Quantum Hall Effect

The quantum Hall effect (QHE) is a fundamental phenomenon that occurs in a two-dimensional electron gas (2DEG) at low temperature and in the presence of a strong magnetic field. It has various applications in fields such as metrology and topological quantum computers. It also provides an extremely precise and independent determination of the fine-structure constant—a quantity of fundamental importance in quantum electrodynamics.

This book attempts to present concepts of QHE to undergraduate and graduate students, post-doctoral researchers, and teachers taking advanced courses on condensed matter physics. The author has integrated all important concepts of QHE such as graphene, the connection between topology and condensed matter physics, and prospects of next-generation storage devices based on the manipulation of spins (spintronic) and presented them in a lucid manner. It offers the advantage of providing a pedagogical presentation to help students with some intermediate steps in derivation.

The book starts with an introduction to the experimental discovery of the QHE that segues into the basics of 2DEG in a magnetic field. The physics of the Landau levels, their properties, and their relevance to the integer QHE are discussed. The importance of conduction and its connection to topological insulators is also emphasised. At a pedagogical level, concepts such as linear response theory, Kubo formula, and topological invariance are explained and their relations to the understanding of QHE, graphene, its symmetries, and its relevance as a quantum Hall insulator are also covered. It ends with an explanation of the role of interparticle interactions to explain fractional QHE with the help of topics such as the Laughlin wave function, fractional charge and statistics, and non-abelian anyons.

Saurabh Basu is Professor in the Department of Physics at IIT Guwahati. His research interests include different fields of theoretical condensed matter physics with a focus on topological materials, higher order topological insulators, ultracold physics, non-Hermitian systems, charge and thermal transport in mesoscale and nanoscale devices, Floquet dynamics, critical phenomena and fractals, and Skyrmions, among others. He has also developed online courses on condensed matter physics, quantum mechanics, superconductivity, and numerical methods which are hosted on SWAYAM, a popular online free e-learning platform initiated by the Government of India. He has also written a few popular science articles for the general audience, which have appeared in different Indian languages.

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The First Topological Insulator

Saurabh Basu

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To my family...

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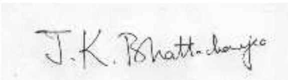
Foreword

Traditionally the different states of matter are described by symmetries that are broken. Typical situations include the freezing of a liquid, which breaks the translational symmetry that the fluid possessed, and the onset of magnetism, where the rotational symmetry is broken by the ordering of the individual magnetic moment vectors. In the early eighties of the previous century a completely new organizational principle of quantum matter was introduced following the discovery of the quantum Hall effect. The robustness of the quantum Hall state was a forerunner of the variety of topologically protected states that forms a large fraction of the condensed matter physics and material science literature at present.

Given the rapid strides that this field has made in the last two decades, it is almost imperative that it should become a part of the senior undergraduate curriculum. This necessitates the existence of a textbook that can address these somewhat esoteric topics at a level which is understandable to those who have not yet decided to specialize in this particular field but very well could, if given a proper exposition. This is a rather difficult task for the author of a textbook of a contemporary topic, and this is where the present book is immensely successful.

I am not a specialist in this subject by any means and found the book to be a comprehensive introduction to the area. I am sure the senior undergraduates and the beginning graduate students will benefit immensely from the book.



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Preface

It is somewhat implicit that the readers are familiar with the first course on *solid state physics*, which mainly deals with electronic systems and teaches us how to distinguish between different forms of matter, such as metals, semiconductors and insulators. An elementary treatise on band structure is introduced in this regard, and in most cases, interacting phenomena, such as magnetism and superconductivity, are taught. The readers are encouraged to look at the classic texts on solid state physics, such as the ones by Kittel, Ashcroft and Mermin.

As a second course, or an advanced course on the subject, more in-depth study of condensed matter physics and its applications to the physical properties of various materials have found a place in the undergraduate curricula for a century or even more. The perspective on teaching the subject has remained unchanged during this period of time. However, the recent developments over the last few decades require a new perspective on teaching and learning about the subject. Quantum Hall effect is one such discovery that has influenced the way condensed matter physics is taught to undergraduate students. The role of topology in condensed matter systems and the fashion in which it is interwoven with the physical observables need to be understood for deeper appreciation of the subject. Thus, to have a quintessential presentation for the undergraduate students, in this book, we have addressed selected topics on the quantum Hall effect, and its close cousin, namely topology, that should comprehensively contribute to the learning of the topics and concepts that have emerged in the not-so-distant past. In this book, we focus on the transport properties of two-dimensional (2D) electronic systems and solely on the role of a constant magnetic field perpendicular to the plane of a electron gas. This brings us to the topic of quantum Hall effect, which is one of the main verticals of the book. The origin of the Landau levels and the passage of the Hall current through edge modes are also discussed. The latter establishes a quantum Hall sample to be the first example of a topological insulator. Hence, our subsequent focus is on the subject topology and its application to quantum Hall systems and in general to condensed matter physics. Introducing the subject from a formal standpoint, we discuss the band structure and topological invariants in 1D. In particular, we talk about the Su–Schrieffer–Heeger and the Kitaev models in 1D, which, apart from being a possible realization for a polyacetylene molecule and a tight-binding chain with *p*-wave superconducting correlations, respectively, have emerged as a paradigmatic tool to study topology in 1D tight-binding systems. We have further discussed a quasi-1D system, namely the Creutz ladder. Owing to its existence intermediate to 1D and 2D, and the inability to put in the conventional classification of the topological insulators, the model, although quite interesting, received less attention. It is important to mention that these models may not be directly related with quantum Hall effect owing to time reversal symmetry being intact. Nevertheless, they are important to understand the topological ideas that are under the lens as well.

