

Mathematics for Future Computing and Communications

For 80 years, mathematics has driven fundamental innovation in computing and communications. This timely book provides a panorama of some recent ideas in mathematics and how they will drive continued innovation in computing, communications and AI in the coming years. It provides a unique insight into how the new techniques that are being developed can be used to provide theoretical foundations for technological progress, just as mathematics was used in earlier times by Turing, von Neumann, Shannon and others.

Edited by leading researchers in the field, chapters cover the application of new mathematics in computer architecture, software verification, quantum computing, compressed sensing, networking, Bayesian inference, machine learning, reinforcement learning and many other areas.

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Cambridge University Press

978-1-316-51358-3 — Mathematics for Future Computing and Communications

Edited by Liao Heng , Bill McColl

Frontmatter

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Mathematics for Future Computing and Communications

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CAMBRIDGE
UNIVERSITY PRESS

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CAMBRIDGE UNIVERSITY PRESS

University Printing House, Cambridge CB2 8BS, United Kingdom

One Liberty Plaza, 20th Floor, New York, NY 10006, USA

477 Williamstown Road, Port Melbourne, VIC 3207, Australia

314–321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi – 110025, India

103 Penang Road, #05–06/07, Visioncrest Commercial, Singapore 238467

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www.cambridge.org

Information on this title: www.cambridge.org/9781316513583

DOI: 10.1017/9781009070218

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First published 2022

Printed in the United Kingdom by TJ Books Limited, Padstow Cornwall

A catalogue record for this publication is available from the British Library.

ISBN 978-1-316-51358-3 Hardback

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Preface

LIAO Heng

The vitality of the computing and communications industry is remarkable. Accomplished experts occasionally fall into the trap of thinking that all major inventions in their field have been made, and that what remains to be discovered are mere refinements to known methods. Advances in the field of computing and communications have proven experts wrong time and time again. Current notable examples include 5G communication networks and AI computing technologies. Such major technological advances are deeply rooted in mathematical science.

How can we ensure the industry preserves the vitality necessary to generate such advances? With this sizeable challenge in mind, we assembled mathematical scientists and engineers from the computing and communications industry to contribute to this volume. The authors are devoted to providing answers to the question presented above. The chapters are intended to provide insights from a variety of different perspectives.

Mathematics is a beautifully self-coherent body of interconnected concepts, but it was not developed in isolation – it has been accompanied all the way by natural science, especially physics. Bertrand Russell once said, “Physics is mathematical not because we know so much about the physical world, but because we know so little; it is only its mathematical properties that we can discover”. Mathematics has been regarded as the universal language for natural science, and likewise, physics has been described as a rich source of inspiration and insight in mathematics. Cross-disciplinary work has led to some of the greatest discoveries of all time. For example, Newton’s pursuit of classical mechanics resulted in the invention of calculus. David Hilbert, best known for setting much of the agenda for twentieth-century mathematics with his famous 23 problems, defined the differential equations of gravity that gave mathematical formulation to Einstein’s theory of general relativity. Eugene Wigner went so far as to describe the intimacy between mathematics and physics as “a miracle”, and his experiences consistently proved him right. Leading research institutes around the world have achieved great success in both mathematics and physics by promoting intimate exchanges across the two fields of study.

In contrast, the relationship between modern computing, communications and mathematics has been slightly less direct. The history can be traced back to Turing, von Neumann and Shannon, the three mathematicians who have come to be seen as the founders of computing and communications. Their work followed a familiar pattern: first, being drawn to a practical challenge (such as sending information across a noisy channel, or performing complex computational calculations) with a particular set of tools available (such as a transistor capable of processing bits); second, formulating the challenge as a mathematical problem; and last, developing mathematical solutions to prove that the theory addresses practical difficulties. In more recent times, Hinton’s work on backpropagation generated new excitement in the study of AI and neural network processing quickly became a major focus of the computing industry. Can the drive to obtain

knowledge from massive amounts of data, to simulate complex phenomena accurately, dealing with intrinsic uncertainty, and communicating at the semantic level – rather than at the bit level – become the inspiration for a new generation of mathematicians? We believe this volume will persuade many others to build fruitful relationships between computing, communications and mathematics.

Many industry visionaries have long realized that the key to business success is to convert scientific methods into technologies, subsequently applying them to product design through the process of research and development (R&D). The bulk of research is often conducted well before the R&D process. Such a realization has led to confusion among leaders about how to support research in mathematical science. We have produced this volume to assuage this confusion by addressing the major challenges we face in the industry, and in doing so, to open up mathematical problems to more fruitful cross-disciplinary discussion. This will help strengthen the confidence and commitment from industry leaders to provide sustained support for mathematical studies, and to direct resources in the most effective directions.

Researchers and developers in the communications and computing industries are mostly trained in highly specialized domains. They often lack an up-to-date knowledge and awareness of mathematical science beyond their own niche. As a result, many opportunities for applying new mathematical methods to solve problems in engineering are lost, simply because they are developed in other fields. Many engineers also lack the training to be able to convert engineering problems into mathematical models, and so must seek help from mathematicians. This volume aims to serve as a map for engineers, to help them navigate the boundary between engineering and mathematics.

Many areas of mathematical science are highly practical. Although some mathematicians primarily focus on proving theorems, others create and apply models to solve real-life problems. It is not uncommon for mathematicians to underestimate the impact of their work. Equally, many mathematicians are not fully aware of the key mathematical problems in any given applied domain. One major purpose of this volume, therefore, is to highlight the main mathematical problems currently being tackled within the computing and communications industries, and to encourage more mathematicians to direct their efforts towards solving them. It is hoped that interdisciplinary research can be promoted to maximize both its academic impact and the benefits it brings for society.

It must be stressed that the role of advancing fundamental research in computing and communications cannot be undertaken by one organization alone. The volume is intended as a beacon to generate a new wave of excitement among the international research community. Ideally, it will motivate policymakers and university executives to fund research and build education programs with a clearer purpose. We hope it will inspire a new generation of young mathematicians to join this grand effort.

The chapter authors are drawn from a very broad group of researchers, from academics working on core areas of mathematics to experts who have made outstanding contributions to the foundation of modern communication networks and advanced computing devices. We greatly appreciate and sincerely thank the contributors for their capacity to envision a new era of mathematical science that will pave the way for the creation of new machines that can perceive, learn, communicate, think and create.