
 tensor algebra, 160  
 topology of compact  
   convergence, 208

transversal, 26

unit group (of an  
   algebra), 49

vector bundle, 215  
   direct product, 217  
   dual bundle, 21, 81  
   pullback, 216  
   set of all sections, 215  
   smooth section, 215  
   Whitney sum, 217, 220

vector bundle morphism, 215  
 vector field, 225  
   (left) invariant, 56, 64  
   along a smooth map, 109  
   complete, 64  
   divergence, 143, 242  
   flow, 226  
   Helmholtz decomposition, 242  
   integral curve, 226  
   Lie bracket, 226  
   local representative, 225  
   principal part, 225

*Index*

267

- related, 226
- second-order, 91
- vector space of, 225
- vector topology, 1
  - generated by seminorms, 3
- Volterra series, 63, 162
- weak integral, 4
- wedge product, 234
- Young integral, 158
- zero-section, 43, 213, 220