Having discussed quantum Hall effect in a 2D electron gas, it is of topical interest to discuss the corresponding scenario on a 2D crystal lattice. Or in a different sense, one may wish to distinguish between the behaviour of the relativistic and the non-relativistic electrons in two dimensions in the presence of a transverse magnetic field. In this context graphene is important, and it is amply elaborated in the text for its reason to be important. The formation of the Landau levels, which is central to the understanding of quantum Hall effect, is discussed, and, quite interestingly, owing to the large spacing between consecutive Landau levels in graphene, one should be able to observe the quantum Hall effect at room temperature. Being able to experimentally measure quantum effects in the classical regime is indeed a significant discovery.

We also deliberate upon the possibility pointed out by Haldane whether magnetic field is indispensable for realizing quantum Hall effect. A related topic that had ignited interest and debate is whether graphene can become a topological insulator. It turns out that being able to break the time reversal symmetry is more fundamental than the presence of an external field. This brings us to the topic of anomalous quantum Hall effect in graphene, which also implies that upon suitably tweaking the Hamiltonian, graphene can become a topological insulator. Addition of the spin of the electrons to the ongoing discussion emerged as a unique possibility to yield another version of the topological insulator, namely the quantum spin Hall insulator, which may lie at the heart of the next-generation spintronic devices.

Thereafter, a crisp introduction to the fractional quantum Hall effect is included. It comprises the discussion of Laughlin states, composite fermions and the hierarchy scenario, which will benefit the students in understanding the role of electronic interactions resulting in fractionally quantized Hall plateaus. We also briefly discuss the particle statistics in 2D, known as the braiding statistics, and touch upon how it aids in solving the riddle of even denominator fractions observed in experiments. A very brief introduction to the fractional quantum Hall effect in graphene has been included at the end.

All the while during the course of the book, we have included rigorous mathematical derivations wherever required, presented experimental details to connect with the ongoing discussions and tried to be as lucid as possible in our presentation of topics and concepts. A whole lot of schematic diagrams are presented for clarity as well. We hope that the students gain from the essence of this book, and it aids their understanding of both the topical and the traditional condensed matter physics. We shall be available and happy to answer queries, provide clarifications to students and researchers, and welcome comments for improvement.

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It is a proud privilege to acknowledge a lot of people who have actively or passively contributed in bringing up the book in the current form. First and foremost, we owe a lot to a number of current and past graduate students of the Department of Physics, IIT Guwahati, such as Ms Shilpi Roy, Mr Sayan Mandal, Mr Dipendu Halder, Mr Koustav Roy and Ms Srijata Lahiri, Ms. Shreya Debnath, Dr Sudin Ganguly, Dr Sk. Noor Nabi, Dr Priyadarshini Kapri and Dr Priyanka Sinha. I am particularly thankful to Shilpi, Sayan and Srijata for contributing in several ways during my preparation of the manuscript. It is also a privilege to acknowledge a number of people who have contributed directly or indirectly to the preparation of the book. They are Prof. Gaurav Dar (BITS Pilani, Goa campus), Prof. A Perumal (HoD, Physics, IIT Guwahati), Dr. Kuntal Bhattacharya (Post-doctoral fellow, IIT Guwahati), Prof. Dilip Pal (IIT Guwahati), Prof. Krishnendu Sengupta (IACS, Kolkata), Prof. Arindam Ghosh (IISc Bangalore), Prof. Tapan Mishra (NISER, Bhubaneswar), Prof. Pankaj Mishra (IIT Guwahati), Prof. Sumiran Pujari (IIT Bombay), Prof. Siddharth Lal (IISER Kolkata) and others. I also thank my daughters Shreya and Shreemoyee for being big critics of me writing this book.

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Quantum Hall Effect

1.1 Introduction

The date of discovery of the quantum Hall effect (QHE) is known pretty accurately. It occurred at 2:00 a.m. on 5 February 1980 at the high magnetic lab in Grenoble, France (see Fig. 1.1). There was an ongoing research on the transport properties of silicon field-effect transistors (FETs). The main motive was to improve the mobility of these FET devices. The devices that were provided by Dorda and Pepper allowed direct measurement of the resistivity tensor. The system is a highly degenerate two-dimensional (2D) electron gas contained in the inversion layer of a metal oxide semiconductor field effect transistor (MOSFET) operated at low temperatures and strong magnetic fields. The original notes appear in Fig. 1.1, where it is clearly stated that the Hall resistivity involves universal constants and hence signals towards the involvement of a very fundamental phenomenon.

In the classical version of the phenomenon discovered by E. Hall in 1879, just over a hundred years before the discovery of its quantum analogue, one may consider a sample with a planar geometry so as to restrict the carriers to move in a 2D plane. Next, turn on a bias voltage so that a current flows in one of the longitudinal directions and a strong magnetic field perpendicular to the plane of the gas (see Fig. 1.2). Because of the Lorentz force, the carriers drift towards a direction transverse to the direction of the current flowing in the sample. At equilibrium, a voltage develops in the transverse direction, which is known as the Hall voltage. The Hall resistivity, R, defined as the Hall voltage divided by the longitudinal current, is found to linearly depend on the magnetic field, B, and inversely on the carrier density, n, through $R = \frac{B}{nq}$ (q is the charge). A related