

## Index

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Terms used in solutions are usually listed only if they are not used in the respective problem statements. Page numbers in bold indicate definitions.

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