

SEISMIC WAVE PROPAGATION THROUGH RANDOM MEDIA

The solid Earth's medium is heterogeneous over a wide range of scales. Seismological observations, including envelope broadening with increasing distance from an earthquake source and the excitation of long-lasting coda waves, provide a means of investigating velocity inhomogeneities in the lithosphere. These phenomena have been studied primarily using radiative transfer theory with random medium modelling. This book presents the mathematical foundations of scalar- and vector-wave scattering in random media, using the Born or Eikonal approximation, which are useful for understanding random inhomogeneity spectra and the scattering characteristics of the solid Earth. A step-by-step Monte Carlo simulation procedure is presented for synthesizing the propagation of energy density for impulsive radiation from a source in random media. Simulation results are then verified by comparison with analytical solutions and finite-difference simulations. Presenting the latest seismological observations and analysis techniques, this is a useful reference for graduate students and researchers in geophysics and physics.

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Monte Carlo Simulation Based on the Radiative
Transfer Theory

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Preface

Recent seismological observations have revealed that the solid Earth medium is heterogeneous at various scales. Tomography based on the travel-time analysis has revealed that velocity inhomogeneities are distributed from regional to global scales. Scattering analyses of seismic waves have revealed random velocity inhomogeneities on small scales. Well loggings in deep boreholes have revealed random inhomogeneities in the shallow crust on a much smaller scale. Rock samples are heterogeneously composed of various grain minerals on a microscopic scale.

A small earthquake generates a seismic wavelet with a short duration, but the recorded seismograms show the broadening of the apparent duration with increasing travel distance. There exist long-lasting coda waves after the direct arrival. Three-component velocity seismograms are complex, reflecting the anisotropic patterns of source radiation and scattering due to medium heterogeneities. These phenomena have been studied primarily based on radiative transfer theory (RTT), which assumes the additivity of seismic energy, neglecting phase information. In practice, Monte Carlo (MC) simulations have often been used to stochastically synthesize the propagation of the seismic wavelet through random velocity fluctuations, which are statistically characterized by their power spectral density function (PSDF).

Our main objective is to introduce the mathematical basics of wave scattering in random media, especially for use in MC simulations in the framework of the RTT. To describe the scattering process, we use the Born or Eikonal approximation based on the relationship between the center wavenumber of the wavelet and the corner wavenumber of the PSDF. We synthesize the propagation of energy density for impulsive radiation from a source in random media. Then, we verify MC simulation results by comparison with analytic solutions and finite-difference simulation results. We also present an ongoing study of the hybrid use of Born and Eikonal approximations in MC simulations.

We intend to write this book for people interested in analyzing seismograms of earthquakes for the structural study of the solid Earth medium and in understanding the source radiation process; however, most of the mathematics used are general and can be applied to wave phenomena in other fields of physics.

We (HS and KE) are grateful to the late Keiiti Aki for introducing us to the fascinating world of seismic wave scattering in the heterogeneous Earth. We have learned a lot and been inspired through collaboration and discussion with Mike Fehler, Ru-Shan Wu, Mitsuyuki Hoshiba, Osamu Nishizawa, Kazushige Obara, Shigeo Kinoshita, Satoshi Matsumoto, Teruo Yamashita, Jun Kawahara, Kiyoshi Yomogida, Michael Korn, Frank Scherbaum, Jayant N. Tripathi, Tae-Woong Chung, Won-Sang Lee, Eduard Carcolé, Michel Campillo, Ludovic Margerin, and Roel Snieder. Special thanks to the staff and former graduate students of the Solid Earth Physics Laboratory at Tohoku University for collaborating with us: Masakazu Ohtake, Takeshi Nishimura, Kazuo Yoshimoto, Hisashi Nakahara, Mare Yamamoto, Katsuhiko Shiomi, Tatsuhiko Saito, Yo Fukushima, Takuto Maeda, Tsutomu Takahashi, Makiko Nishino, Mungiya Kubanza, Toshihiko Hayakawa, Kaoru Sawazaki, and Rintaro Fukushima. We would like to thank Takeshi Furumura and Shunsuke Takemura for their support in numerical simulations.

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Haruo Sato Kentaro Emoto
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