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# Large Deviations for Markov Chains

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*To Martha, Alejandro, and Diego*

# Contents

# Preface

The purpose of this book is to study the large deviations for empirical measures and vector-valued additive functionals of Markov chains with general state space.

Under suitable recurrence conditions, the ergodic theorem for additive functionals of a Markov chain asserts the almost sure convergence of the averages of a real or vector-valued function of the chain to the mean of the function with respect to the invariant measure. In the case of empirical measures, the ergodic theorem states the almost sure convergence in a suitable sense to the invariant measure. The large deviation theorems provide precise asymptotic estimates at logarithmic level of the probabilities of deviating from the preponderant behavior asserted by the ergodic theorems.

In the ergodic theorems, the state space $S$ of the Markov chain is a measurable space, and no topology on $S$ is involved. This setup appears to be the natural one, at least initially, for the study of large deviations, and it is the setup we adopt. In the case of empirical measures, the space of probability measures on $S$ is endowed with the topology induced by a vector subspace of the space of bounded measurable functions, subject to certain restrictions.

Both the perspective and many results in the book have not previously appeared in the literature.

The prerequisites for the study of the book are: standard graduate-level measure and integration theory; basic graduate-level probability theory, including light exposure to large deviations; basic notions of general topology and functional analysis. Some familiarity with general Markov chains is desirable, but we have included several appendices which essentially cover the relevant aspects of the subject.

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