

## SIDNEY COLEMAN'S LECTURES ON RELATIVITY

Sidney Coleman (1937–2007) earned his doctorate at Caltech under Murray Gell-Mann. Before completing his thesis, he was hired by Harvard and remained there his entire career. A celebrated particle theorist, he is perhaps best known for his brilliant lectures, given at Harvard and in a series of summer school courses at Erice, Sicily, several of which are collected in *Aspects of Symmetry* (Cambridge, 1984). Three times in the 1960s he taught a graduate course on special and general relativity; this book is based on lecture notes taken by three of his students and compiled by the editors.

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## Preface

In the spring of 1966, the fall of 1967, and the fall of 1969, Sidney Coleman taught Physics 210 at Harvard. The course title was Relativity, and it was divided into two parts: Special Relativity and General Relativity. Einstein's special theory of relativity (1905) is a metatheory in the sense that it lays down restrictions (Lorentz invariance) that *any* physical theory must obey; it is not the story of any *particular* phenomenon, but rather a description of the spacetime arena in which *all* phenomena take place. If you were to propose some new physical theory, the first question would be, "Is it consistent with special relativity?" General relativity, which emerged in pieces from 1909 to 1917, is Einstein's theory of gravity. It is a generalization of the special theory only in the sense that whereas the special theory allows one to do physics in reference frames moving at uniform velocity, the general theory shows how to do physics in *arbitrary* coordinates, with the inclusion of gravity. But it's a misnomer, really: it should be called the theory of gravity.

This book is not verbatim Coleman—we have no such record—it is based on lecture notes taken by three graduate students enrolled in the class: David Griffiths and David Levin in 1966, and David Politzer in 1969, and we thank them for allowing us to use their work. (We also thank Diana Coleman for approving the project.) The two courses were substantially the same, but where they differed we have followed the treatment that seemed cleaner or more complete. In places we have supplied details, clarified explanations, or systematized the notation, but all the arguments are Coleman's. We have not introduced new material or brought the treatment up to date.<sup>1</sup>

In the first iteration of the course Coleman assigned very few problems, though he sometimes suggested examples, and he expected the students to be exploring

<sup>1</sup> As David Kaiser notes in "A  $\psi$  Is Just a  $\psi$ ? Pedagogy, Practice, and the Reconstitution of general relativity, 1942-1975" (*Stud. Hist. Phil. Phys.* **29**, 321 (1998)), Coleman's approach, with its emphasis on action principles, was already distinctly "modern."

questions of their own devising. Some of the problems in this book are taken from supplementary notes by the three Davids; others are officially assigned problems from the 1969 course.

Physics 210 was intended for graduate students, and the first part presupposes a fairly sophisticated understanding of special relativity; it focuses on foundational issues and advanced topics not often addressed in the classroom. By contrast, the second part is an *introduction* to general relativity ending with gravitational waves, the Schwarzschild metric, and relativistic cosmology. But throughout the book Coleman's uncanny nose for the subtle and the profound is very much in evidence.

Coleman recommended the following references:

- R. Adler, M. Bazin, and M. Schiffer, *Introduction to General Relativity*, McGraw-Hill (1965).
- V. Fock, *The Theory of Space, Time, and Gravitation*, Pergamon Press (1959). For general relativity.
- C. Møller, *The Theory of Relativity*, 2nd ed., Oxford U. P. (1972). Contains most of the material for this class—the nearest thing to a course textbook.
- W. Pauli, *Theory of Relativity*, Pergamon Press (1958) (reprinted by Dover Publications, 1981). A translation of Pauli's famous article written (at the age of 21) for the *Encyklopädie der mathematischen Wissenschaften*.
- E. Schrödinger, *Space-Time Structure*, Cambridge U. P. (1963). For general relativity.
- J. L. Synge, *Relativity: The Special Theory*, 2nd ed., North Holland (1965) and *Relativity: The General Theory*, North Holland (1960). Exhaustive—look here when all else fails.
- H. Weyl, *Space, Time, Matter*, Dover (1952).
- E. H. Wichmann, "Theory of General Relativity," lecture notes for Berkeley Physics 231 (winter 1965).

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